Table 2
Model builders' view of the elementary particle spectrum

Spin 2: One graviton, considered in supergravity, but usually ignored in models that unify just color and flavor.

Spin 3/2: An intriguing "hole" in the spectrum; ignored in unified models, but supergravity Lagrangians suggest it should be filled.

Spin 1: Vector bosons mediating Nature's interactions, including the photon of QED, the charged and neutral weak bosons, and the eight gluons of the strong interactions. Unified models suggest additional vector bosons; for example, in some models there are bosons that mediate proton decay.

Spin 1/2: Quarks and leptons (only the left-handed states are listed)

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t(?) \\ b' \end{pmatrix}_L \qquad \text{Weak doublets}$$

$$\bar{u}_L \quad \bar{d}_L \quad \bar{c}_L \quad \bar{s}_L \quad \bar{b}_L \quad \bar{t}_L(?) \qquad \text{Weak singlets}$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \qquad \text{doublets}$$

$$e_L^+ \quad \mu_L^+ \quad \tau_L^+ \qquad \text{singlets}$$

Are there additional quarks and leptons, or other fermions with higher colors?

Spin 0: None are known for certain. The weak breaking follows a $|\Delta \mathbf{I}^w| = \frac{1}{2}$ rule and the superstrong breaking, $|\Delta \mathbf{I}^w| = 0$, but it is not known whether either of these are associated with explicit scalar particles. One possibility is that the superstrong breaking is due to explicit scalars in the Lagrangian, but the weak breaking is due to composites. The origin of the symmetry breaking is a major puzzle in today's particle theory.

Table 3
Embeddings of SU_3^c in Simple Groups G, subject to the constraint that at least one irrep of G has no more than $\mathbf{1}^c$, $\mathbf{3}^c$ and $\bar{\mathbf{3}}^c$. G^{fl} (fl for flavor) is the largest subgroup defined by $G \supset G^{fl} \times SU_3^c$. The irreps of G satisfying the restriction to the set $\mathbf{1}^c$, $\mathbf{3}^c$, $\bar{\mathbf{3}}^c$, are listed, along with their dimensionality. See ref. [6]

Case	G	G^{fl}	f	Dimensionality
1.	SU_n	$SU_{n_1} \times SU_{n_3} \times U_1$	$\mathbf{n}$	$n = n_1 + 3n_3$
2.	SU_n	$SU_{n-3} \times U_1$	$(\mathbf{n}^k)_a$	$\binom{n}{k}$
3.	SU_n	$SU_{n_1} \times SU_{n_3} \times SU_{n_{\bar{3}}} \times U_1 \times U_1$	$\mathbf{n}$	$n = n_1 + 3n_3 + 3n_{\bar{3}}$
4.	SO_n	$SO_{n_1} \times SU_{n_3} \times U_1$	$\mathbf{n}$	$n = n_1 + 6n_3$
5.	SO_n	$SO_{n-6} \times U_1$	$\mathbf{n}$	$n = n_1 + 6$
			σ, σ' or $\bar{\sigma}$	$2^{[(n-1)/2]}$
6.	Sp_{2n}	$Sp_{2n_1} \times SU_{n_3} \times U_1$	$\mathbf{2n}$	$2n = 2n_1 + 6n_3$
7.	F_4	SU_3	$\mathbf{26}$	26
8.	E_6	$SU_3 \times SU_3$	$\mathbf{27}$	27
9.	E_7	SU_6	$\mathbf{56}$	56

Table 4

Roots of SU_3. The α_i are the conventional choice of simple roots; (ij) are the Dynkin labels of the roots; and the axes associated with the diagonal generators Y, Q and I_3 are indicated and normalized

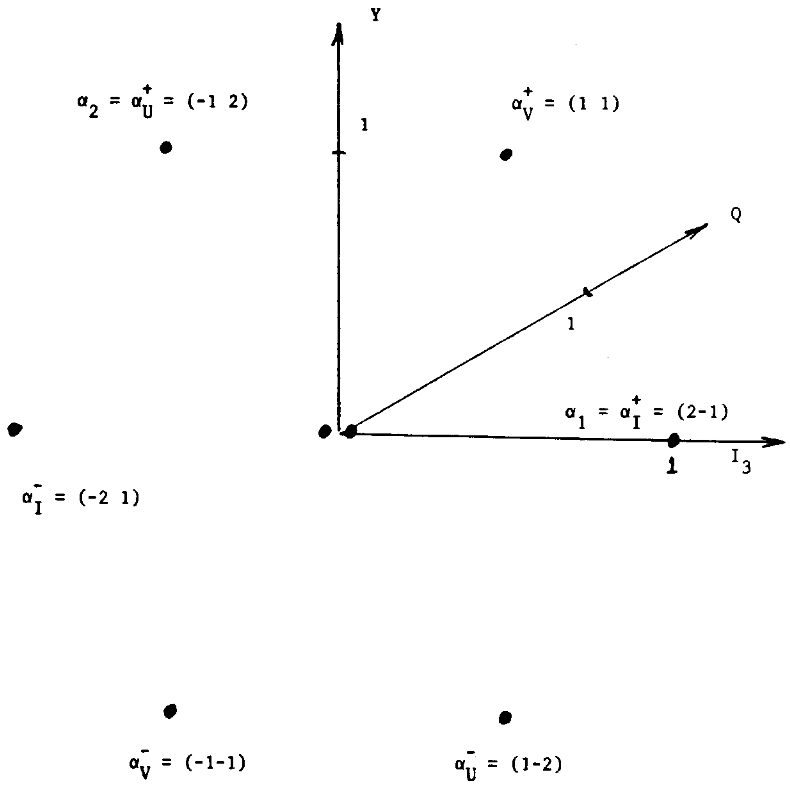


Table 5

Dynkin diagrams for simple Lie algebras. (Black dots represent shorter roots.)

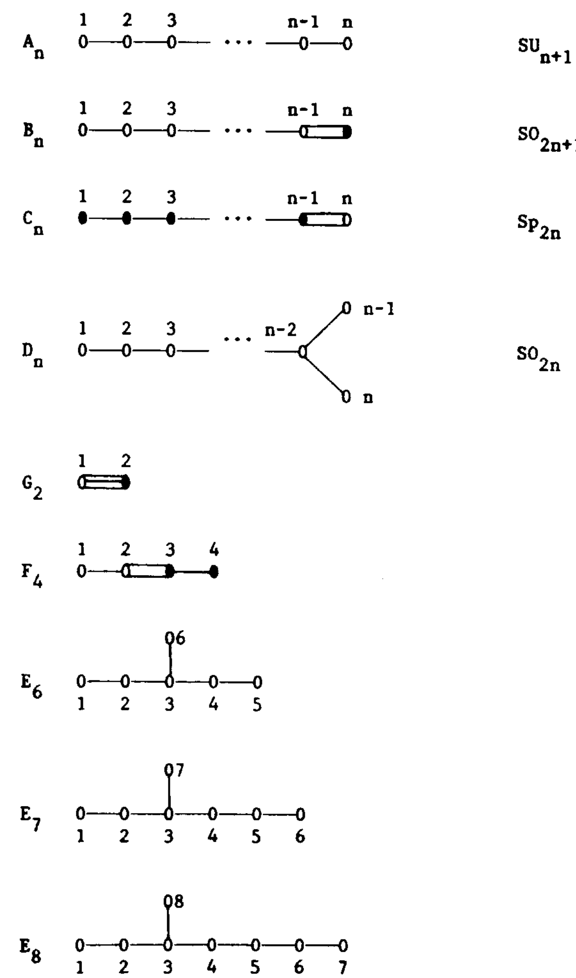


Table 6
Cartan matrices of simple Lie algebras

$$A(\mathrm{A}_n)=\begin{pmatrix} 2 & -1 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ -1 & 2 & -1 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & -1 & 2 & -1 & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & -1 & 2 & -1 \\ 0 & 0 & 0 & \cdot & \cdot & 0 & -1 & 2 \end{pmatrix} \qquad A(\mathrm{G}_2)=\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

$$A(\mathrm{F}_4)=\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -2 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$A(\mathrm{B}_n)=\begin{pmatrix} 2 & -1 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ -1 & 2 & -1 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & -1 & 2 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & 2 & -2 \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & -1 & 2 \end{pmatrix} \qquad A(\mathrm{E}_6)=\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}$$

$A(\mathrm{C}_n)$ is the transpose of $A(\mathrm{B}_n)$, since the short and long roots are interchanged.

$$A(\mathrm{D}_n)=\begin{pmatrix} 2 & -1 & 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ -1 & 2 & -1 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ 0 & -1 & 2 & \cdot & \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & & & & 2 & -1 & -1 \\ 0 & 0 & 0 & & & & -1 & 2 & 0 \\ 0 & 0 & 0 & & & & -1 & 0 & 2 \end{pmatrix} \qquad A(\mathrm{E}_7)=\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$$A(\mathrm{E}_8)=\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Table 7
Metric tensors G for weight space

$$G(\mathrm{A}_n)=\frac{1}{n+1}\begin{pmatrix} 1\cdot n & 1\cdot(n-1) & 1\cdot(n-2) & \cdots & 1\cdot 2 & 1\cdot 1\\ 1\cdot(n-1) & 2\cdot(n-1) & 2\cdot(n-2) & \cdots & 2\cdot 2 & 2\cdot 1\\ 1\cdot(n-2) & 2\cdot(n-2) & 3\cdot(n-2) & \cdots & 3\cdot 2 & 3\cdot 1\\ \hdotsfor{6}\\ 1\cdot 2 & 2\cdot 2 & 3\cdot 2 & \cdots & (n-1)\cdot 2 & (n-1)\cdot 1\\ 1\cdot 1 & 2\cdot 1 & 3\cdot 1 & \cdots & (n-1)\cdot 1 & n\cdot 1 \end{pmatrix}$$

$$G(\mathrm{B}_n)=\frac{1}{2}\begin{pmatrix} 2 & 2 & 2 & \cdots & 2 & 1\\ 2 & 4 & 4 & \cdots & 4 & 2\\ 2 & 4 & 6 & \cdots & 6 & 3\\ \hdotsfor{6}\\ 2 & 4 & 6 & \cdots & 2(n-1) & n-1\\ 1 & 2 & 3 & \cdots & n-1 & n/2 \end{pmatrix}$$

$$G(\mathrm{C}_n)=\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1\\ 1 & 2 & 2 & \cdots & 2 & 2\\ 1 & 2 & 3 & \cdots & 3 & 3\\ \hdotsfor{6}\\ 1 & 2 & 3 & \cdots & n-1 & n-1\\ 1 & 2 & 3 & \cdots & n-1 & n \end{pmatrix}$$

$$G(\mathrm{D}_n)=\frac{1}{2}\begin{pmatrix} 2 & 2 & 2 & \cdots & 2 & 1 & 1\\ 2 & 4 & 4 & \cdots & 4 & 2 & 2\\ 2 & 4 & 6 & \cdots & 6 & 3 & 3\\ \hdotsfor{7}\\ 2 & 4 & 6 & \cdots & 2(n-2) & n-2 & n-2\\ 1 & 2 & 3 & \cdots & n-2 & n/2 & (n-2)/2\\ 1 & 2 & 3 & \cdots & n-2 & (n-2)/2 & n/2 \end{pmatrix}$$

$$G(\mathrm{E}_6)=\frac{1}{3}\begin{pmatrix} 4 & 5 & 6 & 4 & 2 & 3\\ 5 & 10 & 12 & 8 & 4 & 6\\ 6 & 12 & 18 & 12 & 6 & 9\\ 4 & 8 & 12 & 10 & 5 & 6\\ 2 & 4 & 6 & 5 & 4 & 3\\ 3 & 6 & 9 & 6 & 3 & 6 \end{pmatrix}$$

$$G(\mathrm{E}_7)=\frac{1}{2}\begin{pmatrix} 4 & 6 & 8 & 6 & 4 & 2 & 4\\ 6 & 12 & 16 & 12 & 8 & 4 & 8\\ 8 & 16 & 24 & 18 & 12 & 6 & 12\\ 6 & 12 & 18 & 15 & 10 & 5 & 9\\ 4 & 8 & 12 & 10 & 8 & 4 & 6\\ 2 & 4 & 6 & 5 & 4 & 3 & 3\\ 4 & 8 & 12 & 9 & 6 & 3 & 7 \end{pmatrix}$$

$$G(\mathrm{E}_8)=\begin{pmatrix} 4 & 7 & 10 & 8 & 6 & 4 & 2 & 5\\ 7 & 14 & 20 & 16 & 12 & 8 & 4 & 10\\ 10 & 20 & 30 & 24 & 18 & 12 & 6 & 15\\ 8 & 16 & 24 & 20 & 15 & 10 & 5 & 12\\ 6 & 12 & 18 & 15 & 12 & 8 & 4 & 9\\ 4 & 8 & 12 & 10 & 8 & 6 & 3 & 6\\ 2 & 4 & 6 & 5 & 4 & 3 & 2 & 3\\ 5 & 10 & 15 & 12 & 9 & 6 & 3 & 8 \end{pmatrix}$$

$$G(\mathrm{G}_2)=\frac{1}{3}\begin{pmatrix} 6 & 3\\ 3 & 2 \end{pmatrix}$$

$$G(\mathrm{F}_4)=\begin{pmatrix} 2 & 3 & 2 & 1\\ 3 & 6 & 4 & 2\\ 2 & 4 & 3 & 3/2\\ 1 & 2 & 3/2 & 1 \end{pmatrix}$$

Table 8
Root diagrams in the Dynkin basis. "Level of simple roots" is the number of simple roots that must be subtracted from the highest root in order to obtain the simple roots; the next level has the n zero roots corresponding to the Cartan subalgebra

Algebra	Highest root	Level of simple roots	Dimension
A_n	(1 0 0 ... 0 0 1)	$n-1$	$n(n+2)$
B_n	(0 1 0 ... 0 0 0)	$2n-2$	$n(2n+1)$
C_n	(2 0 0 ... 0 0 0)	$2n-2$	$n(2n+1)$
D_n	(0 1 0 ... 0 0 0)	$2n-4$	$n(2n-1)$
G_2	(1 0)	4	14
F_4	(1 0 0 0)	10	52
E_6	(0 0 0 0 0 1)	10	78
E_7	(1 0 0 0 0 0 0)	16	133
E_8	(0 0 0 0 0 0 1 0)	28	248

Table 9
Positive roots in the Dynkin basis of rank 2 and 3 simple algebras, of SU_5 (rank 4) and of SO_{10} (rank 5)

SU_3
(1 1)
(2 −1)(−1 2)

Sp_4
(2 0)
(0 1)
(2 −1)(−2 2)

G_2
(1 0)
(−1 3)
(0 1)
(1 −1)
(2 −3)(−1 2)

SU_4
(1 0 1)
(1 1 −1)(−1 1 1)
(2 −1 0)(−1 2 −1)(0 −1 2)

SO_7
(0 1 0)
(1 −1 2)
(1 0 0)(−1 0 2)
(1 1 −2)(−1 1 0)
(2 −1 0)(−1 2 −2)(0 −1 2)

Sp_6
(2 0 0)
(0 1 0)
(1 −1 1)(−2 2 0)
(1 1 −1)(−1 0 1)
(2 −1 0)(−1 2 −1)(0 −2 2)

SU_5
(1 0 0 1)
(1 0 1 −1)(−1 1 0 1)
(1 1 −1 0)(−1 1 1 −1)(0 −1 1 1)
(2 −1 0 0)(−1 2 −1 0)(0 −1 2 −1)(0 0 −1 2)

SO_{10}
(0 1 0 0 0)
(1 −1 1 0 0)
(−1 0 1 0 0)(1 0 −1 1 1)
(−1 1 −1 1 1)(1 0 0 −1 1)(1 0 0 1 −1)
(0 −1 0 1 1)(−1 1 0 −1 1)(−1 1 0 1 −1)(1 0 1 −1 −1)
(0 −1 1 −1 1)(0 −1 1 1 −1)(−1 1 1 −1 −1)(1 1 −1 0 0)
(0 0 −1 0 2)(0 0 −1 2 0)(0 −1 2 −1 −1)(−1 2 −1 0 0)(2 −1 0 0 0)

Table 10
Level vectors of simple groups. The ordering follows the conventions of table 5

SU_{n+1}	$\bar{R} = [n, 2(n-1), 3(n-2), \ldots, (n-1)2, n]$
SU_5	$\bar{R} = [4, 6, 6, 4]$
SU_6	$\bar{R} = [5, 8, 9, 8, 5]$
SO_{2n+1}	$\bar{R} = [2n, 2(2n-1), 3(2n-2), 4(2n-3), \ldots, (n-1)(n+2), n(n+1)/2]$
SO_9	$\bar{R} = [8, 14, 18, 10]$
Sp_{2n}	$\bar{R} = [(2n-1), 2(2n-2), 3(2n-3), \ldots, (n-1)(n+1), n^2]$
SO_{2n}	$\bar{R} = [(2n-2), 2(2n-3), 3(2n-4), \ldots, (n-2)(n+1), n(n-1)/2, n(n-1)/2]$
SO_8	$\bar{R} = [6, 10, 6, 6]$
SO_{10}	$\bar{R} = [8, 14, 18, 10, 10]$
G_2	$\bar{R} = [10, 6]$
F_4	$\bar{R} = [22, 42, 30, 16]$
E_6	$\bar{R} = [16, 30, 42, 30, 16, 22]$
E_7	$\bar{R} = [34, 66, 96, 75, 52, 27, 49]$
E_8	$\bar{R} = [92, 182, 270, 220, 168, 114, 58, 136]$

Table 11a
Weight diagram for the **16** of SO_{10}

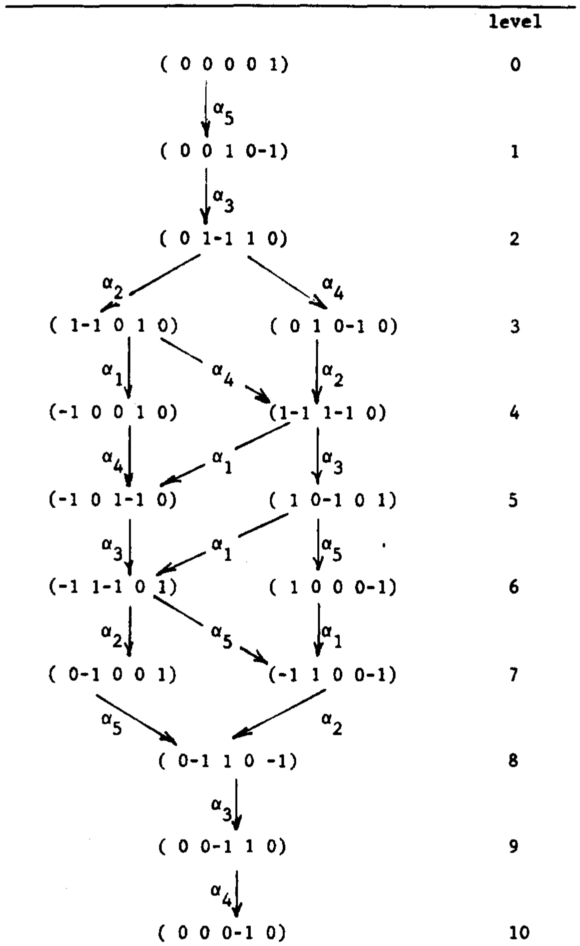


Table 11b
Weight diagram for the **27** of E_6

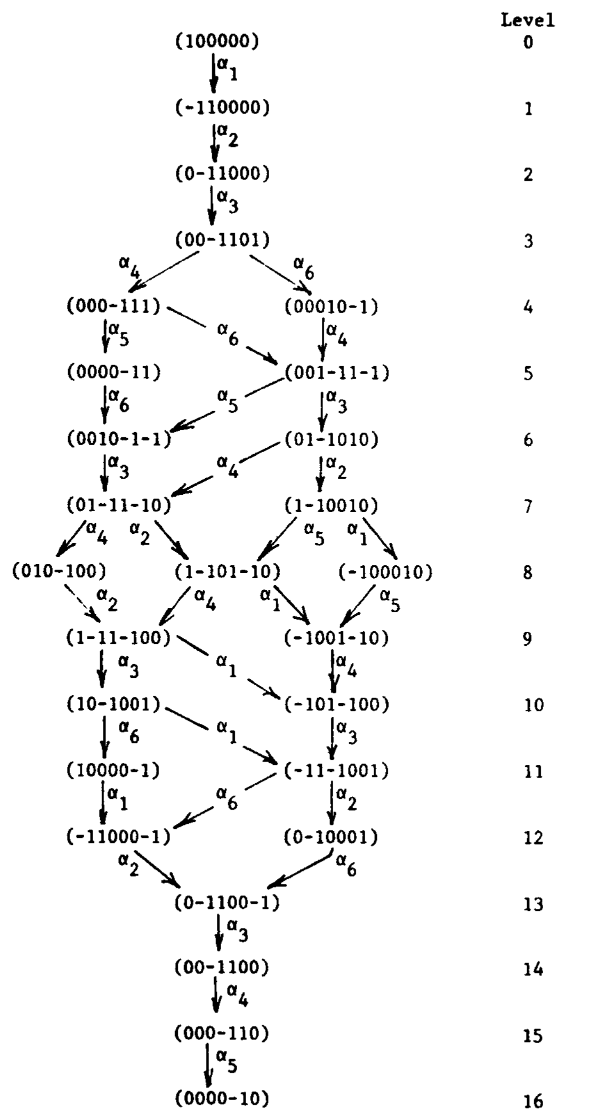


Table 12
Self-conjugate representations of simple groups

Group	Restriction	Real/Pseudoreal (PR)
SU_{l+1} $l = 2, 3, 4; 6, 7, 8; \ldots$	$(a_1, \ldots, a_l) =$ $(a_l, \ldots, a_1)$	Real only
SU_{l+1} $l = 1, 5, 9, 13, \ldots$	$(a_1, \ldots, a_l) =$ $(a_l, \ldots, a_1)$	Real if $a_{(l+1)/2}$ even PR if $a_{(l+1)/2}$ odd
SO_{2l+1} $l = 3, 4; 7, 8; 11, 12; \ldots$	none	Real only
SO_{2l+1} $l = 1, 2; 5, 6; 9, 10; \ldots$	none (α_l is the short root)	Real if a_l even PR if a_l odd
Sp_{2l}	none (α_l is the long root)	Real if $\sum_{i\ \mathrm{odd}} a_i$ even PR if $\sum_{i\ \mathrm{odd}} a_i$ odd
SO_{2l} l odd	$a_{l-1} = a_l$	Real only
SO_{2l} l even, $\frac{1}{2}l$ even	none	Real only
SO_{2l} l even, $\frac{1}{2}l$ odd	none (α_{l-1} and α_l are the spinor roots)	Real if $a_{l-1} + a_l$ even PR if $a_{l-1} + a_l$ odd
G_2	none	Real only
F_4	none	Real only
E_6	$(a_1, a_2, \ldots, a_5, a_6) =$ $(a_5, a_4, \ldots, a_1, a_6)$	Real only
E_7	none [Note, **56** = (0000010)]	Real if $a_4 + a_6 + a_7$ even PR if $a_4 + a_6 + a_7$ odd
E_8	none	Real only

