

Practice Test

Consider the following 8 dimensional representation of an algebra G

$$X^a = \{\sigma_i, \sigma_i\eta_1, \sigma_i\eta_2, \sigma_i\eta_3\tau_1, \sigma_i\eta_3\tau_2, \eta_3, \eta_1\tau_1, \eta_2\tau_1, \eta_1\tau_2, \eta_2\tau_2, \tau_3\}.$$

The matrices σ, τ, η are Pauli matrices acting in three different spaces. We are using a direct product notation, so the total space has dimension $2 \times 2 \times 2 = 8$. There are 21 generators ($i = 1, 2, 3$). You can check that the X^a form a closed algebra.

1. The rank of the Lie algebra is 3. You can choose the Cartan generators to be $H_1 = \sigma_3, H_2 = \eta_3, H_3 = \tau_3$. Find the weights of the 8 dimensional representation given above.
2. Find all the roots.
3. Find the simple roots, and order them according to Georgi's convention.
4. Compute the Cartan Matrix

$$A_{ij} = \frac{2\alpha_i \cdot \alpha_j}{|\alpha_j|^2}.$$

5. Draw the Dynkin diagram. What Lie Algebra is G ?
6. Find the fundamental weights defined by

$$\frac{2\mu_i \cdot \alpha_j}{|\alpha_j|^2} = \delta_{ij}.$$

7. Construct the weights of the $(1, 0, 0)$ representation. What is the dimension of this representation?