Table 13
Simple irreps of simple Lie algebras

Algebra	Dynkin designation	Dimensionality
A_n	(10...0)	$n+1$
	or (0...01)*	$\overline{n+1}$
B_n	(10...0)*	$2n+1$
	(000...01)	2^n
C_n	(10...0)	$2n$
D_n	(10...0)*	$2n$
	(00...01)	2^{n-1}
	or (00...010)*	2^{n-1}
G_2	(01)	7
F_4	(0001)	26
E_6	(100000)	27
	or (000010)*	$\overline{27}$
E_7	(0000010)	56
E_8	(00000010)	248

* This irrep can be constructed from products of the unstarred irrep.

Table 14
Maximal subalgebras of classical simple Lie algebras with rank 8 or less

Rank 1	
$SU_2 \supset U_1$	(R)
(SU_2, SO_3, Sp_2, all isomorphic)	
Rank 2	
$SU_3 \supset SU_2 \times U_1$	(R)
$\supset SU_2$	(S)
$Sp_4 \supset SU_2 \times SU_2$; $SU_2 \times U_1$	(R)
$\supset SU_2$	(S)
(SO_5 isomorphic to Sp_4, $SO_4 \sim SU_2 \times SU_2$)	
Rank 3	
$SU_4 \supset SU_3 \times U_1$; $SU_2 \times SU_2 \times U_1$	(R)
$\supset Sp_4$; $SU_2 \times SU_2$	(S)
$SO_7 \supset SU_4$; $SU_2 \times SU_2 \times SU_2$; $Sp_4 \times U_1$	(R)
$\supset G_2$	(S)
$Sp_6 \supset SU_3 \times U_1$; $SU_2 \times Sp_4$	(R)
$\supset SU_2$; $SU_2 \times SU_2$	(S)
(SO_6 is isomorphic to SU_4)	
Rank 4	
$SU_5 \supset SU_4 \times U_1$; $SU_2 \times SU_3 \times U_1$	(R)
$\supset Sp_4$	(S)
$SO_9 \supset SO_8$; $SU_2 \times SU_2 \times Sp_4$; $SU_2 \times SU_4$; $SO_7 \times U_1$	(R)
$\supset SU_2$; $SU_2 \times SU_2$	(S)
$Sp_8 \supset SU_4 \times U_1$; $SU_2 \times Sp_6$; $Sp_4 \times Sp_4$	(R)
$\supset SU_2$; $SU_2 \times SU_2 \times SU_2$	(S)
$SO_8 \supset SU_2 \times SU_2 \times SU_2 \times SU_2$; $SU_4 \times U_1$	(R)
$\supset SU_3$; SO_7; $SU_2 \times Sp_4$	(S)

Table 14 (continued)

Rank 5		
SU_6	$\supset SU_5\times U_1$; $SU_2\times SU_4\times U_1$; $SU_3\times SU_3\times U_1$	(R)
	$\supset SU_3$; SU_4; Sp_6; $SU_2\times SU_3$	(S)
SO_{11}	$\supset SO_{10}$; $SU_2\times SO_8$; $Sp_4\times SU_4$; $SU_2\times SU_2\times SO_7$; $SO_9\times U_1$	(R)
	$\supset SU_2$	(S)
Sp_{10}	$\supset SU_5\times U_1$; $SU_2\times Sp_8$; $Sp_4\times Sp_6$	(R)
	$\supset SU_2$; $SU_2\times Sp_4$	(S)
SO_{10}	$\supset SU_5\times U_1$; $SU_2\times SU_2\times SU_4$; $SO_8\times U_1$	(R)
	$\supset Sp_4$; SO_9; $SU_2\times SO_7$; $Sp_4\times Sp_4$	(S)
Rank 6		
SU_7	$\supset SU_6\times U_1$; $SU_2\times SU_5\times U_1$; $SU_3\times SU_4\times U_1$	(R)
	$\supset SO_7$	(S)
SO_{13}	$\supset SO_{12}$; $SU_2\times SO_{10}$; $Sp_4\times SO_8$; $SU_4\times SO_7$; $SU_2\times SU_2\times SO_9$; $SO_{11}\times U_1$	(R)
	$\supset SU_2$	(S)
Sp_{12}	$\supset SU_6\times U_1$; $SU_2\times Sp_{10}$; $Sp_4\times Sp_8$; $Sp_6\times Sp_6$	(R)
	$\supset SU_2$; $SU_2\times SU_4$; $SU_2\times Sp_4$	(S)
SO_{12}	$\supset SU_6\times U_1$; $SU_2\times SU_2\times SO_8$; $SU_4\times SU_4$; $SO_{10}\times U_1$	(R)
	$\supset SU_2\times Sp_6$; $SU_2\times SU_2\times SU_2$; SO_{11}; $SU_2\times SO_9$; $Sp_4\times SO_7$	(S)
Rank 7		
SU_8	$\supset SU_7\times U_1$; $SU_2\times SU_6\times U_1$; $SU_3\times SU_5\times U_1$; $SU_4\times SU_4\times U_1$	(R)
	$\supset SO_8$; Sp_8; $SU_2\times SU_4$	(S)
SO_{15}	$\supset SO_{14}$; $SU_2\times SO_{12}$; $Sp_4\times SO_{10}$; $SO_7\times SO_8$; $SU_4\times SO_9$; $SU_2\times SU_2\times SO_{11}$; $SO_{13}\times U_1$	(R)
	$\supset SU_2$; SU_4; $SU_2\times Sp_4$	(S)
Sp_{14}	$\supset SU_7\times U_1$; $SU_2\times Sp_{12}$; $Sp_4\times Sp_{10}$; $Sp_6\times Sp_8$	(R)
	$\supset SU_2$; $SU_2\times SO_7$	(S)
SO_{14}	$\supset SU_7\times U_1$; $SU_2\times SU_2\times SO_{10}$; $SU_4\times SO_8$; $SO_{12}\times U_1$	(R)
	$\supset Sp_4$; Sp_6; G_2; SO_{13}; $SU_2\times SO_{11}$; $Sp_4\times SO_9$; $SO_7\times SO_7$	(S)
Rank 8		
SU_9	$\supset SU_8\times U_1$; $SU_2\times SU_7\times U_1$; $SU_3\times SU_6\times U_1$; $SU_4\times SU_5\times U_1$	(R)
	$\supset SO_9$; $SU_3\times SU_3$	(S)
SO_{17}	$\supset SO_{16}$; $SU_2\times SO_{14}$; $Sp_4\times SO_{12}$; $SO_7\times SO_{10}$; $SO_8\times SO_9$; $SU_4\times SO_{11}$; $SU_2\times SU_2\times SO_{13}$; $SO_{15}\times U_1$	(R)
	$\supset SU_2$	(S)
Sp_{16}	$\supset SU_8\times U_1$; $SU_2\times Sp_{14}$; $Sp_4\times Sp_{12}$; $Sp_6\times Sp_{10}$; $Sp_8\times Sp_8$	(R)
	$\supset SU_2$; Sp_4; $SU_2\times SO_8$	(S)
SO_{16}	$\supset SU_8\times U_1$; $SU_2\times SU_2\times SO_{12}$; $SU_4\times SO_{10}$; $SO_8\times SO_8$; $SO_{14}\times U_1$	(R)
	$\supset SO_9$; $SU_2\times Sp_8$; $Sp_4\times Sp_4$; SO_{15}; $SU_2\times SO_{13}$; $Sp_4\times SO_{11}$; $SO_7\times SO_9$	(S)

Table 15
Maximal subalgebras of exceptional algebras; branching rules for the fundamental representation

$G_2 \supset SU_3$	$7 = 1 + 3 + \bar{3}$	(R)
$\supset SU_2 \times SU_2$	$7 = (2, 2) + (1, 3)$	(R)
$\supset SU_2$	$7 = 7$	(S)
$F_4 \supset SO_9$	$26 = 1 + 9 + 16$	(R)
$\supset SU_3 \times SU_3$	$26 = (8, 1) + (3, 3) + (\bar{3}, \bar{3})$	(R)
$\supset SU_2 \times Sp_6$	$26 = (2, 6) + (1, 14)$	(R)
$\supset SU_2$	$26 = 9 + 17$	(S)
$\supset SU_2 \times G_2$	$26 = (5, 1) + (3, 7)$	(S)
$E_6 \supset SO_{10} \times U_1$	$27 = 1 + 10 + 16$	(R)
$\supset SU_2 \times SU_6$	$27 = (2, \bar{6}) + (1, 15)$	(R)
$\supset SU_3 \times SU_3 \times SU_3$	$27 = (\bar{3}, 3, 1^c) + (3, 1, 3) + (1, \bar{3}, \bar{3})$	(R)
$\supset SU_3$	$27 = 27$	(S)
$\supset G_2$	$27 = 27$	(S)
$\supset Sp_8$	$27 = 27$	(S)
$\supset F_4$	$27 = 1 + 26$	(S)
$\supset SU_3 \times G_2$	$27 = (\bar{6}, 1) + (3, 7)$	(S)
$E_7 \supset E_6 \times U_1$	$56 = 1 + 1 + 27 + \overline{27}$	(R)
$\supset SU_8$	$56 = 28 + \overline{28}$	(R)
$\supset SU_2 \times SO_{12}$	$56 = (2, 12) + (1, 32)$	(R)
$\supset SU_3 \times SU_6$	$56 = (3, 6) + (\bar{3}, \bar{6}) + (1, 20)$	(R)
$\supset SU_2$	$56 = 10 + 18 + 28$	(S)
$\supset SU_2$	$56 = 6 + 12 + 16 + 22$	(S)
$\supset SU_3$	$56 = 28 + \overline{28}$	(S)
$\supset SU_2 \times SU_2$	$56 = (5, 2) + (3, 6) + (7, 4)$	(S)
$\supset SU_2 \times G_2$	$56 = (4, 7) + (2, 14)$	(S)
$\supset SU_2 \times F_4$	$56 = (4, 1) + (2, 26)$	(S)
$\supset G_2 \times Sp_6$	$56 = (1, 14') + (7, 6)$	(S)
$E_8 \supset SO_{16}$	$248 = 120 + 128$	(R)
$\supset SU_5 \times SU_5$	$248 = (24, 1) + (1, 24) + (10, 5) + (\overline{10}, \bar{5}) + (5, \overline{10}) + (\bar{5}, 10)$	(R)
$\supset SU_3 \times E_6$	$248 = (8, 1) + (1, 78) + (3, 27) + (\bar{3}, \overline{27})$	(R)
$\supset SU_2 \times E_7$	$248 = (3, 1) + (1, 133) + (2, 56)$	(R)
$\supset SU_9$	$248 = 80 + 84 + \overline{84}$	(R)
$\supset SU_2$	$248 = 3 + 15 + 23 + 27 + 35 + 39 + 47 + 59$	(S)
$\supset SU_2$	$248 = 3 + 11 + 15 + 19 + 23 + 27 + 29 + 35 + 39 + 47$	(S)
$\supset SU_2$	$248 = 3 + 7 + 11 + 15 + 17 + 19 + 23 + 23 + 27 + 29 + 35 + 39$	(S)
$\supset G_2 \times F_4$	$248 = (14, 1) + (1, 52) + (7, 26)$	(S)
$\supset SU_2 \times SU_3$	$248 = (3, 1) + (1, 8) + (7, 8) + (5, 10) + (5, \overline{10}) + (3, 27)$	(S)
$\supset Sp_4$	$248 = 10 + 84 + 154$	(S)

Table 16
Extended Dynkin diagrams for simple Lie algebras. (The extended root is marked by x; black dots represent shorter roots.)

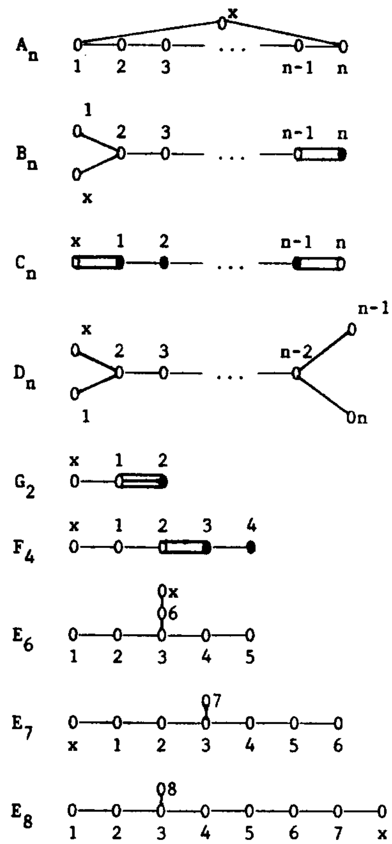


Table 17
Symmetric subgroups of simple groups

G	G_S	rank(A)	Action of C on irrep R
SU_n	SO_n	$n-1$	$\bar{R}$
SU_{p+q}	$SU_p \times SU_q \times U_1$	min(p, q)[a]	R
SU_{2n}	Sp_{2n}	$n-1$	$\bar{R}$
SO_{p+q}	$SO_p \times SO_q$	min(p, q)[a]	R (p or q even) $\bar{R}$ or R′ (p and q odd)[b]
SO_{2n}	$SU_n \times U_1$	$[n/2]$	R
Sp_{2n}	$SU_n \times U_1$	n	R
Sp_{2p+2q}	$Sp_{2p} \times Sp_{2q}$	min(p, q)[a]	R
G_2	$SU_2 \times SU_2$	2	R
F_4	$SU_2 \times Sp_6$	4	R
	SO_9	1	R
E_6	Sp_8	6	$\bar{R}$
	$SU_2 \times SU_6$	4	R
	$SO_{10} \times U_1$	2	R
	F_4	2	$\bar{R}$
E_7	SU_8	7	R
	$SU_2 \times SO_{12}$	4	R
	$E_6 \times U_1$	3	R
E_8	SO_{16}	8	R
	$SU_2 \times E_7$	4	R

[a] The case p or q equal unity defines a symmetric subgroup with SU_1 or SO_1 empty; the Lie algebra of Sp_2 is isomorphic to that of SU_2.

[b] If $p+q=4n+2$, C reflects complex spinor irreps into their conjugates; if $p+q=4n$, C reflects the real or pseudoreal spinor irreps into the nonequivalent spinor of the same dimension.

Table 18
E_6 and its subgroups with $U_1^{em} \times SU_3^c$

Group	No. max. subgroups	Satisfactory maximal subgroups	Unsatisfactory maximal subgroups
E_6	8	F_4, $SO_{10} \times U_1$, $SU_2 \times SU_6$, $SU_3 \times SU_3 \times SU_3$	G_2, SU_3, $SU_3 \times G_2$, $[Sp_8]$
F_4	5	SO_9, $SU_3 \times SU_3$	SU_2, $SU_2 \times G_2$, $[SU_2 \times Sp_6]$
SO_9	5	SO_8, $SU_2 \times SU_4$	SU_2, $SU_2 \times SU_2$, $SU_2 \times SU_2 \times Sp_4$, $SO_7 \times U_1$*
SO_8	4	SO_7, $SU_4 \times U_1$	SU_3, $SU_2 \times Sp_4$, $SU_2 \times SU_2 \times SU_2 \times SU_2$
SO_7	3	SU_4	G_2, $SU_2 \times SU_2 \times SU_2$, $Sp_4 \times U_1$
SU_4	3	$SU_3 \times U_1$	Sp_4, $SU_2 \times SU_2$
SO_{10}	6	$SU_5 \times U_1$, $SU_2 \times SU_2 \times SU_4$, SO_9, $SU_2 \times SO_7$, $SO_8 \times U_1$	Sp_4, $Sp_4 \times Sp_4$
SU_6	7	$SU_5 \times U_1$, $SU_2 \times U_1 \times SU_4$, $SU_3 \times SU_3 \times U_1$	SU_3, $SU_2 \times SU_3$, $[SU_4]$, $[Sp_6]$
SU_5	3	$SU_4 \times U_1$, $SU_2 \times U_1 \times SU_3$	Sp_4

* See discussion for table 40.

Table 19
Physical roots and axes in E_6 weight space

	Dynkin basis	Dual basis		Dynkin basis	Dual basis
	(1 1) (0 0 0 0 0 1)	[1 2 3 2 1 2]	I_3^w axis	$\frac{1}{2}$(1 0 0 0 1 −1)	$\frac{1}{2}$[1 1 1 1 1 0]
Color roots	(2 −1) (0 1 0 0 −1 0)	[1 2 2 1 0 1]	Y^w axis	$\frac{1}{3}$(3 −4 6 −6 1 −1)	$\frac{1}{3}$[1 −1 1 −3 −1 0]
	(−1 2) (0 −1 0 0 1 1)	[0 0 1 1 1 1]	Q^t axis	3(1 −1 0 1 −1 0)	[1 −1 0 1 −1 0]
Weak isospin root	(1 0 0 0 1 −1)	[1 1 1 1 1 0]	Q^r axis	(−3 −1 4 1 −1 −4)	[−1 1 4 3 1 0]
Q^{em} axis	$\frac{1}{3}$(3 −2 3 −3 2 −2)	$\frac{1}{3}$[2 1 2 0 1 0]	$\frac{1}{5}(3Y^w+4Q^r-10I_3)$	(−1 −1 2 −1 −1 0)	[−1 −1 0 −1 −10]

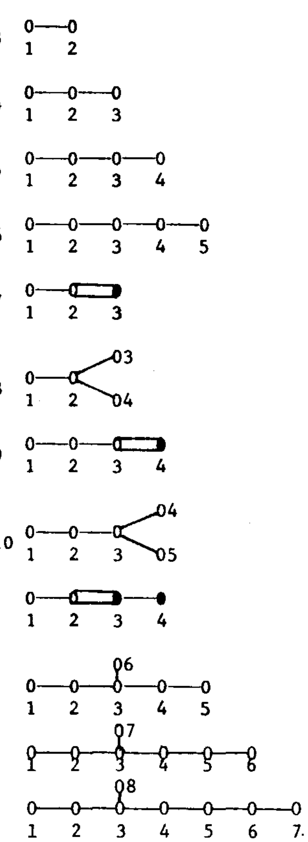
Table 20
Nonzero E_6 roots

Root	Level	Color	Q^{em}	I_3^w	Q^t	$SU_5(SO_{10})$	$\bar{B}\cdot\alpha$	$\bar{L}\cdot\alpha$
Color SU_3 roots								
(0 0 0 0 0 1)	0	(1 1)	0	0	0	24(45)	0	0
(0 1 0 0 −1 0)	4	(2 −1)	0	0	0	24(45)	0	0
(0 −1 0 0 1 1)	7	(−1 2)	0	0	0	24(45)	0	0
Left-handed SU_3 roots								
(1 0 0 0 1 −1)	6	(0 0)	1	1	0	24(45)	0	± 1
(−1 1 0 0 1 −1)	7	(0 0)	0	1/2	−3	$\bar{5}$(16)	$3c$	$3d\pm 2$
(−2 1 0 0 0 0)	12	(0 0)	−1	−1/2	−3	$\bar{5}$(16)	$3c$	$3d\pm 1$
Right-handed SU_3 roots								
(0 −1 1 1 −1 −1)	9	(0 0)	0	0	3	1($\overline{16}$)	$a+b-2c$	$-d+2e\mp 2$
(0 0 −1 2 −1 0)	10	(0 0)	−1	0	3	$\overline{10}$($\overline{16}$)	$-a+2b-c$	$-2d+e\mp 1$
(0 −1 2 −1 0 −1)	10	(0 0)	1	0	0	10(45)	$2a-b-c$	$d+3\mp 1$
SU_5 antilepto-diquarks								
(1 −1 1 −1 1 0)	4	(0 1)	4/3	1/2	0	24(45)	$a-b-c$	0
(1 0 1 −1 0 −1)	8	(1 −1)	4/3	1/2	0	24(45)	$a-b-c$	0
(1 −1 1 −1 1 −1)	15	(−1 0)	4/3	1/2	0	24(45)	$a-b-c$	0
(0 −1 1 −1 0 1)	9	(0 1)	1/3	−1/2	0	24(45)	$a-b-c$	∓ 1
(0 0 1 −1 −1 0)	13	(1 −1)	1/3	−1/2	0	24(45)	$a-b-c$	∓ 1
(0 −1 1 −1 0 0)	20	(−1 0)	1/3	−1/2	0	24(45)	$a-b-c$	∓ 1
SO_{10}/SU_5 leptoquarks								
(0 0 1 0 0 −1)	1	(1 0)	2/3	1/2	0	10(45)	a	$d+e$
(0 −1 1 0 1 −1)	8	(−1 1)	2/3	1/2	0	10(45)	a	$d+e$
(0 0 1 0 0 −2)	12	(0 −1)	2/3	1/2	0	10(45)	a	$d+e$
(−1 0 1 0 −1 0)	6	(1 0)	−1/3	−1/2	0	10(45)	a	$d+e\mp 1$
(−1 −1 1 0 0 0)	13	(−1 1)	−1/3	−1/2	0	10(45)	a	$d+e\mp 1$
(−1 0 1 0 −1 −1)	17	(0 −1)	−1/3	−1/2	0	10(45)	a	$d+e\mp 1$
(−1 0 0 1 0 0)	4	(0 1)	−2/3	0	0	10(45)	$b+c$	$d+e$
(−1 1 0 1 −1 −1)	8	(1 −1)	−2/3	0	0	10(45)	$b+c$	$d+e$
(−1 0 0 1 0 −1)	15	(−1 0)	−2/3	0	0	10(45)	$b+c$	$d+e$
E_6/SO_{10} leptoquarks								
(0 1 0 −1 1 0)	3	(1 0)	2/3	1/2	−3	10(16)	$-b+2c$	$2d-e\pm 2$
(0 0 0 −1 2 0)	10	(−1 1)	2/3	1/2	−3	10(16)	$-b+2c$	$2d-e\pm 2$
(0 1 0 −1 1 −1)	14	(0 −1)	2/3	1/2	−3	10(16)	$-b+2c$	$2d-e\pm 2$
(−1 1 0 −1 0 1)	8	(1 0)	−1/3	−1/2	−3	10(16)	$-b+2c$	$2d-e\pm 1$
(−1 0 0 −1 1 1)	15	(−1 1)	−1/3	−1/2	−3	10(16)	$-b+2c$	$2d-e\pm 1$
(−1 1 0 −1 0 0)	19	(0 −1)	−1/3	−1/2	−3	10(16)	$-b+2c$	$2d-e\pm 1$
(−1 0 1 −1 1 0)	5	(0 1)	1/3	0	−3	$\bar{5}$(16)	$a-b+2c$	$3d\pm 1$
(−1 1 1 −1 0 −1)	9	(1 −1)	1/3	0	−3	$\bar{5}$(16)	$a-b+2c$	$3d\pm 1$
(−1 0 1 −1 1 −1)	16	(−1 0)	1/3	0	−3	$\bar{5}$(16)	$a-b+2c$	$3d\pm 1$
(−1 1 −1 0 1 1)	6	(0 1)	−2/3	0	−3	10(16)	$-a+3c$	$2d-e\pm 2$
(−1 2 −1 0 0 0)	10	(1 −1)	−2/3	0	−3	10(16)	$-a+3c$	$2d-e\pm 2$
(−1 1 −1 0 1 0)	17	(−1 0)	−2/3	0	−3	10(16)	$-a+3c$	$2d-e\pm 2$

Table 21
Weights and content of the **27** of E_6

Weight	Level	Color	Q^{em}	I_3^w	Q'	$SU_5(SO_{10})$	SO_{10} weight
(0 0 0 1 0 −1)	4	(0 0)	0	1/2	1	$\bar{5}$(16)	(1 −1 0 1 0)
(−1 0 0 1 −1 0)	9	(0 0)	−1	−1/2	1	$\bar{5}$(16)	(1 0 0 0 −1)
(1 −1 1 −1 0 0)	9	(0 0)	1	0	1	10(16)	(−1 0 1 −1 0)
(1 0 −1 0 0 1)	10	(0 0)	0	0	1	1(16)	(−1 1 −1 0 1)
(0 0 1 −1 1 −1)	5	(0 0)	1	1/2	−2	5(10)	(0 −1 1 0 0)
(−1 0 1 −1 0 0)	10	(0 0)	0	−1/2	−2	5(10)	(0 0 1 −1 −1)
(0 1 −1 0 1 0)	6	(0 0)	0	1/2	−2	$\bar{5}$(10)	(0 0 −1 1 1)
(−1 1 −1 0 0 1)	11	(0 0)	−1	−1/2	−2	$\bar{5}$(10)	(0 1 −1 0 0)
(1 −1 0 1 −1 0)	8	(0 0)	0	0	4	1(1)	(0 0 0 0 0)
(1 0 0 0 0 0)	0	(1 0)	2/3	1/2	1	10(16)	(0 0 0 0 1)
(1 −1 0 0 1 0)	7	(−1 1)	2/3	1/2	1	10(16)	(−1 0 0 1 0)
(1 0 0 0 0 −1)	11	(0 −1)	2/3	1/2	1	10(16)	(0 −1 0 0 1)
(0 0 0 0 −1 1)	5	(1 0)	−1/3	−1/2	1	10(16)	(0 1 0 −1 0)
(0 −1 0 0 0 1)	12	(−1 1)	−1/3	−1/2	1	10(16)	(−1 1 0 0 −1)
(0 0 0 0 −1 0)	16	(0 −1)	−1/3	−1/2	1	10(16)	(0 0 0 −1 0)
(−1 1 0 0 0 0)	1	(1 0)	−1/3	0	−2	5(10)	(1 0 0 0 0)
(−1 0 0 0 1 0)	8	(−1 1)	−1/3	0	−2	5(10)	(0 0 0 1 −1)
(−1 1 0 0 0 −1)	12	(0 −1)	−1/3	0	−2	5(10)	(1 −1 0 0 0)
(0 0 0 −1 1 1)	4	(0 1)	1/3	0	−2	$\bar{5}$(10)	(−1 1 0 0 0)
(0 1 0 −1 0 0)	8	(1 −1)	1/3	0	−2	$\bar{5}$(10)	(0 0 0 −1 1)
(0 0 0 −1 1 0)	15	(−1 0)	1/3	0	−2	$\bar{5}$(10)	(−1 0 0 0 0)
(0 −1 1 0 0 0)	2	(0 1)	1/3	0	1	$\bar{5}$(16)	(0 0 1 0 −1)
(0 0 1 0 −1 −1)	6	(1 −1)	1/3	0	1	$\bar{5}$(16)	(1 −1 1 −1 0)
(0 −1 1 0 0 −1)	13	(−1 0)	1/3	0	1	$\bar{5}$(16)	(0 −1 1 0 −1)
(0 0 −1 1 0 1)	3	(0 1)	−2/3	0	1	10(16)	(0 1 −1 1 0)
(0 1 −1 1 −1 0)	7	(1 −1)	−2/3	0	1	10(16)	(1 0 −1 0 1)
(0 0 −1 1 0 0)	14	(−1 0)	−2/3	0	1	10(16)	(0 0 −1 1 0)

Table 22
Ordering of simple roots of Dynkin diagrams; orders of independent Casimir invariants for tables 23 to 53. (Shorter roots are denoted by black dots.)



	Orders of Independent Casimir Invariants
SU_3	2,3
SU_4	2,3,4
SU_5	2,3,4,5
SU_6	2,3,4,5,6
SO_7	2,4,6
SO_8	2,4,4,6
SO_9	2,4,6,8
SO_{10}	2,4,5,6,8
F_4	2,6,8,12
E_6	2,5,6,8,9,12
E_7	2,6,8,10,12,14,18
E_8	2,8,12,14,18,20,24,30

Table 23
SU_3 irreps of dimension less than 65

Dynkin label	Dimension (name)	l (index)	Triality	SU_2 singlets	SO_3 singlets
(10)	3	1	1	1	0
(20)	6	5	2*	1	1
(11)	8	6	0	1**	0
(30)	10	15	0	1	0
(21)	15	20	1	1	0
(40)	15′	35	1	1	1
(05)	21	70	1	1	0
(13)	24	50	1	1	0
(22)	27	54	0	1**	1
(60)	28	126	0	1	1
(41)	35	105	0	1	0
(70)	36	210	1	1	0
(32)	42	119	1	1	0
(08)	45	330	1	1	1
(51)	48	196	1	1	0
(90)	55	495	0	1	0
(24)	60	230	1	1	1
(16)	63	336	1	1	0
(33)	64	240	0	1**	0

*Note standard convention that 6 = (20).
**$SU_2 \times U_1$ singlet.

Table 24
SU_3 tensor products; triality 0 and 1 combinations shown

$\bar{3}\times\bar{3} = 3_a + \bar{6}_s$
$3\times\bar{3} = 1 + 8$
$6\times 3 = 8 + 10$
$6\times\bar{3} = 3 + 15$
$6\times 6 = \bar{6}_s + 15_a + 15'_s$
$6\times\bar{6} = 1 + 8 + 27$
$8\times 3 = 3 + \bar{6} + 15$
$8\times\bar{6} = 3 + \bar{6} + 15 + 24$
$8\times 8 = 1_s + 8_s + 8_a + 10_a + \overline{10}_a + 27_s$
$10\times 3 = 15 + 15'$
$\overline{10}\times 3 = \bar{6} + 24$
$10\times\bar{6} = 3 + 15 + 42$
$\overline{10}\times\bar{6} = 15 + 21 + 24$
$10\times 8 = 8 + 10 + 27 + 35$
$10\times 10 = 10_a + 27_s + 28_s + 35_a$
$\overline{10}\times 10 = 1 + 8 + 27 + 64$
$\overline{15}\times\bar{3} = \bar{6} + 15 + 24$
$15\times\bar{3} = 8 + 10 + 27$
$\overline{15}\times 6 = 3 + \bar{6} + 15 + 24 + 42$
$15\times 6 = 8 + 10 + \overline{10} + 27 + 35$
$15\times 8 = 3 + \bar{6} + 15_1 + 15_2 + 15' + 24 + 42$
$15\times 10 = \bar{6} + 15 + 15' + 24 + 42 + 48$
$15\times\overline{10} = 3 + \bar{6} + 15' + 24 + 42 + 60$
$\overline{15}\times\overline{15} = 3_a + \bar{6}_s + 15_s + 15_a + 15'_s + 21_a + 24_s + 24_a + 42_a + 60_s$
$15\times\overline{15} = 1 + 8_1 + 8_2 + 10 + \overline{10} + 27_1 + 27_2 + 35 + \overline{35} + 64$

Table 25
SU_4 irreps of dimension less than 180

Dynkin label	Dimension (name)	l (index)	Quadrality	SU_3 singlets
(100)	4	1	1	1
(010)	6	2	2	0
(200)	10	6	2	1
(101)	15	8	0	1*
(011)	20	13	1	0
(020)	20′	16	0	0
(003)	20″	21	1	1
(400)	35	56	0	1
(201)	36	33	1	1
(210)	45	48	0	0
(030)	50	70	2	0
(500)	56	126	1	1
(120)	60	71	1	0
(111)	64	64	2	0
(301)	70	98	2	1
(202)	84	112	0	1*
(310)	84′	133	1	0
(600)	84″	252	2	1
(040)	105	224	0	0
(104)	120	238	1	1
(007)	120′	462	1	1
(220)	126	210	2	0
(112)	140	203	1	0
(031)	140′	259	1	0
(410)	140″	308	2	0
(302)	160	296	1	1
(800)	165	792	0	1
(121)	175	280	0	0

*$SU_3 \times U_1$ singlet.

Table 26
SU_4 tensor products; quadrality 0, 1 and 2 shown

$4 \times 4 = 6_a + 10_s$
$4 \times \bar{4} = 1 + 15$
$6 \times \bar{4} = 4 + 20$
$6 \times 6 = 1_s + 15_a + 20'_s$
$\overline{10} \times \bar{4} = 20 + 20''$
$10 \times \bar{4} = 4 + 36$
$10 \times 6 = 15 + 45$
$10 \times 10 = 20'_s + 35_s + 45_a$
$10 \times \overline{10} = 1 + 15 + 84$
$15 \times 4 = 4 + 20 + 36$
$15 \times 6 = 6 + 10 + \overline{10} + 64$
$15 \times 10 = 6 + 10 + 64 + 70$
$15 \times 15 = 1_s + 15_s + 15_a + 20'_s + 45_a + \overline{45}_a + 84_s$
$\overline{20} \times 4 = 15 + 20' + 45$
$20 \times 4 = 6 + \overline{10} + 64$
$\overline{20} \times 6 = 4 + 20 + 36 + 60$
$\overline{20} \times 10 = 20 + 36 + 60 + 84'$
$\overline{20} \times \overline{10} = 4 + 20 + 36 + 140$
$20 \times 15 = 4 + 20_1 + 20_2 + 20'' + 36 + 60 + 140$
$20 \times 20 = 6_a + 10_s + \overline{10}_s + 50_a + 64_s + 64_a + 70_a + \overline{126}_s$
$20 \times \overline{20} = 1 + 15_1 + 15_2 + 20' + 45 + \overline{45} + 84 + 175$
$20' \times 4 = 20 + 60$
$20' \times 6 = 6 + 50 + 64$
$20' \times 10 = 10 + 64 + 126$
$20' \times 15 = 15 + 20' + 45 + \overline{45} + 175$
$20' \times 20 = 4 + 20 + 36 + 60 + 140 + 140'$
$20' \times 20' = 1_s + 15_a + 20'_s + 84_s + 105_s + 175_a$

Table 27
Branching rules for $SU_4 \supset SU_3 \times U_1$

(100) = 4 = 1(1) + 3(−1/3) (establishes normalization of U_1 generator)
(010) = 6 = 3(2/3) + $\bar{3}$(−2/3)
(200) = 10 = 1(2) + 3(2/3) + 6(−2/3)
(101) = 15 = 1(0) + 3(−4/3) + $\bar{3}$(4/3) + 8(0)
(011) = 20 = 3(−1/3) + $\bar{3}$(−5/3) + $\bar{6}$(−1/3) + 8(1)
(020) = 20′ = $\bar{6}$(−4/3) + 6(4/3) + 8(0)
(003) = 20″ = 1(−3) + $\bar{3}$(−5/3) + $\bar{6}$(−1/3) + $\overline{10}$(1)
(400) = 35 = 1(4) + 3(8/3) + 6(4/3) + 10(0) + 15′(−4/3)
(201) = 36 = 1(1) + 3(−1/3) + $\bar{3}$(7/3) + 6(−5/3) + 8(1) + 15(−1/3)
(210) = 45 = 3(8/3) + $\bar{3}$(4/3) + 6(4/3) + 6(4/3) + 8(0) + 10(0) + 15(−4/3)
(030) = 50 = 10(2) + $\overline{10}$(−2) + 15(2/3) + $\overline{15}$(−2/3)
(500) = 56 = 1(5) + 3(11/3) + 6(7/3) + 10(1) + 15′(−1/3) + $\overline{21}$(−5/3)
(120) = 60 = $\bar{6}$(−1/3) + 6(7/3) + 8(1) + 10(1) + 15(−1/3) + $\overline{15}$(−5/3)
(111) = 64 = 3(2/3) + $\bar{3}$(−2/3) + $\bar{6}$(2/3) + 6(−2/3) + 8(2) + 8(−2) + 15(2/3) + $\overline{15}$(−2/3)

Table 28
SU_5 irreps of dimension less than 800

Dynkin label	Dimension (name)	l (index)	Quintality	SU_4 singlets	$SU_2 \times SU_3$ singlets
(1000)	5	1	1	1	0
(0100)	10	3	2	0	1
(2000)	15	7	2	1	0
(1001)	24	10	0	1*	1*
(0003)	35	28	2	1	0
(0011)	40	22	2	0	0
(0101)	45	24	1	0	0
(0020)	50	35	1	0	1
(2001)	70	49	1	1	0
(0004)	70′	84	1	1	0
(0110)	75	50	0	0	1*
(0012)	105	91	1	0	0
(2010)	126	105	0	0	0
(5000)	126′	210	0	1	0
(3001)	160	168	2	1	0
(1101)	175	140	2	0	1
(1200)	175′	175	0	0	0
(0300)	175″	210	1	0	1
(2002)	200	200	0	1*	1*
(1020)	210	203	2	0	0
(6000)	210′	462	1	1	0
(3100)	224	280	0	0	0
(1110)	280	266	1	0	0
(3010)	280′	336	1	0	0
(0210)	315	357	2	0	1
(1004)	315′	462	2	1	0
(7000)	330	924	2	1	0
(2200)	420	574	1	0	0
(4100)	420′	714	1	0	0
(1012)	450	510	2	0	0
(3002)	450′	615	1	1	0
(1102)	480	536	1	0	0
(0040)	490	882	2	0	1
(0008)	495	1716	2	1	0
(4010)	540	882	2	0	0
(0202)	560	728	2	0	0
(1300)	560′	868	2	0	0
(1005)	560″	1092	1	1	0
(2110)	700	910	2	0	0
(1030)	700′	1050	0	0	0
(0009)	715	3003	1	1	0
(1021)	720	924	1	0	1
(5100)	720′	1596	2	0	0

(0130)	980	1666	1	0	1
(1111)	1024	1280	0	0	1*
(0121)	1050	1540	2	0	0
(0211)	1120	1624	1	0	0
(0220)	1176	1960	0	0	1*

Table 29
SU_5 tensor products

$5\times 5 = 10_a + 15_s$

$5\times \bar{5} = 1 + 24$

$\overline{10}\times \bar{5} = 10 + 40$

$10\times \bar{5} = 5 + 45$

$\overline{10}\times \overline{10} = 5_s + 45_a + 50_s$

$\overline{10}\times 10 = 1 + 24 + 75$

$\overline{15}\times \bar{5} = 35 + 40$

$15\times \bar{5} = 5 + 70$

$\overline{15}\times \overline{10} = 45 + 105$

$15\times \overline{10} = 24 + 126$

$\overline{15}\times \overline{15} = 50_s + 70'_s + 105_a$

$\overline{15}\times 15 = 1 + 24 + 200$

$24\times 5 = 5 + 45 + 70$

$24\times 10 = 10 + 15 + 40 + 175$

$24\times 15 = 10 + 15 + 160 + 175$

$24\times 24 = 1_s + 24_s + 24_a + 75_s + 126_a + \overline{126}_a + 200_s$

$\overline{40}\times \bar{5} = 10 + 15 + 175$

$40\times \bar{5} = 45 + 50 + 105$

$\overline{40}\times 10 = 24 + 75 + 126 + 175'$

$\overline{40}\times \overline{10} = 5 + 45 + 70 + 280$

$\overline{40}\times 15 = 75 + 126 + 175' + 224$

$\overline{40}\times \overline{15} = 5 + 45 + 70 + 480$

$40\times 24 = 10 + 35 + 40_1 + 40_2 + 175 + 210 + 450$

$\overline{40}\times \overline{40} = 45_a + 50_s + 70_s + 175''_a + 280_a + 280_s + 280'_a + 420_s$

$40\times \overline{40} = 1 + 24_1 + 24_2 + 75 + 126 + \overline{126} + 200 + 1024$

$45\times 5 = 10 + 40 + 175$

$\overline{45}\times 5 = 24 + 75 + 126$

$\overline{45}\times 10 = 5 + 45 + 50 + 70 + 280$

$\overline{45}\times \overline{10} = 10 + 15 + 40 + 175 + 210$

$\overline{45}\times 15 = 45 + 70 + 280 + 280'$

$\overline{45}\times \overline{15} = 10 + 40 + 175 + 450$

$45\times 24 = 5 + 45_1 + 45_2 + 50 + 70 + 105 + 280 + 480$

$\overline{45}\times \overline{40} = 10 + 15 + 40 + 160 + 175_1 + 175_2 + 210 + 315 + 700$

$\overline{45}\times 40 = 45 + 50_1 + 50_2 + 70 + 105 + 280 + 480 + 720$

$45\times 45 = 10_a + 15_s + 35_s + 40_s + 40_a + 175_s + 175_a + 210_s + 315_a + 450_a + 560_s$

$\overline{45}\times 45 = 1 + 24_1 + 24_2 + 75_1 + 75_2 + 126 + \overline{126} + 175' + \overline{175}' + 200 + 1024$

$50\times 5 = 40 + 210$

$\overline{50}\times 5 = 75 + 175'$

$\overline{50}\times \overline{10} = 10 + 175 + 315$

$\overline{50}\times 10 = 45 + 175'' + 280$

$\overline{50}\times \overline{15} = 15 + 175 + 560$

$\overline{50}\times 15 = 50 + 280 + 420$

$50\times 24 = 45 + 50 + 105 + 280 + 720$

$\overline{50}\times \overline{40} = 40 + 175 + 210 + 315 + 560' + 700$

$\overline{50}\times 40 = 5 + 45 + 70 + 280 + 480 + 1120$

$50\times 45 = 10 + 40 + 175 + 210 + 315 + 450 + 1050$

$50\times \overline{45} = 24 + 75 + 126 + \overline{126} + \overline{175}' + 700' + 1024$

$50\times 50 = 15_s + 175_a + 210_s + 490_s + 560_s + 1050_a$

$\overline{50}\times 50 = 1 + 24 + 75 + 200 + 1024 + 1176$

$75\times 5 = 45 + 50 + 280$

$75\times 10 = 10 + 40 + 175 + 210 + 315$

$75\times 15 = 40 + 175 + 210 + 700$

$75\times 24 = 24 + 75_1 + 75_2 + 126 + \overline{126} + 175' + \overline{175}' + 1024$

$75\times 40 = 10 + 15 + 40 + 175_1 + 175_2 + 210 + 315 + 450 + 560 + 1050$

$75\times 45 = 5 + 45_1 + 45_2 + 50 + 70 + 105 + 175'' + 280_1 + 280_2 + 480 + 720 + 1120$

$75\times 50 = 5 + 45 + 50 + 70 + 280 + 480 + 720 + 980 + 1120$

$75\times 75 = 1_s + 24_s + 24_a + 75_s + 75_a + 126_a + \overline{126}_a + 175'_s + \overline{175}'_s + 200_s + 700'_s + \overline{700}'_s + 1024_s + 1024_a + 1176_s$

Table 30
Branching rules for SU_5

$SU_5 \supset SU_4 \times U_1$
(1000) = 5 = 1(4) + 4(−1)
(0100) = 10 = 4(3) + 6(−2)
(2000) = 15 = 1(8) + 4(3) + 10(−2)
(1001) = 24 = 1(0) + 4(−5) + $\bar{4}$(5) + 15(0)
(0003) = 35 = 1(−12) + $\bar{4}$(−7) + $\overline{10}$(−2) + 20″(3)
(0011) = 40 = $\bar{4}$(−7) + 6(−2) + $\overline{10}$(−2) + 20(3)
(0101) = 45 = 4(−1) + 6(−6) + 15(4) + 20(−1)
(0020) = 50 = $\overline{10}$(−6) + 20(−1) + 20′(4)
(2001) = 70 = 1(4) + 4(−1) + $\bar{4}$(9) + 10(−6) + 15(4) + 36(−1)
(0004) = 70′ = 1(−16) + $\bar{4}$(−11) + $\overline{10}$(−6) + 20″(−1) + $\overline{35}$(4)
(0110) = 75 = 15(0) + 20(−5) + $\overline{20}$(5) + 20′(0)

$SU_5 \supset SU_2 \times SU_3 \times U_1$
5 = (2, 1)(3) + (1, 3)(−2)
10 = (1, 1)(6) + (1, $\bar{3}$)(−4) + (2, 3)(1)
15 = (3, 1)(6) + (2, 3)(1) + (1, 6)(−4)
24 = (1, 1)(0) + (3, 1)(0) + (2, 3)(−5) + (2, $\bar{3}$)(5) + (1, 8)(0)
35 = (4, 1)(−9) + (3, $\bar{3}$)(−4) + (2, $\bar{6}$)(1) + (1, $\overline{10}$)(6)
40 = (2, 1)(−9) + (2, 3)(1) + (1, $\bar{3}$)(−4) + (3, $\bar{3}$)(−4) + (1, 8)(6) + (2, $\bar{6}$)(1)
45 = (2, 1)(3) + (1, 3)(−2) + (3, 3)(−2) + (1, $\bar{3}$)(8) + (2, $\bar{3}$)(−7) + (1, $\bar{6}$)(−2) + (2, 8)(3)
50 = (1, 1)(−12) + (1, 3)(−2) + (2, $\bar{3}$)(−7) + (3, $\bar{6}$)(−2) + (1, 6)(8) + (2, 8)(3)
70 = (2, 1)(3) + (4, 1)(3) + (1, 3)(−2) + (3, 3)(−2) + (3, $\bar{3}$)(8) + (2, 6)(−7) + (2, 8)(3) + (1, 15)(−2)
70′ = (5, 1)(−12) + (4, $\bar{3}$)(−7) + (3, $\bar{6}$)(−2) + (2, $\overline{10}$)(3) + (1, $\overline{15'}$)(8)
75 = (1, 1)(0) + (1, 3)(10) + (2, 3)(−5) + (1, $\bar{3}$)(−10) + (2, $\bar{3}$)(5) + (2, $\bar{6}$)(−5) + (2, 6)(5) + (1, 8)(0) + (3, 8)(0)

Table 31
SU_6 irreps of dimension less than 1000

Dynkin label	Dimension	l (index)	Sextality	SU_5 singlets	$SU_2 \times SU_4$ singlets	$SU_3 \times SU_3$ singlets
(10000)	6	1	1	1	0	0
(01000)	15	4	2	0	1	0
(00100)	20	6	3	0	0	2
(20000)	21	8	2	1	0	0
(10001)	35	12	0	1*	1*	1*
(30000)	56	36	3	1	0	0
(11000)	70	33	3	0	0	0
(01001)	84	38	1	0	0	0
(00101)	105	52	2	0	0	0
(00020)	105′	64	2	0	1	0
(20001)	120	68	1	1	0	0
(00004)	126	120	2	1	0	0
(00200)	175	120	0	0	0	2 + 1*
(01010)	189	108	0	0	1*	1*
(00110)	210	131	1	0	0	0
(00012)	210′	152	2	0	0	0
(00005)	252	330	1	1	0	0
(20010)	280	192	0	0	0	0
(30001)	315	264	2	1	0	0
(00102)	336	248	1	0	0	0
(11001)	384	256	2	0	1	0
(20002)	405	324	0	1*	1*	1*

Table 31 (continued)

Dynkin label	Dimension	l (index)	Sextality	SU_5 singlets	$SU_2 \times SU_4$ singlets	$SU_3 \times SU_3$ singlets
(00021)	420	358	1	0	0	0
(00006)	462	792	0	1	0	0
(00030)	490	504	0	0	1	0
(00013)	504	516	1	0	0	0
(10101)	540	378	3	0	0	2
(02001)	560	456	3	0	0	0
(40001)	700	810	3	1	0	0
(30010)	720	696	1	0	0	0
(70000)	792	1716	1	1	0	0
(11010)	840	668	1	0	0	0
(10200)	840′	764	1	0	0	0
(30100)	840″	864	0	0	0	0
(11100)	896	768	0	0	0	0
(00300)	980	1134	3	0	0	4

(10110)	1050	880	2	0	0	0
(21001)	1134	1053	3	0	0	0
(22000)	1134′	1296	0	0	0	0
(02010)	1176	1120	2	0	1	0
(02100)	1176′	1204	1	0	0	0
(11002)	1260	1146	1	0	0	0
(01200)	1470	1568	2	0	0	0
(10102)	1701	1620	2	0	0	0
(13000)	1764	2310	1	0	0	0
(04000)	1764′	2688	2			
(02002)	1800	1920	2	0	0	0
(01110)	1960	1932	3	0	0	2
(10021)	2205	2352	2	0	1	0
(21010)	2430	2592	2	0	0	0
(21100)	2520	2868	1	0	0	0
(20200)	2520′	2976	2	0	0	0
(10030)	2520″	3156	1	0	0	0
(01102)	3240	3564	3	0	0	0
(11011)	3675	3780	0	0	1*	1*
(10201)	3969	4536	0	0	0	2 + 1*
(00400)	4116	7056	0	0	0	4 + 1*
(10111)	4410	4767	1	0	0	0
(12100)	4410′	5712	2	0	0	0
(00301)	4410″	6216	2	0	0	0
(01021)	4536	5508	3	0	0	0
(00130)	4704	7056	3	0	0	0
(02011)	5040	6024	1			
(01030)	5040′	7104	2			
(02101)	5670	7128	0			
(00211)	5880	7812	3			
(02020)	6720	9216	0			
(01201)	6804	8910	1			
(00220)	7056	10752	2			
(00310)	7056′	11256	1			
(01111)	8064	9984	2			
(01120)	10080	14352	1			
(01210)	11340	16848	0			

Table 32
SU_6 tensor products

$6\times 6 = 15_a + 21_s$

$6\times \bar{6} = 1 + 35$

$15\times 6 = 20 + 70$

$15\times \bar{6} = 6 + 84$

$\overline{15}\times 15 = 1 + 35 + 189$

$\overline{15}\times \overline{15} = 15_s + 105_a + 105'_s$

$20\times \bar{6} = 15 + 105$

$20\times \overline{15} = 6 + 84 + 210$

$20\times 20 = 1_a + 35_s + 175_s + 189_a$

$21\times 6 = 56 + 70$

$21\times \bar{6} = 6 + 120$

$\overline{21}\times \overline{15} = 105 + 210'$

$21\times \overline{15} = 35 + 280$

$\overline{21}\times 20 = 84 + 336$

$\overline{21}\times 21 = 1 + 35 + 405$

$\overline{21}\times \overline{21} = 105'_s + 126_s + 210'_a$

$35\times 6 = 6 + 84 + 120$

$35\times 15 = 15 + 21 + 105 + 384$

$35\times 20 = 20 + 70 + \overline{70} + 540$

$35\times 21 = 15 + 21 + 315 + 384$

$35\times 35 = 1_s + 35_s + 35_a + 189_s + 280_a + \overline{280}_a + 405_s$

$70\times \bar{6} = 15 + 21 + 384$

$\overline{70}\times \bar{6} = 105 + 105' + 210'$

$70\times \overline{15} = 6 + 84 + 120 + 840$

$\overline{70}\times \overline{15} = 84 + 210 + 336 + 420$

$70\times 20 = 35 + 189 + 280 + 896$

$70\times \overline{21} = 6 + 84 + 120 + 1260$

$\overline{70}\times \overline{21} = 210 + 336 + 420 + 504$

$70\times 35 = 20 + 56 + 70_1 + 70_2 + 540 + 560 + 1134$

$70\times 70 = 175_s + 189_a + 280_s + 490_a + 840''_a + 896_s + 896_a + 1134'_s$

$\overline{70}\times 70 = 1 + 35_1 + 35_2 + 189 + 280 + \overline{280} + 405 + 3675$

$84\times 6 = 15 + 105 + 384$

$\overline{84}\times 6 = 35 + 189 + 280$

$84\times 15 = 20 + 70 + \overline{70} + 540 + 560$

$\overline{84}\times 15 = 6 + 84 + 120 + 210 + 840$

$\overline{84}\times 20 = 15 + 21 + 105 + 105' + 384 + 1050$

$84\times 21 = 20 + 70 + 540 + 1134$

$\overline{84}\times 21 = 84 + 120 + 840 + 720$

$84\times 35 = 6 + 84_1 + 84_2 + 120 + 210 + 336 + 840 + 1260$

$\overline{84}\times 70 = 15 + 21 + 105 + 315 + 384_1 + 384_2 + 1050 + 1176 + 2430$

$\overline{84}\times \overline{70} = 15\times 105_1 + 105_2 + 105' + 210' + 384 + 1050 + 1701 + 2205$

$84\times 84 = 15_a + 21_s + 105_s + 105_a + 105'_a + 210'_s + 384_s + 384_a + 1050_s + 1176_a + 1701_a + 1800_s$

$\overline{84}\times 84 = 1 + 35_1 + 35_2 + 175 + 189_1 + 189_2 + 280 + \overline{280} + 405 + 896 + \overline{896} + 3675$

Table 33
SU_6 branching rules

$SU_6 \supset SU_5 \times U_1$

$(10000) = 6 = 1(-5) + 5(1)$
$(01000) = 15 = 5(-4) + 10(2)$
$(00100) = 20 = 10(-3) + \overline{10}(3)$
$(20000) = 21 = 1(-10) + 5(-4) + 15(2)$
$(10001) = 35 = 1(0) + 5(6) + \bar{5}(-6) + 24(0)$
$(30000) = 56 = 1(-15) + 5(-9) + 15(-3) + \overline{35}(3)$
$(11000) = 70 = 5(-9) + 10(-3) + 15(-3) + \overline{40}(3)$
$(01001) = 84 = 5(1) + 10(7) + 24(-5) + 45(1)$
$(00101) = 105 = 10(2) + \overline{10}(8) + 40(2) + 45(-4)$
$(00020) = 105' = \overline{15}(8) + 40(2) + 50(-4)$

$SU_6 \supset SU_2 \times SU_4 \times U_1$

$6 = (2,1)(2) + (1,4)(-1)$
$15 = (1,1)(4) + (1,6)(-2) + (2,4)(1)$
$20 = (1,4)(3) + (1,\bar{4})(-3) + (2,6)(0)$
$21 = (3,1)(4) + (2,4)(1) + (1,10)(-2)$
$35 = (1,1)(0) + (3,1)(0) + (1,15)(0) + (2,4)(-3) + (2,\bar{4})(3)$
$56 = (4,1)(6) + (3,4)(3) + (2,10)(0) + (1,\overline{20}'')(-3)$
$70 = (2,1)(6) + (1,4)(3) + (3,4)(3) + (2,6)(0) + (2,10)(0) + (1,\overline{20})(-3)$
$84 = (2,1)(2) + (1,4)(5) + (1,\bar{4})(-1) + (3,4)(-1) + (2,6)(-4) + (1,20)(-1) + (2,15)(2)$
$105 = (2,4)(1) + (2,\bar{4})(-5) + (1,6)(-2) + (3,6)(-2) + (1,\overline{10})(-2) + (1,15)(4) + (2,20)(1)$
$105' = (1,1)(-8) + (1,6)(-2) + (2,\bar{4})(-5) + (3,\overline{10})(-2) + (1,20')(4) + (2,20)(1)$

$SU_6 \supset SU_3 \times SU_3 \times U_1$

$6 = (3,1)(1) + (1,3)(-1)$
$15 = (\bar{3},1)(2) + (1,\bar{3})(-2) + (3,3)(0)$
$20 = (1,1)(3) + (1,1)(-3) + (3,\bar{3})(-1) + (\bar{3},3)(1)$
$21 = (6,1)(2) + (1,6)(-2) + (3,3)(0)$
$35 = (1,1)(0) + (8,1)(0) + (1,8)(0) + (3,\bar{3})(2) + (\bar{3},3)(-2)$
$56 = (10,1)(3) + (1,10)(-3) + (3,6)(-1) + (6,3)(1)$
$70 = (8,1)(3) + (1,8)(-3) + (3,\bar{3})(-1) + (\bar{3},3)(1) + (3,6)(-1) + (6,3)(1)$
$84 = (3,1)(1) + (1,3)(-1) + (\bar{6},1)(1) + (1,\bar{6})(-1) + (\bar{3},\bar{3})(3) + (\bar{3},\bar{3})(-3) + (3,8)(1) + (8,3)(-1)$
$105 = (\bar{3},1)(2) + (\bar{3},1)(-4) + (1,\bar{3})(-2) + (1,\bar{3})(4) + (3,3)(0) + (\bar{6},3)(0) + (3,\bar{6})(0) + (8,\bar{3})(-2) + (\bar{3},8)(2)$
$105' = (6,1)(-4) + (1,6)(4) + (3,3)(0) + (8,\bar{3})(-2) + (\bar{3},8)(2) + (\bar{6},\bar{6})(0)$

Table 34
SO_7 irreps of dimension less than 650 and branching rules ($SO_7 \supset SU_4$)

Dynkin label	Dimension (name)	$l/2$ (index)	Branching into SU_4 irreps
(100)	7	1	$1+6$
(001)	8	1	$4+\bar{4}$
(010)	21	5	$6+15$
(200)	27	9	$1+6+20'$
(002)	35	10	$10+\overline{10}+15$
(101)	48	14	$4+\bar{4}+20+\overline{20}$
(300)	77	44	$1+6+20'+50$
(110)	105	45	$6+15+20'+64$
(011)	112	46	$20+\overline{20}+36+\overline{36}$
(003)	112'	54	$20''+\overline{20}''+36+\overline{36}$
(020)	168	96	$20'+64+84$
(201)	168'	85	$4+\bar{4}+20+\overline{20}+60+\overline{60}$
(400)	182	156	$1+6+20'+50+105$
(102)	189	90	$10+\overline{10}+15+45+\overline{45}+64$
(004)	294	210	$35+\overline{35}+70+\overline{70}+84$
(210)	330	220	$6+15+20'+50+64+175$
(012)	378	234	$45+\overline{45}+64+70+\overline{70}+84$
(500)	378'	450	$1+6+20'+50+105+196$
(301)	448	344	$4+\bar{4}+20+\overline{20}+60+\overline{60}+140'+\overline{140}'$
(111)	512	320	$20+\overline{20}+36+\overline{36}+60+\overline{60}+140+\overline{140}$
(103)	560	390	$20''+\overline{20}''+36+\overline{36}+84'+\overline{84}'+140+\overline{140}$
(202)	616	440	$10+\overline{10}+15+45+\overline{45}+64+126+\overline{126}+175$

Table 35
SO_7 tensor products

$$7\times 7 = 1_s + 21_a + 27_s$$
$$8\times 7 = 8 + 48$$
$$8\times 8 = 1_s + 7_a + 21_a + 35_s$$
$$21\times 7 = 7 + 35 + 105$$
$$21\times 8 = 8 + 48 + 112$$
$$21\times 21 = 1_s + 21_a + 27_s + 35_s + 168_s + 189_a$$
$$27\times 7 = 7 + 77 + 105$$
$$27\times 8 = 48 + 168'$$
$$27\times 21 = 21 + 27 + 189 + 330$$
$$27\times 27 = 1_s + 21_a + 27_s + 168_s + 182_s + 330_a$$
$$35\times 7 = 21 + 35 + 189$$
$$35\times 8 = 8 + 48 + 112 + 112'$$
$$35\times 21 = 7 + 21 + 35 + 105 + 189 + 378$$
$$35\times 27 = 35 + 105 + 189 + 616$$
$$35\times 35 = 1_s + 7_a + 21_a + 27_s + 35_s + 105_s + 168_s + 189_a + 294_s + 378_a$$
$$48\times 7 = 8 + 48 + 112 + 168'$$
$$48\times 8 = 7 + 21 + 27 + 35 + 105 + 189$$
$$48\times 21 = 8 + 48_1 + 48_2 + 112 + 112' + 168' + 512$$
$$48\times 27 = 8 + 48 + 112 + 168' + 448 + 512$$
$$48\times 35 = 8 + 48_1 + 48_2 + 112_1 + 112_2 + 112' + 168' + 512 + 560$$
$$48\times 48 = 1_s + 7_a + 21_{a1} + 21_{a2} + 27_s + 35_{s1} + 35_{s2} + 77_a + 105_s + 105_a + 168_s + 189_s + 189_a + 330_a + 378_a + 616_s$$

Table 36
SO_8 irreps of dimension less than 1300

Dynkin label	Dimension (name)	Congruency class	$l/8$ (index)	Branching into SO_7 irreps
(1000)	8_v	(01)	1	8
(0001)	8_s	(10)	1	1 + 7
(0010)	8_c	(11)	1	8
(0100)	28	(00)	6	7 + 21
(2000)	35_v	(00)	10	35
(0002)	35_s	(00)	10	1 + 7 + 27
(0020)	35_c	(00)	10	35
(0011)	56_v	(01)	15	8 + 48
(1010)	56_s	(10)	15	21 + 35
(1001)	56_c	(11)	15	8 + 48
(3000)	112_v	(01)	54	112′
(0003)	112_s	(10)	54	1 + 7 + 27 + 77
(0030)	112_c	(11)	54	112′
(1100)	160_v	(01)	60	48 + 112
(0101)	160_s	(10)	60	7 + 21 + 27 + 105
(0110)	160_c	(11)	60	48 + 112
(1002)	224_{sv}	(01)	100	8 + 48 + 168′
(1020)	224_{cv}	(01)	100	112 + 112′
(2001)	224_{vs}	(10)	100	35 + 189
(2010)	224_{vc}	(11)	100	112 + 112′
(0012)	224_{sc}	(11)	100	8 + 48 + 168′
(0021)	224_{cs}	(10)	100	35 + 189

Table 36 (continued)

Dynkin label	Dimension (name)	Congruency class	$l/8$ (index)	Branching into SO_7 irreps
(4000)	294_v	(00)	210	294
(0004)	294_s	(00)	210	1 + 7 + 27 + 77 + 182
(0040)	294_c	(00)	210	294
(0200)	300	(00)	150	27 + 105 + 168
(1011)	350	(00)	150	21 + 35 + 105 + 189
(2100)	567_v	(00)	324	189 + 378
(0102)	567_s	(00)	324	7 + 21 + 27 + 77 + 105 + 330
(0120)	567_c	(00)	324	189 + 378
(3001)	672_{vs}	(11)	444	112′ + 560
(3010)	672_{vc}	(10)	444	294 + 378
(1003)	672_{sv}	(11)	444	8 + 48 + 168′ + 448
(1030)	672_{cv}	(10)	444	294 + 378
(0013)	672_{sc}	(01)	444	8 + 48 + 168′ + 448
(0031)	672_{cs}	(01)	444	112′ + 560
(5000)	$672'_v$	(01)	660	672
(0005)	$672'_s$	(10)	660	1 + 7 + 27 + 77 + 182 + 378′
(0050)	$672'_c$	(11)	660	672
(0111)	840_v	(01)	465	48 + 112 + 168′ + 512
(1110)	840_s	(10)	465	105 + 168 + 189 + 378
(1101)	840_c	(11)	465	48 + 112 + 168′ + 512
(0022)	$840'_v$	(00)	540	35 + 189 + 616
(2020)	$840'_s$	(00)	540	168 + 294 + 378
(2002)	$840'_c$	(00)	540	35 + 189 + 616
(2011)	1296_v	(01)	810	112 + 112′ + 512 + 560
(1012)	1296_s	(10)	810	21 + 35 + 105 + 189 + 330 + 616
(1021)	1296_c	(11)	810	112 + 112′ + 512 + 560

Table 37
SO_8 tensor products

$8_i \times 8_i = 1_s + 28_a + (35_i)_s$ (i = v, s, or c)

$8_i \times 8_j = 8_k + 56_k$ (i, j, k cyclic)

$28 \times 8_i = 8_i + 56_i + 160_i$

$28 \times 28 = 1_s + 28_a + (35_v)_s + (35_s)_s + (35_c)_s + 300_s + 350_a$

$35_i \times 8_i = 8_i + 112_i + 160_i$

$35_i \times 8_j = 56_j + 224_{ij}$ ($i \neq j$)

$35_i \times 28 = 28 + 35_i + 350 + 567_i$

$35_i \times 35_i = 1_s + 28_a + (35_i)_s + (294_i)_s + 300_s + (567_i)_a$

$35_i \times 35_j = 35_k + 350 + 840'_k$ (i, j, k cyclic)

$56_i \times 8_i = 28 + 35_j + 35_k + 350$ ($i \neq j \neq k \neq i$)

$56_i \times 8_j = 8_k + 56_k + 160_k + 224_{jk}$

$56_i \times 28 = 8_i + 56_{i1} + 56_{i2} + 160_i + 224_{ji} + 224_{ki} + 840_i$

$56_i \times 35_i = 56_i + 160_i + 224_{ji} + 224_{ki} + 1296_i$ ($i \neq j \neq k \neq i$)

$56_i \times 35_j = 8_i + 56_i + 160_i + 224_{ji} + 672_{jk} + 840_i$ ($i \neq j \neq k \neq i$)

$56_i \times 56_i = 1_s + 28_{a1} + 28_{a2} + (35_v)_s + (35_s)_s + (35_c)_s + 300_s + 350_s + 350_a + (567_j)_a + (567_k)_a + (840'_i)_s$ ($i \neq j \neq k \neq i$)

$56_i \times 56_j = 8_k + 56_{k1} + 56_{k2} + 112_k + 160_{k1} + 160_{k2} + 224_{ik} + 224_{jk} + 840_k + 1296_k$ (i, j, k cyclic)

$(28^3)_s = 28_1 + 28_2 + 28_3 + 350 + 567_v + 567_s + 567_c + 1925$

Table 38
SO_9 irreps of dimension less than 5100

Dynkin label	Dimension (name)	$l/2$ (index)	SO_8 singlets	$SU_2 \times SU_4$ singlets
(1000)	9	1	1	0
(0001)	16	2	0	0
(0100)	36	7	0	0
(2000)	44	11	1	1
(0010)	84	21	0	1
(0002)	126	35	0	0
(1001)	128	32	0	0
(3000)	156	65	1	0
(1100)	231	77	0	0
(0101)	432	150	0	0
(4000)	450	275	1	1
(0200)	495	220	0	1
(2001)	576	232	0	0
(1010)	594	231	0	0
(0003)	672	308	0	0
(0011)	768	320	0	0
(2100)	910	455	0	0
(1002)	924	385	0	0
(5000)	1122	935	1	0
(0110)	1650	825	0	0
(3001)	1920	1120	0	0
(0020)	1980	1155	0	1
(2010)	2457	1365	0	1
(6000)	2508	2717	1	1
(1101)	2560	1280	0	0
(1200)	2574	1573	0	0
(0102)	2772	1463	0	0
(0004)	2772′	1848	0	0
(3100)	2772″	1925	0	0
(2002)	3900	2275	0	0
(0300)	4004	3003	0	0
(0012)	4158	2541	0	0
(1003)	4608	2816	0	0
(0201)	4928	3080	0	0
(1011)	5040	2870	0	0

Table 39
SO_9 tensor products

$9\times 9 = 1_s + 36_a + 44_s$

$16\times 9 = 16 + 128$

$16\times 16 = 1_s + 9_s + 36_a + 84_a + 126_s$

$36\times 9 = 9 + 84 + 231$

$36\times 16 = 16 + 128 + 432$

$36\times 36 = 1_s + 36_a + 44_s + 126_s + 495_s + 594_a$

$44\times 9 = 9 + 156 + 231$

$44\times 16 = 128 + 576$

$44\times 36 = 36 + 44 + 594 + 910$

$44\times 44 = 1_s + 36_a + 44_s + 450_s + 495_s + 910_a$

$84\times 9 = 36 + 126 + 594$

$84\times 16 = 16 + 128 + 432 + 768$

$84\times 36 = 9 + 84 + 126 + 231 + 924 + 1650$

$84\times 44 = 84 + 231 + 924 + 2457$

$84\times 84 = 1_s + 36_a + 44_s + 84_a + 126_s + 495_s + 594_a + 924_s + 1980_s + 2772_a$

$126\times 9 = 84 + 126 + 924$

$126\times 16 = 16 + 128 + 432 + 672 + 768$

$126\times 36 = 36 + 84 + 126 + 594 + 924 + 2772$

$126\times 44 = 126 + 594 + 924 + 3900$

$126\times 84 = 9 + 36 + 84 + 126 + 231 + 594 + 924 + 1650 + 2772 + 4158$

$126\times 126 = 1_s + 9_s + 36_a + 44_s + 84_a + 126_s + 231_a + 495_s + 594_a + 924_s + 1650_s + 1980_s + 2772_a + 2772'_s + 4158_a$

$128\times 9 = 16 + 128 + 432 + 576$

$128\times 16 = 9 + 36 + 44 + 84 + 126 + 231 + 594 + 924$

$128\times 36 = 16 + 128_1 + 128_2 + 432 + 576 + 768 + 2560$

$128\times 44 = 16 + 128 + 432 + 576 + 1920 + 2560$

$128\times 84 = 16 + 128_1 + 128_2 + 432_1 + 432_2 + 576 + 672 + 768 + 2560 + 5040$

$128\times 126 = 16 + 128_1 + 128_2 + 432_1 + 432_2 + 576 + 672 + 768_1 + 768_2 + 2560 + 4608 + 5040$

$128\times 128 = 1_s + 9_s + 36_{a1} + 36_{a2} + 44_s + 84_{a1} + 84_{a2} + 126_{s1} + 126_{s2} + 156_s + 231_s + 231_a + 495_a + 594_s + 594_a + 910_a + 924_s + 924_a + 1650_s + 2457_a + 2772_a + 3900_s$

Table 40
Branchings of SO_9 representations

$SO_9 \supset SO_8$

$9 = 1 + 8_v$

$16 = 8_c + 8_s$

$36 = 8_v + 28$

$44 = 1 + 8_v + 35_v$

$84 = 28 + 56_v$

$126 = 35_c + 35_s + 56_v$

$128 = 8_c + 8_s + 56_c + 56_s$

$156 = 1 + 8_v + 35_v + 112_v$

$231 = 8_v + 28 + 35_v + 160_v$

$432 = 56_c + 56_s + 160_c + 160_s$

$450 = 1 + 8_v + 35_v + 112_v + 294_v$

$495 = 35_v + 160_v + 300$

$576 = 8_c + 8_s + 56_c + 56_s + 224_{vc} + 224_{vs}$

$594 = 28 + 56_v + 160_v + 350$

$SO_9 \supset SU_2\times SU_4$

$9 = (3, 1) + (1, 6)$

$16 = (2, 4) + (2, \bar{4})$

$36 = (3, 1) + (1, 15) + (3, 6)$

$44 = (1, 1) + (5, 1) + (3, 6) + (1, 20')$

$84 = (1, 1) + (1, 10) + (1, \overline{10}) + (3, 6) + (3, 15)$

$126 = (1, 6) + (1, 15) + (3, 10) + (3, \overline{10}) + (3, 15)$

$128 = (2, 4) + (2, \bar{4}) + (4, 4) + (4, \bar{4}) + (2, 20) + (2, \overline{20})$

$156 = (3, 1) + (1, 6) + (7, 1) + (5, 6) + (1, 50) + (3, 20')$

$231 = (3, 1) + (5, 1) + (1, 6) + (3, 6) + (5, 6) + (3, 15) + (3, 20') + (1, 64)$

$432 = (2, 4) + (2, \bar{4}) + (4, 4) + (4, \bar{4}) + (2, 20) + (2, \overline{20}) + (4, 20) + (4, \overline{20}) + (2, 36) + (2, \overline{36})$

$450 = (1, 1) + (5, 1) + (9, 1) + (3, 6) + (7, 6) + (1, 20') + (5, 20') + (3, 50) + (1, 105)$

$495 = (1, 1) + (5, 1) + (3, 6) + (5, 6) + (1, 20') + (5, 20') + (3, 15) + (3, 64) + (1, 84)$

$576 = (2, 4) + (2, \bar{4}) + (4, 4) + (4, \bar{4}) + (6, 4) + (6, \bar{4}) + (2, 20) + (2, \overline{20}) + (4, 20) + (4, \overline{20}) + (2, 60) + (2, \overline{60})$

$594 = (3, 1) + (1, 6) + (3, 6) + (5, 6) + (1, 15) + (3, 15) + (5, 15) + (3, 10) + (3, \overline{10}) + (3, 20') + (1, 45) + (1, \overline{45}) + (3, 64)$

Table 41
SO_{10} irreps of dimension less tan 12000

Dynkin label	Dimension (name)	Congruency class	$l/2$ (index)	SU_5 singlets	$SU_2 \times SU_2 \times SU_4$ singlets	SO_9 singlets	$SU_2 \times SO_7$ singlets
(10000)	10	2	1	0	0	1	0
(00001)	16	1	2	1	0	0	0
(01000)	45	0	8	1*	0	0	0
(20000)	54	0	12	0	1	1	1
(00100)	120	2	28	0	0	0	1
(00002)	126	2	35	1	0	0	0
(10010)	144	1	34	0	0	0	0
(00011)	210	0	56	1	1	0	0
(30000)	210′	2	77	0	0	1	0
(11000)	320	2	96	0	0	0	0
(01001)	560	1	182	1	0	0	0
(40000)	660	0	352	0	1	1	1
(00030)	672	1	308	1	0	0	0
(20001)	720	1	266	0	0	0	0
(02000)	770	0	308	1*	1	0	1
(10100)	945	0	336	0	0	0	0
(10002)	1050	0	420	0	0	0	0
(00110)	1200	1	470	0	0	0	0
(21000)	1386	0	616	0	0	0	0
(00012)	1440	1	628	1	0	0	0
(10011)	1728	2	672	0	0	0	0
(50000)	1782	2	1287	0	0	1	0
(30010)	2640	1	1386	0	0	0	0
(00004)	2772	0	1848	1	0	0	0
(01100)	2970	2	1353	0	0	0	0
(01002)	3696	2	1848	1	0	0	0
(11010)	3696′	1	1694	0	0	0	0
(00200)	4125	0	2200	0	1	0	1
(60000)	4290	0	4004	0	1	1	1
(20100)	4312	2	2156	0	0	0	1
(12000)	4410	2	2401	0	0	0	0
(31000)	4608	2	2816	0	0	0	0
(20002)	4950	2	2695	0	0	0	0
(10003)	5280	1	3124				
(01011)	5940	0	2904				
(00102)	6930	0	4004				
(00013)	6930′	2	4389				
(03000)	7644	0	5096				
(40001)	7920	1	5566				
(02001)	8064	1	4592				
(20011)	8085	0	4312				
(10101)	8800	1	4620				
(00022)	8910	0	5544				
(70000)	9438	2	11011				
(00005)	9504	1	8580				
(00111)	10560	2	5984				
(10021)	11088	1	6314				

Table 42
SO_{10} tensor products

$10\times 10 = 1_s + 45_a + 54_s$

$\overline{16}\times 10 = 16 + 144$

$16\times 16 = 10_s + 120_a + 126_s$

$\overline{16}\times 16 = 1 + 45 + 210$

$45\times 10 = 10 + 120 + 320$

$45\times 16 = 16 + 144 + 560$

$45\times 45 = 1_s + 45_a + 54_s + 210_s + 770_s + 945_a$

$54\times 10 = 10 + 210' + 320$

$54\times 16 = 144 + 720$

$54\times 45 = 45 + 54 + 945 + 1386$

$54\times 54 = 1_s + 45_a + 54_s + 660_s + 770_s + 1386_a$

$120\times 10 = 45 + 210 + 945$

$120\times \overline{16} = 16 + 144 + 560 + 1200$

$120\times 45 = 10 + 120 + 126 + \overline{126} + 320 + 1728 + 2970$

$120\times 54 = 120 + 320 + 1728 + 4312$

$120\times 120 = 1_s + 45_a + 54_s + 210_s + 210_a + 770_s + 945_a + 1050_s + \overline{1050}_s + 4125_s + 5940_a$

$126\times 10 = 210 + 1050$

$\overline{126}\times \overline{16} = 144 + 672 + 1200$

$126\times \overline{16} = 16 + 560 + 1440$

$126\times 45 = 120 + 126 + 1728 + 3696$

$126\times 54 = \overline{126} + 1728 + 4950$

$126\times 120 = 45 + 210 + 945 + 1050 + 5940 + 6930$

$126\times 126 = 54_s + 945_a + 1050_s + 2772_s + 4125_s + 6930_a$

$\overline{126}\times 126 = 1 + 45 + 210 + 770 + 5940 + 8910$

$\overline{144}\times 10 = 16 + 144 + 560 + 720$

$\overline{144}\times 16 = 45 + 54 + 210 + 945 + 1050$

$\overline{144}\times \overline{16} = 10 + 120 + 126 + 320 + 1728$

$144\times 45 = 16 + 144_1 + 144_2 + 560 + 720 + 1200 + 3696'$

$144\times 54 = 16 + 144 + 560 + 720 + 2640 + 3696'$

$\overline{144}\times 120 = 16 + 144_1 + 144_2 + 560_1 + 560_2 + 720 + 1200 + 1440 + 3696' + 8800$

$\overline{144}\times 126 = 144 + 560 + 720 + 1200 + 1440 + 5280 + 8800$

$\overline{144}\times \overline{126} = 16 + 144 + 560 + 1200 + 1440 + 3696' + 11088$

$144\times 144 = 10_s + 120_{a1} + 120_{a2} + 126_s + \overline{126}_s + 210'_s + 320_s + 320_a + 1728_s + 1728_a + 2970_s + 3696_a + 4312_a + 4950_s$

$\overline{144}\times 144 = 1 + 45_1 + 45_2 + 54 + 210_1 + 210_2 + 770 + 945_1 + 945_2 + 1050 + \overline{1050} + 1386 + 5940 + 8085$

$210\times 10 = 120 + 126 + \overline{126} + 1728$

$210\times 16 = 16 + 144 + 560 + 1200 + 1440$

$210\times 45 = 45 + 210_1 + 210_2 + 945 + 1050 + \overline{1050} + 5940$

$210\times 54 = 210 + 945 + \overline{1050} + 1050 + 8085$

$210\times 120 = 10 + 120_1 + 120_2 + 126 + \overline{126} + 320 + 1728_1 + 1728_2 + 2970 + 3696 + \overline{3696} + 10560$

$210\times 126 = 10 + 120 + 126 + 320 + 1728 + 2970 + 3696 + 6930' + 10560$

$210\times 144 = 16 + 144_1 + 144_2 + 560_1 + 560_2 + 672 + 720 + 1200_1 + 1200_2 + 1440 + 3696' + 8800 + 11088$

$210\times 210 = 1_s + 45_s + 45_a + 54_s + 210_s + 210_a + 770_s + 945_{a1} + 945_{a2} + \overline{1050}_s + 1050_s + 4125_s + 5940_s + 5940_a + 6930_a + \overline{6930}_a + 8910_s$

Table 43
Branching rules for SO_{10}

$SO_{10} \supset SU_5 \times U_1$

$(10000) = 10 = 5(2) + \bar{5}(-2)$

$(00001) = 16 = 1(-5) + \bar{5}(3) + 10(-1)$

$(01000) = 45 = 1(0) + 10(4) + \overline{10}(-4) + 24(0)$

$(20000) = 54 = 15(4) + \overline{15}(-4) + 24(0)$

$(00100) = 120 = 5(2) + \bar{5}(-2) + 10(-6) + \overline{10}(6) + 45(2) + \overline{45}(-2)$

$(00002) = 126 = 1(-10) + \bar{5}(-2) + 10(-6) + \overline{15}(6) + 45(2) + \overline{50}(-2)$

$(10010) = 144 = \bar{5}(3) + 5(7) + 10(-1) + 15(-1) + 24(-5) + 40(-1) + \overline{45}(3)$

$(00011) = 210 = 1(0) + 5(-8) + \bar{5}(8) + 10(4) + \overline{10}(-4) + 24(0) + 40(-4) + \overline{40}(4) + 75(0)$

$(30000) = 210' = 35(-6) + \overline{35}(6) + 70(2) + \overline{70}(-2)$

$(11000) = 320 = 5(2) + \bar{5}(-2) + 40(-6) + \overline{40}(6) + 45(2) + \overline{45}(-2) + 70(2) + \overline{70}(-2)$

$(01001) = 560 = 1(-5) + \bar{5}(3) + \overline{10}(-9) + 10(-1)_1 + 10(-1)_2 + 24(-5) + 40(-1) + 45(7) + \overline{45}(3) + \overline{50}(3) + \overline{70}(3) + 75(-5) + 175(-1)$

$SO_{10} \supset SU_2 \times SU_2 \times SU_4$

$10 = (2, 2, 1) + (1, 1, 6)$

$16 = (2, 1, 4) + (1, 2, \bar{4})$

$45 = (3, 1, 1) + (1, 3, 1) + (1, 1, 15) + (2, 2, 6)$

$54 = (1, 1, 1) + (3, 3, 1) + (1, 1, 20') + (2, 2, 6)$

$120 = (2, 2, 1) + (1, 1, 10) + (1, 1, \overline{10}) + (3, 1, 6) + (1, 3, 6) + (2, 2, 15)$

$126 = (1, 1, 6) + (3, 1, \overline{10}) + (1, 3, 10) + (2, 2, 15)$

$144 = (2, 1, 4) + (1, 2, \bar{4}) + (3, 2, \bar{4}) + (2, 3, 4) + (2, 1, 20) + (1, 2, \overline{20})$

$210 = (1, 1, 1) + (1, 1, 15) + (2, 2, 6) + (3, 1, 15) + (1, 3, 15) + (2, 2, 10) + (2, 2, \overline{10})$

$210' = (2, 2, 1) + (1, 1, 6) + (4, 4, 1) + (3, 3, 6) + (2, 2, 20') + (1, 1, 50)$

$320 = (2, 2, 1) + (1, 1, 6) + (4, 2, 1) + (2, 4, 1) + (3, 1, 6) + (1, 3, 6) + (2, 2, 15) + (3, 3, 6) + (1, 1, 64) + (2, 2, 20')$

$560 = (2, 1, 4) + (1, 2, \bar{4}) + (4, 1, 4) + (1, 4, \bar{4}) + (2, 3, 4) + (3, 2, \bar{4}) + (2, 1, 20) + (1, 2, \overline{20}) + (2, 1, 36) + (1, 2, \overline{36}) + (2, 3, 20) + (3, 2, \overline{20})$

$SO_{10} \supset SO_9$

$10 = 1 + 9$

$16 = 16$

$45 = 9 + 36$

$54 = 1 + 9 + 44$

$120 = 36 + 84$

$126 = 126$

$144 = 16 + 128$

$210 = 84 + 126$

$210' = 1 + 9 + 44 + 156$

$320 = 9 + 36 + 44 + 231$

$560 = 128 + 432$

$SO_{10} \supset SU_2 \times SO_7$

$10 = (3, 1) + (1, 7)$

$16 = (2, 8)$

$45 = (3, 1) + (1, 21) + (3, 7)$

$54 = (1, 1) + (5, 1) + (3, 7) + (1, 27)$

$120 = (1, 1) + (3, 7) + (1, 35) + (3, 21)$

$126 = (1, 21) + (3, 35)$

$144 = (2, 8) + (4, 8) + (2, 48)$

$210 = (1, 7) + (1, 35) + (3, 21) + (3, 35)$

$210' = (3, 1) + (7, 1) + (1, 7) + (5, 7) + (3, 27) + (1, 77)$

$320 = (3, 1) + (5, 1) + (1, 7) + (3, 7) + (5, 7) + (3, 21) + (3, 27) + (1, 105)$

$560 = (2, 8) + (4, 8) + (2, 48) + (4, 48) + (2, 112)$

Table 44
F_4 irreps of dimension less than 100000

Dynkin label	Dimension (name)	l/6 (index)	SO_9 singlets	$SU_3 \times SU_3$ singlets
(0001)	26	1	1	0
(1000)	52	3	0	0
(0010)	273	21	0	1
(0002)	324	27	1	1
(1001)	1053	108	0	0
(2000)	1053'	135	0	1
(0100)	1274	147	0	1
(0003)	2652	357	1	1
(0011)	4096	512	0	0
(1010)	8424	1242	0	1
(1002)	10829	1666	0	1
(3000)	12376	2618		
(0004)	16302	3135	1	1
(2001)	17901	3213		
(0101)	19278	3213	0	1
(0020)	19448	3366	0	2
(1100)	29172	5610		
(0012)	34749	6237	0	1
(1003)	76076	16093		
(0005)	81081	20790		

Table 45
F_4 tensor products

$26 \times 26 = 1_s + 26_s + 52_a + 273_a + 324_s$
$52 \times 26 = 26 + 273 + 1053$
$52 \times 52 = 1_s + 52_a + 324_s + 1053'_s + 1274_a$
$273 \times 26 = 26 + 52 + 273 + 324 + 1053 + 1274 + 4096$
$273 \times 52 = 26 + 273 + 324 + 1053 + 4096 + 8424$
$273 \times 273 = 1_s + 26_s + 52_a + 273_{a1} + 273_{a2} + 324_{s1} + 324_{s2} + 1053_s + 1053_a + 1053'_s + 1274_a + 2652_s + 4096_s + 4096_a + 8424_s + 10829_a + 19278_a + 19448_s$
$324 \times 26 = 26 + 273 + 324 + 1053 + 2652 + 4096$
$324 \times 52 = 52 + 273 + 324 + 1274 + 4096 + 10829$
$324 \times 273 = 26 + 52 + 273_1 + 273_2 + 324 + 1053_1 + 1053_2 + 1274 + 2652 + 4096_1 + 4096_2 + 8424 + 10829 + 19278 + 34749$
$324 \times 324 = 1_s + 26_s + 52_a + 273_a + 324_{s1} + 324_{s2} + 1053_a + 1053'_s + 1274_a + 2652_s + 4096_s + 4096_a + 8424_s + 10829_a + 16302_s + 19448_s + 34749_a$

Table 46
Branchings of F_4 representations

$F_4 \supset SO_9$
$(0001) = 26 = 1 + 9 + 16$
$(1000) = 52 = 16 + 36$
$(0010) = 273 = 9 + 16 + 36 + 84 + 128$
$(0002) = 324 = 1 + 9 + 16 + 44 + 126 + 128$
$(1001) = 1053 = 16 + 36 + 84 + 126 + 128 + 231 + 432$
$(2000) = 1053' = 126 + 432 + 495$
$(0100) = 1274 = 36 + 84 + 128 + 432 + 594$

$F_4 \supset SU_3 \times SU_3$
$26 = (8, 1) + (3, 3) + (\bar{3}, \bar{3})$
$52 = (8, 1) + (1, 8) + (6, \bar{3}) + (\bar{6}, 3)$
$273 = (1, 1) + (8, 1) + (3, 3) + (\bar{3}, \bar{3}) + (10, 1) + (\overline{10}, 1) + (6, \bar{3}) + (\bar{6}, 3) + (3, \bar{6}) + (\bar{3}, 6) + (15, 3) + (\overline{15}, \bar{3}) + (8, 8)$
$324 = (1, 1) + (8, 1) + (1, 8) + (3, 3) + (\bar{3}, \bar{3}) + (6, \bar{3}) + (\bar{6}, 3) + (27, 1) + (6, 6) + (\bar{6}, \bar{6}) + (15, 3) + (\overline{15}, \bar{3}) + (8, 8)$

Table 47
E_6 irreps of dimension less than 100000

Dynkin label	Dimension (name)	$l/6$ (index)	Triality	F_4 singlets	SO_{10} singlets	$SU_2 \times SU_6$ singlets	$SU_3 \times SU_3 \times SU_3$ singlets
(100000)	27	1	1	1	1	0	0
(000001)	78	4	0	0	1*	0	0
(000100)	351	25	1	0	0	0	0
(000020)	351′	28	1	1	1	0	0
(100010)	650	50	0	1	1*	1	2
(100001)	1728	160	1	0	1	0	0
(000002)	2430	270	0	0	1*	1	1
(001000)	2925	300	0	0	0	0	1
(300000)	3003	385	0	1	1	0	1
(000110)	5824	672	0	0	0	0	0
(010010)	7371	840	1	0	0	0	0
(200010)	7722	946	1	1	1	0	0
(000101)	17550	2300	1	0	0	0	0
(000021)	19305	2695	1	0	1	0	0
(400000)	19305′	3520	1	1	1	0	0
(020000)	34398	5390	1	0	0	0	0
(100011)	34749	4752	0	0	1*	0	
(000003)	43758	7854	0				
(100002)	46332	7260	1				
(101000)	51975	7700	1	0	0	0	0
(210000)	54054	8932	1	0	0	0	0
(100030)	61425	10675	1				
(010100)	70070	10780	0	0	0	1	
(010020)	78975	12825	0	0	0	0	0
(200020)	85293	14580	0	1	1*	1	2
(100110)	112320	18080	1				

*$SO_{10} \times U_1$ singlet.

Table 48
E_6 tensor products

$\overline{27} \times \overline{27} = 27_s + 351_a + 351'_s$

$\overline{27} \times 27 = 1 + 78 + 650$

$78 \times 27 = 27 + 351 + 1728$

$78 \times 78 = 1_s + 78_a + 650_s + 2430_s + 2925_a$

$351 \times \overline{27} = 78 + 650 + 2925 + 5824$

$\overline{351} \times \overline{27} = 27 + 351 + 1728 + 7371$

$351 \times 78 = 27 + 351 + 351' + 1728 + 7371 + 17550$

$\overline{351} \times \overline{351} = 27_s + 351_a + 351'_s + 1728_s + 1728_a + 7371_a + \overline{7722_s} + 17550_s + 34398_s + 51975_a$

$351 \times \overline{351} = 1 + 78 + 650_1 + 650_2 + 2430 + 2925 + 5824 + \overline{5824} + 34749 + 70070$

$\overline{351'} + 27 = 650 + 3003 + \overline{5824}$

$\overline{351'} + \overline{27} = 27 + 1728 + 7722$

$351' \times 78 = 351 + 351' + 7371 + 19305$

$\overline{351'} \times \overline{351} = 351 + 1728 + 7371 + 7722 + 51975 + 54054$

$351' \times \overline{351} = 78 + 650 + 2925 + 5824 + 34749 + 78975$

$\overline{351'} \times \overline{351'} = 351'_s + 7371_a + 7722_s + 19305'_s + 34398_s + 54054_a$

$351' \times \overline{351'} = 1 + 78 + 650 + 2430 + 34749 + 85293$

$650 \times 27 = 27 + 351 + 351' + 1728 + 7371 + \overline{7722}$

$650 \times 78 = 78 + 650_1 + 650_2 + 2925 + 5824 + \overline{5824} + 34749$

$650 \times 351 = 27 + 351_1 + 351_2 + 351' + 1728_1 + 1728_2 + 7371_1 + 7371_2 + 7722 + 17550 + 19305 + 51975 + 112320$

$650 \times 351' = 27 + 351 + 351' + 1728 + 7371 + 7722 + 17550 + 19305 + 61425 + 112320$

$650 \times 650 = 1_s + 78_s + 78_a + 650_{s1} + 650_{s2} + 650_a + 2430_s + 2925_{a1} + 2925_{a2} + 3003_s + \overline{3003_s} + 5824_s + \overline{5824_s} + 5824_a + \overline{5824_a} + 34749_s + 34749_a + 70070_s + 78975_a + \overline{78975_a} + 85293_s$

$\overline{1728} \times \overline{27} = 351 + 351' + 1728 + 7371 + 17550 + 19305$

$\overline{1728} \times 27 = 78 + 650 + 2430 + 2925 + 5824 + 34749$

$1728 \times 78 = 27 + 351 + 1728_1 + 1728_2 + 7371 + 7722 + 17550 + 46332 + 51975$

$\overline{1728} \times \overline{351} = 27 + 351' + 351_1 + 351_2 + 1728_1 + 1728_2 + 7371_1 + 7371_2 + 7722 + 17550_1 + 17550_2 + 19305 + 46332 + 51975 + 112320 + 314496$

$\overline{1728} \times \overline{1728}_a = 351_1 + 351_2 + 1728 + 7371_1 + 7371_2 + 17550 + 19305_1 + 19305_2 + 51975_1 + 51975_2 + 112320 + 314496 + 393822 + 494208$

$\overline{1728} \times \overline{1728}_s = 27 + 351'_1 + 351'_2 + 1728 + 7371_1 + 7371_2 + 7722 + 17550_1 + 17550_2 + 19305 + 34398 + 46332 + 61425 + 112320 + 314496 + 386100 + 459459$

Table 49
Branchings of E_6 representations

$E_6 \supset F_4$

$(100000) = 27 = 1 + 26$

$(000001) = 78 = 26 + 52$

$(000100) = 351 = 26 + 52 + 273$

$(000020) = 351' = 1 + 26 + 324$

$(100010) = 650' = 1 + 26_1 + 26_2 + 273 + 324$

$(100001) = 1728 = 26 + 52 + 273 + 324 + 1053$

$(000002) = 2430 = 324 + 1053 + 1053'$

$(001000) = 2925 = 52 + 273_1 + 273_2 + 1053 + 1274$

$E_6 \supset SO_{10} \times U_1$ (Value of U_1 generator in parenthesis)

$27 = 1(4) + 10(-2) + 16(1)$

$78 = 1(0) + 45(0) + 16(-3) + \overline{16}(3)$

$351 = 10(-2) + \overline{16}(-5) + 16(1) + 45(4) + 120(-2) + 144(1)$

$351' = 1(-8) + 10(-2) + \overline{16}(-5) + 54(4) + \overline{126}(-2) + 144(1)$

$650 = 1(0) + 10(6) + 10(-6) + 16(-3) + \overline{16}(3) + 45(0) + 54(0) + 144(-3) + \overline{144}(3) + 210(0)$

$1728 = 1(4) + 10(-2) + 16_1(1) + 16_2(1) + \overline{16}(7) + 45(4) + 120(-2) + 126(-2) + 144(1) + \overline{144}(-5) + 210(4) + 320(-2) + 560(1)$

$2430 = 1(0) + 16(-3) + \overline{16}(3) + 45(0) + 126(-6) + \overline{126}(6) + 210(0) + 560(-3) + \overline{560}(3) + 770(0)$

$2925 = 16(-3) + \overline{16}(3) + 45_1(0) + 45_2(0) + 120_1(6) + 120_2(-6) + 144(-3) + \overline{144}(3) + 210(0) + 560(-3) + \overline{560}(3) + 945(0)$

$E_6 \supset SU_2 \times SU_6$

$27 = (2, \bar{6}) + (1, 15)$

$78 = (3, 1) + (1, 35) + (2, 20)$

$351 = (2, \bar{6}) + (1, 21) + (3, 15) + (1, 105) + (2, \overline{84})$

$351' = (1, 15) + (3, 21) + (2, \overline{84}) + (1, 105')$

$650 = (1, 1) + (1, 35) + (2, 20) + (3, 35) + (2, 70) + (2, \overline{70}) + (1, 189)$

$1728 = (2, \bar{6}) + (1, 15) + (4, \bar{6}) + (3, 15) + (1, 105) + (2, \overline{84}) + (2, \overline{120}) + (3, 105) + (1, 384) + (2, \overline{210})$

$2430 = (1, 1) + (5, 1) + (2, 20) + (4, 20) + (3, 35) + (1, 189) + (1, 405) + (3, 175) + (2, 540)$

$2925 = (3, 1) + (1, 35) + (2, 20) + (3, 35) + (4, 20) + (2, 70) + (2, \overline{70}) + (1, 175) + (1, 280) + (1, \overline{280}) + (3, 189) + (2, 540)$

$E_6 \supset SU_3^{w} \times SU_3 \times SU_3^{c}$

$27 = (\bar{3}, 3, 1) + (3, 1, 3) + (1, \bar{3}, \bar{3})$

$78 = (8, 1, 1) + (1, 8, 1) + (1, 1, 8) + (3, 3, \bar{3}) + (\bar{3}, \bar{3}, 3)$

$351 = (\bar{3}, 3, 1) + (\bar{3}, \bar{6}, 1) + (6, 3, 1) + (3, 1, 3) + (\bar{6}, 1, 3) + (3, 8, 3) + (1, \bar{3}, \bar{3}) + (1, 6, \bar{3}) + (8, \bar{3}, \bar{3}) + (3, 1, \bar{6}) + (1, \bar{3}, 6) + (\bar{3}, 3, 8)$

$351' = (\bar{3}, 3, 1) + (6, \bar{6}, 1) + (3, 1, 3) + (3, 8, 3) + (1, \bar{3}, \bar{3}) + (8, \bar{3}, \bar{3}) + (\bar{6}, 1, \bar{6}) + (1, 6, 6) + (\bar{3}, 3, 8)$

$650 = (1, 1, 1) + (1, 1, 1) + (8, 1, 1) + (1, 8, 1) + (1, 1, 8) + (3, 3, \bar{3}) + (3, 3, \bar{3}) + (\bar{3}, \bar{3}, 3) + (\bar{3}, \bar{3}, 3) + (6, \bar{3}, 3) + (\bar{3}, 6, 3) + (\bar{6}, 3, \bar{3}) + (3, \bar{6}, \bar{3})$
$\quad + (\bar{3}, \bar{3}, \bar{6}) + (3, 3, 6) + (8, 8, 1) + (8, 1, 8) + (1, 8, 8)$

$1728 = (\bar{3}, 3, 1) + (\bar{3}, 3, 1) + (\bar{3}, \bar{6}, 1) + (6, 3, 1) + (\bar{3}, 15, 1) + (\overline{15}, 3, 1) + (3, 1, 3) + (3, 1, 3) + (\bar{6}, 1, 3) + (3, 8, 3) + (3, 8, 3) + (15, 1, 3) + (\bar{6}, 8, 3)$
$\quad + (1, \bar{3}, \bar{3}) + (1, \bar{3}, \bar{3}) + (1, 6, \bar{3}) + (8, \bar{3}, \bar{3}) + (8, \bar{3}, \bar{3}) + (1, \overline{15}, \bar{3}) + (8, 6, \bar{3}) + (3, 1, \bar{6}) + (3, 8, \bar{6}) + (1, \bar{3}, 6) + (8, \bar{3}, 6) + (\bar{3}, 3, 8) + (\bar{3}, 3, 8) + (\bar{3}, \bar{6}, 8)$
$\quad + (6, 3, 8) + (3, 1, 15) + (1, \bar{3}, \overline{15})$

Table 50

Guide to projection matrices for $E_6 \supset \cdots \supset U_1^{em} \times SU_3^c$. The U_1 factors may be found in table 18. The factor X in $P(X \supset Y)$ is chosen to be simple; it is underlined when ambiguity is possible

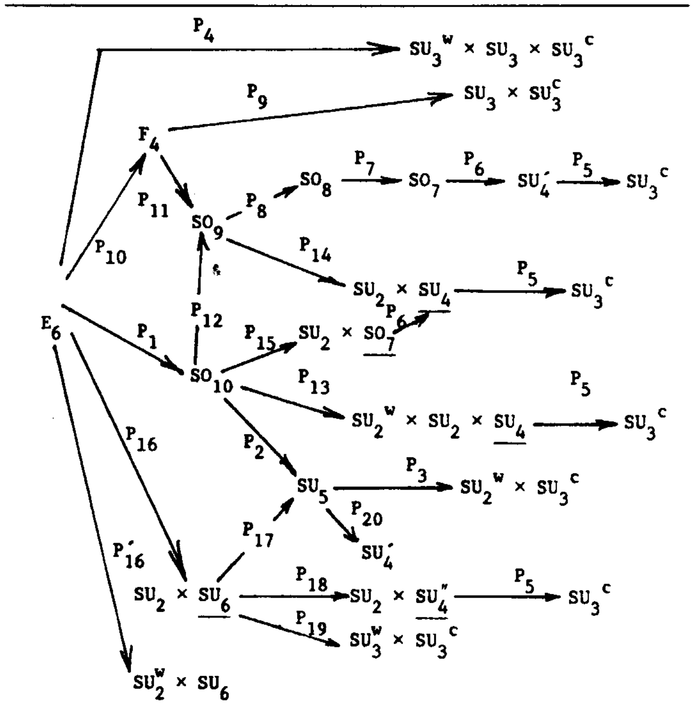


Table 51

Matrix elements of the simple root lowering operators for the 3, 6, 8 and 10 of SU_3. Internal lines in the octet each have value $1/\sqrt{2}$

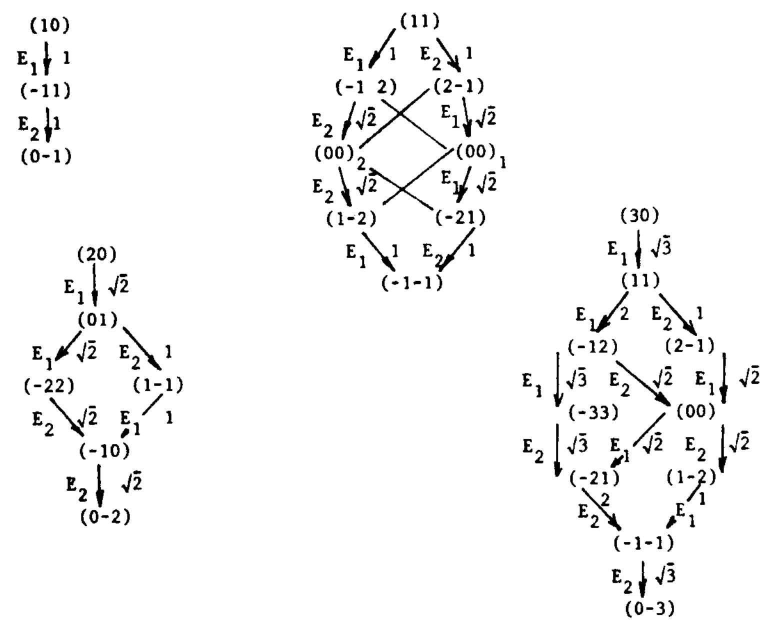


Table 52
Irreps, products and branching rules for E_7

Dynkin label	Dimension (name)	$l/12$ index	Branching into E_6 irreps U_1 factors suppressed
(0000010)	56	1	$1+1+27+\overline{27}$
(1000000)	133	3	$1+27+\overline{27}+78$
(0000001)	912	30	$27+\overline{27}+78+78+351+\overline{351}$
(0000020)	1463	55	$1+1+1+27+27+\overline{27}+\overline{27}+351'+\overline{351}'+650$
(0000100)	1539	54	$1+27+27+\overline{27}+\overline{27}+78+351+\overline{351}+650$
(1000010)	6480	270	Branching rules to other
(2000000)	7371	351	regular subgroups below.
(0100000)	8645	390	
(0000030)	24320	1440	
(0001000)	27664	1430	
(0000011)	40755	2145	
(0000110)	51072	2832	
(1000001)	86184	4995	

$56\times 56 = 1_a + 133_s + 1463_s + 1539_a$
$133\times 56 = 56+912+6480$
$133\times 133 = 1_s + 133_a + 1539_s + 7371_s + 8645_a$
$912\times 56 = 133+1539+8645+40755$
$912\times 133 = 56+912+6480+27664+86184$
$912\times 912 = 1_a + 133_s + 1463_s + 1539_a + 7371_a + 8645_s + 40755_a + 152152_s + 253935_s + 365750_a$
$1463\times 56 = 56+6480+24320+51072$
$1463\times 133 = 1463+1539+40755+150822$
$1463\times 912 = 912+6480+27664+51072+362880+885248$
$1463\times 1463 = 1_s + 133_a + 1463_a + 1539_s + 7371_s + 150822_s + 152152_a + 293930_s + 617253_s + 915705_a$
$1539\times 56 = 56+912+6480+27664+51072$
$1539\times 133 = 133+1463+1539+8645+40755+152152$
$1539\times 912 = 56+912+6480_1+6480_2+27664+51072+86184+362880+861840$

$E_7 \supset SU_8$
$(0000010) = 56 = 28+\overline{28}$
$(1000000) = 133 = 63+70$
$(0000001) = 912 = 36+\overline{36}+420+\overline{420}$
$(0000020) = 1463 = 1+70+336+\overline{336}+720$
$(0000100) = 1539 = 63+378+\overline{378}+720$

$E_7 \supset SU_2\times SO_{12}$
$56 = (2,12)+(1,32)$
$133 = (3,1)+(2,32')+(1,66)$
$912 = (2,12)+(3,32)+(1,352)+(2,220)$
$1463 = (1,66)+(3,77)+(1,462)+(2,352')$
$1539 = (1,1)+(2,32')+(1,77)+(3,66)+(1,495)+(2,352')$

$E_7 \supset SU_3\times SU_6$
$56 = (3,6)+(\bar{3},\bar{6})+(1,20)$
$133 = (8,1)+(1,35)+(3,\overline{15})+(\bar{3},15)$
$912 = (3,6)+(\bar{3},\bar{6})+(6,\bar{6})+(\bar{6},6)+(1,70)+(1,\overline{70})+(8,20)+(3,84)+(\bar{3},\overline{84})$
$1463 = (1,1)+(1,35)+(3,\overline{15})+(\bar{3},15)+(6,21)+(\bar{6},\overline{21})+(1,175)+(8,35)+(3,\overline{105})+(\bar{3},105)$
$1539 = (1,1)+(8,1)+(1,35)+(3,\overline{15})+(\bar{3},15)+(3,\overline{21})+(\bar{3},21)+(6,15)+(\bar{6},\overline{15})+(1,189)+(3,\overline{105})+(\bar{3},105)+(8,35)$

Table 53
Irreps, products and branching rules for E_8

Dynkin label	Dimension (name)	$l/60$ (index)
(00000010)	248	1
(10000000)	3875	25
(00000020)	27000	225
(00000100)	30380	245
(00000001)	147250	1425
(10000010)	779247	8379
(00000030)	1763125	22750
(00100000)	2450240	29640
(00000110)	4096000	51200
(20000000)	4881384	65610
(01000000)	6696000	88200
(00000011)	26411008	372736

$$248 \times 248 = 1_s + 248_a + 3875_s + 27000_s + 30380_a$$
$$3875 \times 248 = 248 + 3875 + 30380 + 147250 + 779247$$
$$3875 \times 3875 = 1_s + 248_a + 3875_s + 27000_s + 30380_a + 147250_s + 779247_a + 2450240_s + 4881384_s + 6696000_a$$

Branching rules to regular maximal subgroups; SO_{16} and SU_9 Dynkin labels given.

$E_8 \supset SO_{16}$
$248 = (01000000)120 + (00000001)128$
$3875 = (20000000)135 + (00010000)1820 + (10000010)1920$

$E_8 \supset SU_9$
$248 = (10000001)80 + (00100000)84 + (00000100)\overline{84}$
$3875 = (10000001)80 + (11000000)240 + (00000011)\overline{240} + (00010001)1050 + (10001000)\overline{1050} + (01000010)1215$

$E_8 \supset SU_2 \times E_7$
$248 = (3, 1) + (1, 133) + (2, 56)$
$3875 = (1, 1) + (2, 56) + (3, 133) + (1, 1539) + (2, 912)$

$E_8 \supset SU_3 \times E_6$
$248 = (8, 1) + (1, 78) + (3, 27) + (\bar{3}, \overline{27})$
$3875 = (1, 1) + (8, 1) + (3, 27) + (\bar{3}, \overline{27}) + (\bar{6}, 27) + (6, \overline{27}) + (8, 78) + (1, 650) + (3, 351) + (\bar{3}, \overline{351})$

$E_8 \supset SU_5 \times SU_5$
$248 = (1, 24) + (24, 1) + (5, \overline{10}) + (\bar{5}, 10) + (10, 5) + (\overline{10}, \bar{5})$
$3875 = (1, 1) + (1, 24) + (24, 1) + (5, \overline{10}) + (\bar{5}, 10) + (10, 5) + (\overline{10}, \bar{5}) + (1, 75) + (75, 1) + (5, \overline{15}) + (\bar{5}, 15) + (15, 5) + (\overline{15}, \bar{5}) + (5, \overline{40}) + (\bar{5}, 40) + (40, 5) + (\overline{40}, \bar{5})$
$+ (10, 45) + (\overline{10}, \overline{45}) + (45, \overline{10}) + (\overline{45}, 10) + (24, 24)$

Table 54
Irreps, products and branching rules for SU_8

Dynkin (name)	Dimension (name)	Octality	l (index)	Branching into SO_8 irreps
(1000000)	8	1	1	
(0100000)	28	2	6	28
(2000000)	36	2	10	$1+35_v$
(0010000)	56	3	15	56_v
(1000001)	63	0	16	$28+35_v$
(0001000)	70	4	20	35_s+35_c
(3000000)	120	3	55	8_v+112_v
(1100000)	168	3	61	8_v+160_v
(0100001)	216	1	75	56_v+160_v
(2000001)	280	1	115	$8_v+112_v+160_v$
(4000000)	330	4	220	$1+35_v+294_v$
(0200000)	336	4	160	$1+35_v+300$
(1010000)	378	4	156	$28+350$
(0010001)	420	2	170	35_s+35_c+350
(0001001)	504	3	215	$56_v+224_{sv}+224_{cv}$
(2100000)	630	4	340	$28+35_v+567_v$
(0100010)	720	0	320	$35_s+35_c+300+350$
(0000005)	792	3	715	$8_v+112_v+672'_v$
(3000001)	924	2	550	$28+35_v+294_v+567_v$
(2000010)	945	0	480	$28+350+567_v$
(0000110)	1008	3	526	$8_v+160_v+840_v$
(0000200)	1176	2	700	$1+35_v+300+840'_v$

(Note that the projection of 8 to 8_v is a convention and may be changed to 8 to 8_s or 8 to 8_c.)

$8\times 8=28_a+36_s$
$\overline{8}\times 8=1+63$
$28\times 8=56+168$
$28\times\overline{8}=8+216$
$28\times 28=70_s+336_s+378_a$
$\overline{28}\times 28=1+63+720$
$36\times 8=120+168$
$36\times\overline{8}=8+280$
$36\times 28=378+630$
$36\times\overline{28}=63+945$
$36\times 36=330_s+336_s+630_a$
$\overline{36}\times 36=1+63+1232$
$56\times 8=70+278$
$56\times\overline{8}=28+420$
$\overline{56}\times\overline{28}=56+504+1008$
$56\times\overline{28}=8+216+1344$
$\overline{56}\times\overline{36}=504+1512'$
$56\times\overline{36}=216+1800$
$\overline{56}\times\overline{56}=28_a+420_s+1176_s+1512_a$
$\overline{56}\times 56=1+63+720+2352$
$63\times 8=8+216+280$
$63\times 28=28+36+420+1280$
$63\times 36=28+36+924+1280$
$63\times 56=56+168+504+2800$
$63\times 63=1_s+63_s+63_a+720_s+945_a+\overline{945}_a+1232_s$

Branching rules to $SU_3\times SU_5\times U_1$ irreps; U_1 generator in parentheses:
$(1000000)=8=(3,1)(-5)+(1,5)(3)$
$(0100000)=28=(\overline{3},1)(-10)+(1,10)(6)+(3,5)(-2)$

Table 54 (continued)

(2000000) = 36 = (6, 1)(−10) + (1, 15)(6) + (3, 5)(−2)
(0010000) = 56 = (1, 1)(−15) + (1, $\overline{10}$)(9) + ($\bar{3}$, 5)(−7) + (3, 10)(1)
(1000001) = 63 = (1, 1)(0) + (8, 1)(0) + (3, $\bar{5}$)(−8) + ($\bar{3}$, 5)(8) + (1, 24)(0)
(0001000) = 70 = (1, 5)(−12) + (1, $\bar{5}$)(12) + (3, $\overline{10}$)(4) + ($\bar{3}$, 10)(−4)
(3000000) = 120 = (10, 1)(−15) + ($\bar{6}$, 5)(−7) + (3, 15)(1) + (1, $\overline{35}$)(9)
(1100000) = 168 = (8, 1)(−15) + ($\bar{3}$, 5)(−7) + (6, 5)(−7) + (3, 10)(1) + (1, $\overline{40}$)(9) + (3, 15)(1)
(0100001) = 216 = (3, 1)(−5) + (1, 5)(3) + ($\bar{6}$, 1)(−5) + ($\bar{3}$, $\bar{5}$)(−13) + ($\bar{3}$, 10)(11) + (8, 5)(3) + (1, 45)(3) + (3, 24)(−5)
(2000001) = 280 = (3, 1)(−5) + (1, 5)(3) + (15, 1)(−5) + (6, $\bar{5}$)(−13) + (8, 5)(3) + (3, 24)(−5) + ($\bar{3}$, 15)(11) + (1, 70)(3)
(4000000) = 330 = (15′, 1)(−20) + (10, 5)(−12) + (1, $\overline{70}'$)(12) + (6, 15)(−4) + (3, $\overline{35}$)(4)
(0200000) = 336 = ($\bar{6}$, 1)(−20) + ($\bar{3}$, 10)(−4) + (8, 5)(−12) + (1, $\overline{50}$)(12) + (3, $\overline{40}$)(4) + (6, 15)(−4)
(1010000) = 378 = (3, 1)(−20) + (1, 5)(−12) + (3, $\overline{10}$)(4) + ($\bar{3}$, 10)(−4) + (8, 5)(−12) + (1, $\overline{45}$)(12) + ($\bar{3}$, 15)(−4) + (6, 10)(−4) + (3, $\overline{40}$)(4)
(0010001) = 420 = ($\bar{3}$, 1)(−10) + (1, $\bar{5}$)(−18) + (1, 10)(6) + (3, 5)(−2) + ($\bar{6}$, 5)(−2) + ($\bar{3}$, $\overline{10}$)(14) + (1, 40)(6) + ($\bar{3}$, 24)(−10) + (8, 10)(6) + (3, 45)(−2)
(0001001) = 504 = (1, $\overline{10}$)(9) + ($\bar{3}$, 5)(−7) + ($\bar{3}$, $\bar{5}$)(17) + (1, $\overline{15}$)(9) + (1, 24)(−15) + (3, 10)(1) + ($\bar{6}$, 10)(1) + (8, $\overline{10}$)(9) + (3, 40)(1) + ($\bar{3}$, 45)(−7)

Table 55
Irreps, products and branching rules for SO_{14}

Irreps and $SO_{14} \supset SU_2 \times SU_2 \times SO_{10}$ branching rules:
(1000000) = 14 = (2, 2, 1) + (1, 1, 10)
(0100000) = 91 = (3, 1, 1) + (1, 3, 1) + (1, 1, 45) + (2, 2, 10)
(0010000) = 364 = (2, 2, 1) + (3, 1, 10) + (1, 3, 10) + (1, 1, 120) + (2, 2, 45)
(0001000) = 1001 = (1, 1, 1) + (2, 2, 10) + (3, 1, 45) + (1, 3, 45) + (1, 1, 210) + (2, 2, 120)
(0000100) = 2002 = (1, 1, 10) + (1, 1, 126) + (1, 1, $\overline{126}$) + (2, 2, 45) + (3, 1, 120) + (1, 3, 120) + (2, 2, 210)
(0000011) = 3003 = (1, 1, 45) + (1, 1, 120) + (2, 2, 120) + (3, 1, 210) + (1, 3, 210) + (2, 2, 126) + (2, 2, $\overline{126}$)
(0000002) = 1716 = (1, 1, 120) + (3, 1, 126) + (1, 3, $\overline{126}$) + (2, 2, 210)
(0000001) = 64 = (2, 1, 16) + (1, 2, $\overline{16}$)

Products of spinors:
$64 \times 64 = 14_a + 364_s + 1716_s + 2002_a$
$64 \times \overline{64} = 1 + 91 + 1001 + 3003$

Table 56
Irreps, products and branching rules for SO_{18}

Irreps and $SO_{18} \supset SO_8 \times SO_{10}$ branching rules:
(100000000) = 18 = (8_v, 1) + (1, 10)
(010000000) = 153 = (28, 1) + (1, 45) + (8_v, 10)
(010000000) = 816 = (56_v, 1) + (1, 120) + (28, 10) + (8_v, 45)
(000100000) = 3060 = (35_s, 1) + (35_c, 1) + (1, 210) + (56_v, 10) + (28, 45) + (8_v, 120)
(000010000) = 8568 = (56_v, 1) + (1, 126) + (1, $\overline{126}$) + (35_s, 10) + (35_c, 10) + (56_v, 45) + (28, 120) + (8_v, 210)
(000001000) = 18564 = (28, 1) + (1, 210) + (56_v, 10) + (8_v, 126) + (8_v, $\overline{126}$) + (35_s, 45) + (35_c, 45) + (28, 210) + (56_v, 120)
(000000100) = 31824 = (8_v, 1) + (1, 120) + (28, 10) + (8_v, 210) + (28, 126) + (28, $\overline{126}$) + (56_v, 45) + (56_v, 210) + (35_s, 120) + (35_c, 120)
(000000011) = 43758 = (1, 1) + (1, 45) + (8_v, 10) + (8_v, 120) + (28, 45) + (28, 210) + (56_v, 120) + (56_v, 126) + (56_v, $\overline{126}$) + (35_s, 210) + (35_c, 210)
(000000002) = 24310 = (1, 10) + (8_v, 45) + (28, 120) + (56_v, 210) + (35_s, 126) + (35_c, $\overline{126}$)
(000000001) = 256 = (8_s, 16) + (8_c, $\overline{16}$)

Products of spinors:
$256 \times 256 = 18_s + 816_a + 8568_s + 31824_a + 24310_s$
$256 \times \overline{256} = 1 + 153 + 3060 + 18564 + 43758$

Table 57
Irreps, products and branching rules for SO_{22}

Irreps and $SO_{22} \supset SO_{12} \times SO_{10}$ branching rules:
(10000000000) = 22 = (1, 10) + (12, 1)
(01000000000) = 231 = (1, 45) + (66, 1) + (12, 10)
(00100000000) = 1540 = (1, 210) + (220, 1) + (66, 10) + (12, 45)
(00010000000) = 7315 = (1, 210) + (495, 1) + (12, 120) + (220, 10) + (66, 45)
(00001000000) = 26334 = (1, 126) + (1, $\overline{126}$) + (792, 1) + (12, 210) + (495, 10) + (66, 120) + (220, 45)
(00000100000) = 74613 = (1, 210) + (462, 1) + (462′, 1) + (12, 126) + (12, $\overline{126}$) + (792, 10) + (66, 210) + (495, 45) + (220, 120)
(00000010000) = 170544 = (1, 120) + (792, 1) + (12, 210) + (462, 10) + (462′, 10) + (66, 126) + (66, $\overline{126}$) + (792, 45) + (220, 210) + (495, 120)
(00000001000) = 319770 = (1, 45) + (495, 1) + (12, 120) + (792, 10) + (66, 210) + (462, 45) + (462′, 45) + (220, 126) + (220, $\overline{126}$) + (792, 120) + (495, 210)
(00000000100) = 497420 = (1, 10) + (220, 1) + (12, 45) + (495, 10) + (66, 120) + (792, 45) + (220, 210) + (462, 120) + (462′, 120) + (495, 126) + (495, $\overline{126}$) + (792, 210)
(00000000011) = 646646 = (1, 1) + (12, 10) + (66, 1) + (66, 45) + (220, 10) + (220, 120) + (495, 45) + (495, 210) + (792, 120) + (792, 126) + (792, $\overline{126}$) + (462, 210) + (462′, 210)
(00000000002) = 352716 = (12, 1) + (66, 10) + (220, 45) + (495, 120) + (792, 210) + (462, 126) + (462′, $\overline{126}$)
(00000000001) = 1024 = (32, 16) + (32′, $\overline{16}$)

Products of spinors:
$1024 \times 1024 = 22_a + 1540_s + 26334_a + 170544_s + 352716_s + 497420_a$
$1024 \times \overline{1024} = 1 + 231 + 7315 + 74613 + 319770 + 646646$

Table 58
Branching rules to all maximal subgroups

This table is designed to facilitate analyses such as the search for maximal little groups, and also it represents a summary of the group theory aspects of the review. The branching rules of a few low-lying irreps to the irreps of *every* maximal subgroup are listed for simple groups up to rank 6. Many results repeat those in tables 14, 15, and the branching rule tables, but here there is no restriction to subgroups that can contain flavor and color. When this table or the preceding ones are insufficient, the reader should refer to the much longer tables of ref. [57], although in many practical cases a quick calculation based on the results of this table will fill in the missing information. The format is to give both the Dynkin designation and the dimensionality (name) as (Dynkin)*r*, except when the subgroup is SU_2, $SU_2 \times SU_2$, or more products of SU_2's, in which case only the dimensionality is listed. The eigenvalues of the U_1 generator, when relevant, are given in parentheses after the irrep names, and are normalized to be integers.

Rank 2: $SU_3 \supset SU_2 \times U_1$ (R)
(10)3 = 1(−2) + 2(1)
(20)6 = 1(−4) + 2(−1) + 3(2)
(11)8 = 1(0) + 2(3) + 2(−3) + 3(0)

$SU_3 \supset SU_2$ (S)
(10)3 = 3
(20)6 = 1 + 5
(11)8 = 3 + 5

$Sp_4 \supset SU_2 \times SU_2$ (R)
(10)4 = (2, 1) + (1, 2)
(01)5 = (1, 1) + (2, 2)
(20)10 = (3, 1) + (1, 3) + (2, 2)

$Sp_4 \supset SU_2 \times U_1$ (R)
(10)4 = 2(1) + 2(−1)
(01)5 = 1(2) + 1(−2) + 3(0)
(20)10 = 1(0) + 3(0) + 3(2) + 3(−2)

Table 58 (continued)

$Sp_4 \supset SU_2$ (S)
(10)4 = 4
(01)5 = 5
(20)10 = 3 + 7

$G_2 \supset SU_3$ (R)
(01)7 = (00)1 + (10)3 + (01)$\bar{3}$
(10)14 = (10)3 + (01)$\bar{3}$ + (11)8

$G_2 \supset SU_2 \times SU_2$ (R)
(01)7 = (1, 3) + (2, 2)
(10)14 = (1, 3) + (3, 1) + (2, 4)

$G_2 \supset SU_2$ (S)
(01)7 = 7
(10)14 = 3 + 11

Rank 3: $SU_4 \supset SU_3 \times U_1$ (R)
(100)4 = (00)1(3) + (10)3(−1)
(010)6 = (10)3(2) + (01)$\bar{3}$(−2)
(101)15 = (00)1(0) + (10)3(−4) + (01)$\bar{3}$(4) + (11)8(0)

$SU_4 \supset SU_2 \times SU_2 \times U_1$ (R)
(100)4 = (2, 1)(1) + (1, 2)(−1)
(010)6 = (1, 1)(2) + (1, 1)(−2) + (2, 2)(0)
(101)15 = (1, 1)(0) + (3, 1)(0) + (1, 3)(0) + (2, 2)(2) + (2, 2)(−2)

$SU_4 \supset Sp_4$ (S)
(100)4 = (10)4
(010)6 = (00)1 + (01)5
(101)15 = (01)5 + (20)10

$SU_4 \supset SU_2 \times SU_2$ (S)
(100)4 = (2, 2)
(010)6 = (1, 3) + (3, 1)
(101)15 = (1, 3) + (3, 1) + (3, 3)

$SO_7 \supset SU_4$ (R)
(100)7 = (000)1 + (010)6
(001)8 = (100)4 + (001)$\bar{4}$
(010)21 = (010)6 + (101)15

$SO_7 \supset SU_2 \times SU_2 \times SU_2$ (R)
(100)7 = (1, 1, 3) + (2, 2, 1)
(001)8 = (1, 2, 2) + (2, 1, 2)
(010)21 = (1, 1, 3) + (1, 3, 1) + (3, 1, 1) + (2, 2, 3)

$SO_7 \supset Sp_4 \times U_1$ (R)
(100)7 = (00)1(2) + (00)1(−2) + (01)5(0)
(001)8 = (10)4(1) + (10)4(−1)
(010)21 = (00)1(0) + (01)5(2) + (01)5(−2) + (20)10(0)

$SO_7 \supset G_2$ (S)
(100)7 = (01)7
(001)8 = (00)1 + (01)7
(010)21 = (01)7 + (10)14

Table 58 (continued)

$Sp_6 \supset SU_3 \times U_1$ (R)
(100)6 = (10)3(1) + (01)$\bar{3}$(−1)
(010)14 = (10)3(−2) + (01)$\bar{3}$(2) + (11)8(0)
(001)14′ = (00)1(3) + (00)1(−3) + (20)6(−1) + (02)$\bar{6}$(1)
(200)21 = (00)1(0) + (20)6(2) + (02)$\bar{6}$(−2) + (11)8(0)

$Sp_6 \supset SU_2 \times Sp_4$(R)
(100)6 = (1)(00)(2, 1) + (0)(10)(1, 4)
(010)14 = (0)(00)(1, 1) + (0)(01)(1, 5) + (1)(10)(2, 4)
(001)14′ = (0)(10)(1, 4) + (1)(01)(2, 5)
(200)21 = (2)(0)(3, 1) + (0)(20)(1, 10) + (1)(10)(2, 4)

$Sp_6 \supset SU_2$ (S)
(100)6 = 6
(010)14 = 5 + 9
(001)14′ = 4 + 10
(200)21 = 3 + 7 + 11

$Sp_6 \supset SU_2 \times SU_2$ (S)
(100)6 = (2, 3)
(010)14 = (1, 5) + (3, 3)
(001)14′ = (4, 1) + (2, 5)
(200)21 = (1, 3) + (3, 1) + (3, 5)

Rank 4: $SU_5 \supset SU_4 \times U_1$ (R)
(1000)5 = (000)1(4) + (100)4(−1)
(0100)10 = (100)4(3) + (010)6(−2)
(1001)24 = (000)1(0) + (100)4(−5) + (001)$\bar{4}$(5) + (101)15(0)

$SU_5 \supset SU_2 \times SU_3 \times U_1$ (R)
(1000)5 = (1)(00)(2, 1)(3) + (0)(10)(1, 3)(−2)
(0100)10 = (0)(00)(1, 1)(6) + (0)(01)(1, $\bar{3}$)(−4) + (1)(10)(2, 3)(1)
(1001)24 = (0)(00)(1, 1)(0) + (2)(00)(3, 1)(0) + (1)(10)(2, 3)(−5) + (1)(01)(2, $\bar{3}$)(5) + (0)(11)(1, 8)(0)

$SU_5 \supset Sp_4$ (S)
(1000)5 = (01)5
(0100)10 = (20)10
(1001)24 = (20)10 + (02)14

$SO_9 \supset SO_8$(R)
(1000)9 = (0000)1 + (1000)8_v
(0001)16 = (0010)8_c + (0001)8_s
(0100)36 = (1000)8_v + (0100)28

$SO_9 \supset SU_2 \times SU_2 \times Sp_4$ (R)
(1000)9 = (1)(1)(00)(2, 2, 1) + (0)(0)(01)(1, 1, 5)
(0001)16 = (0)(1)(10)(1, 2, 4) + (1)(0)(10)(2, 1, 4)
(0100)36 = (2)(0)(00)(3, 1, 1) + (0)(2)(00)(1, 3, 1) + (0)(0)(20)(1, 1, 10) + (1)(1)(01)(2, 2, 5)

$SO_9 \supset SU_2 \times SU_4$ (R)
(1000)9 = (2)(000)(3, 1) + (0)(010)(1, 6)
(0001)16 = (1)(100)(2, 4) + (1)(001)(2, $\bar{4}$)
(0100)36 = (2)(000)(3, 1) + (0)(101)(1, 15) + (2)(010)(3, 6)

Table 58 (continued)

$SO_9 \supset SO_7 \times U_1$ (R)
(1000)9 = (000)1(2) + (000)1(−2) + (100)7(0)
(0001)16 = (001)8(1) + (001)8(−1)
(0100)36 = (000)1(0) + (100)7(2) + (100)7(−2) + (010)21(0)

$SO_9 \supset SU_2$ (S)
(1000)9 = 9
(0001)16 = 5 + 11
(0100)36 = 3 + 7 + 11 + 15

$SO_9 \supset SU_2 \times SU_2$ (S)
(1000)9 = (3, 3)
(0001)16 = (2, 4) + (4, 2)
(0100)36 = (1, 3) + (3, 1) + (3, 5) + (5, 3)

$Sp_8 \supset SU_4 \times U_1$ (R)
(1000)8 = (100)4(1) + (001)$\bar{4}$(−1)
(2000)36 = (000)1(0) + (200)10(2) + (002)$\overline{10}$(−2) + (101)15(0)
(0001)42 = (000)1(4) + (000)1(−4) + (200)10(−2) + (002)$\overline{10}$(2) + (020)20′(0)

$Sp_8 \supset SU_2 \times Sp_6$ (R)
(1000)8 = (1)(000)(2, 1) + (0)(100)(1, 6)
(2000)36 = (2)(000)(3, 1) + (0)(200)(1, 21) + (1)(100)(2, 6)
(0001)42 = (0)(010)(1, 14) + (1)(001)(2, 14′)

$Sp_8 \supset Sp_4 \times Sp_4$ (R)
(1000)8 = (00)(10)(1, 4) + (10)(00)(4, 1)
(2000)36 = (00)(20)(1, 10) + (20)(00)(10, 1) + (10)(10)(4, 4)
(0001)42 = (00)(00)(1, 1) + (10)(10)(4, 4) + (01)(01)(5, 5)

$Sp_8 \supset SU_2$ (S)
(1000)8 = 8
(2000)36 = 3 + 7 + 11 + 15
(0001)42 = 5 + 9 + 11 + 17

$Sp_8 \supset SU_2 \times SU_2 \times SU_2$ (S)
(1000)8 = (2, 2, 2)
(2000)36 = (1, 1, 3) + (1, 3, 1) + (3, 1, 1) + (3, 3, 3)
(0001)42 = (1, 1, 5) + (1, 5, 1) + (5, 1, 1) + (3, 3, 3)

$SO_8 \supset SU_2 \times SU_2 \times SU_2 \times SU_2$ (R)
(1000)8_v = (2, 2, 1, 1) + (1, 1, 2, 2)
(0001)8_s = (1, 2, 1, 2) + (2, 1, 2, 1)
(0010)8_c = (1, 2, 2, 1) + (2, 1, 1, 2)
(0100)28 = (1, 1, 1, 3) + (1, 1, 3, 1) + (1, 3, 1, 1) + (3, 1, 1, 1) + (2, 2, 2, 2)

$SO_8 \supset SU_4 \times U_1$ (R)
(1000)8_v = (100)4(1) + (001)$\bar{4}$(−1)
(0001)8_s = (000)1(2) + (000)1(−2) + (010)6(0)
(0010)8_c = (100)4(−1) + (001)$\bar{4}$(1)
(0100)28 = (000)1(0) + (010)6(2) + (010)6(−2) + (101)15(0)

$SO_8 \supset SU_3$ (S)
(1000)8_v = (11)8
(0001)8_s = (11)8
(0010)8_c = (11)8
(0100)28 = (11)8 + (30)10 + (03)$\overline{10}$

Table 58 (continued)

$SO_8 \supset SO_7$ (S)
(1000)8_v = (001)8
(0001)8_s = (000)1 + (100)7
(0010)8_c = (001)8
(0100)28 = (100)7 + (010)21

$SO_8 \supset SU_2 \times Sp_4$ (S)
(1000)8_v = (1)(10)(2, 4)
(0010)8_c = (1)(10)(2, 4)
(0001)8_s = (0)(01)(1, 5) + (2)(00)(3, 1)
(0100)28 = (2)(00)(3, 1) + (0)(20)(1, 10) + (2)(01)(3, 5)

$F_4 \supset SO_9$ (R)
(0001)26 = (0000)1 + (1000)9 + (0001)16
(1000)52 = (0001)16 + (0100)36

$F_4 \supset SU_3 \times SU_3$ (R)
(0001)26 = (11)(00)(8, 1) + (10)(10)(3, 3) + (01)(01)($\bar{3}$, $\bar{3}$)
(1000)52 = (11)(00)(8, 1) + (00)(11)(1, 8) + (20)(01)(6, $\bar{3}$) + (02)(10)($\bar{6}$, 3)

$F_4 \supset SU_2 \times Sp_6$ (R)
(0001)26 = (1)(100)(2, 6) + (0)(010)(2, 14)
(1000)52 = (2)(000)(3, 1) + (0)(200)(1, 21) + (1)(001)(2, 14′)

$F_4 \supset SU_2$ (S)
(0001)26 = 9 + 17
(1000)52 = 3 + 11 + 15 + 23

$F_4 \supset SU_2 \times G_2$ (S)
(0001)26 = (4)(00)(5, 1) + (2)(01)(3, 7)
(1000)52 = (2)(00)(3, 1) + (0)(10)(1, 14) + (4)(01)(5, 7)

Rank 5: $SU_6 \supset SU_5 \times U_1$ (R)
(10000)6 = (0000)1(−5) + (1000)5(1)
(00100)20 = (0100)10(−3) + (0010)$\overline{10}$(3)
(10001)35 = (0000)1(0) + (1000)5(6) + (0001)$\bar{5}$(−6) + (1001)24(0)

$SU_6 \supset SU_2 \times SU_4 \times U_1$ (R)
(10000)6 = (1)(000)(2, 1)(2) + (0)(100)(1, 4)(−1)
(00100)20 = (0)(100)(1, 4)(3) + (0)(001)(1, $\bar{4}$)(−3) + (1)(010)(2, 6)(0)
(10001)35 = (0)(000)(1, 1)(0) + (2)(000)(3, 1)(0) + (0)(101)(1, 15)(0) + (1)(100)(2, 4)(−3) + (1)(001)(2, $\bar{4}$)(3)

$SU_6 \supset SU_3 \times SU_3 \times U_1$ (R)
(10000)6 = (00)(10)(1, 3)(−1) + (10)(00)(3, 1)(1)
(00100)20 = (00)(00)(1, 1)(3) + (00)(00)(1, 1)(−3) + (10)(01)(3, $\bar{3}$)(−1) + (01)(10)($\bar{3}$, 3)(1)
(10001)35 = (00)(00)(1, 1)(0) + (00)(11)(1, 8)(0) + (11)(00)(8, 1)(0) + (10)(01)(3, $\bar{3}$)(2) + (01)(10)($\bar{3}$)(−2)

$SU_6 \supset SU_3$ (S)
(10000)6 = (20)6
(00100)20 = (30)10 + (03)$\overline{10}$
(10001)35 = (11)8 + (22)27

$SU_6 \supset SU_4$ (S)
(10000)6 = (010)6
(00100)20 = (200)10 + (002)$\overline{10}$
(10001)35 = (101)15 + (020)20′

Table 58 (continued)

$SU_6 \supset Sp_6$ (S)

(10000)6 = (100)6

(00100)20 = (100)6 + (001)14′

(10001)35 = (010)14 + (200)21

$SU_6 \supset SU_2 \times SU_3$ (S)

(10000)6 = (1)(10)(2, 3)

(00100)20 = (3)(00)(4, 1) + (1)(11)(2, 8)

(10001)35 = (2)(00)(3, 1) + (0)(11)(1, 8) + (2)(11)(3, 8)

$SO_{11} \supset SO_{10}$ (R)

(10000)11 = (00000)1 + (10000)10

(00001)32 = (00001)16 + (00010)$\overline{16}$

(01000)55 = (10000)10 + (01000)45

$SO_{11} \supset SU_2 \times SO_8$ (R)

(10000)11 = (2)(0000)(3, 1) + (0)(1000)(1, 8_v)

(00001)32 = (1)(0001)(2, 8_s) + (1)(0010)(2, 8_c)

(01000)55 = (2)(0000)(3, 1) + (0)(0100)(1, 28) + (2)(1000)(3, 8_v)

$SO_{11} \supset Sp_4 \times SU_4$ (R)

(10000)11 = (01)(000)(5, 1) + (00)(010)(1, 6)

(00001)32 = (10)(100)(4, 4) + (10)(001)(4, $\bar{4}$)

(01000)55 = (20)(000)(10, 1) + (00)(101)(1, 15) + (01)(010)(5, 6)

$SO_{11} \supset SU_2 \times SU_2 \times SO_7$ (R)

(10000)11 = (1)(1)(000)(2, 2, 1) + (0)(0)(100)(1, 1, 7)

(00001)32 = (0)(1)(001)(1, 2, 8) + (1)(0)(001)(2, 1, 8)

(01000)55 = (2)(0)(000)(3, 1, 1) + (0)(2)(000)(1, 3, 1) + (0)(0)(010)(1, 1, 21) + (1)(1)(100)(2, 2, 7)

$SO_{11} \supset SO_9 \times U_1$ (R)

(10000)11 = (0000)1(2) + (0000)1(−2) + (1000)9(0)

(00001)32 = (0001)16(1) + (0001)16(−1)

(01000)55 = (0000)1(0) + (1000)9(2) + (1000)9(−2) + (0100)36(0)

$SO_{11} \supset SU_2$ (S)

(10000)11 = 11

(00001)32 = 6 + 10 + 16

(01000)55 = 3 + 7 + 11 + 15 + 19

$Sp_{10} \supset SU_5 \times U_1$ (R)

(10000)10 = (1000)5(1) + (0001)$\bar{5}$(−1)

(20000)55 = (0000)1(0) + (2000)15(2) + (0002)$\overline{15}$(−2) + (1001)24(0)

$Sp_{10} \supset SU_2 \times Sp_8$ (R)

(10000)10 = (1)(0000)(2, 1) + (0)(1000)(1, 8)

(20000)55 = (2)(0000)(3, 1) + (0)(2000)(1, 36) + (1)(1000)(2, 8)

$Sp_{10} \supset Sp_4 \times Sp_6$ (R)

(10000)10 = (10)(000)(4, 1) + (00)(100)(1, 6)

(20000)55 = (20)(000)(10, 1) + (00)(200)(1, 21) + (10)(100)(4, 6)

$Sp_{10} \supset SU_2$ (S)

(10000)10 = 10

(20000)55 = 3 + 7 + 11 + 15 + 19

Table 58 (continued)

$Sp_{10} \supset SU_2 \times Sp_4$ (S)
(10000)10 = (1)(01)(2, 5)
(20000)55 = (2)(00)(3, 1) + (0)(20)(1, 10) + (2)(02)(3, 14)

$SO_{10} \supset SU_5 \times U_1$ (R)
(10000)10 = (1000)5(2) + (0001)$\bar{5}$(−2)
(00001)16 = (0000)1(−5) + (0001)$\bar{5}$(3) + (0100)10(−1)
(01000)45 = (0000)1(0) + (0100)10(4) + (0010)$\overline{10}$(−4) + (1001)24(0)

$SO_{10} \supset SU_2 \times SU_2 \times SU_4$ (R)
(10000)10 = (1)(1)(000)(2, 2, 1) + (0)(0)(010)(1, 1, 6)
(00001)16 = (1)(0)(100)(2, 1, 4) + (0)(1)(001)(1, 2, $\bar{4}$)
(01000)45 = (2)(0)(000)(3, 1, 1) + (0)(2)(000)(1, 3, 1) + (0)(0)(101)(1, 1, 15) + (1)(1)(010)(2, 2, 6)

$SO_{10} \supset SO_8 \times U_1$ (R)
(10000)10 = (0000)1(2) + (0000)1(−2) + (1000)8_v(0)
(00001)16 = (0010)8_c(1) + (0001)8_s(−1)
(01000)45 = (0000)1(0) + (1000)8_v(2) + (1000)8_v(−2) + (0100)28(0)

$SO_{10} \supset Sp_4$ (S)
(10000)10 = (20)10
(00001)16 = (11)16
(01000)45 = (20)10 + (21)35

$SO_{10} \supset SO_9$(S)
(10000)10 = (0000)1 + (1000)9
(00001)16 = (0001)16
(01000)45 = (1000)9 + (0100)36

$SO_{10} \supset SU_2 \times SO_7$ (S)
(10000)10 = (2)(000)(3, 1) + (0)(100)(1, 7)
(00001)16 = (1)(001)(2, 8)
(01000)45 = (2)(000)(3, 1) + (0)(010)(1, 21) + (2)(100)(3, 7)

$SO_{10} \supset Sp_4 \times Sp_4$ (S)
(10000)10 = (00)(01)(1, 5) + (01)(00)(5, 1)
(00001)16 = (10)(10)(4, 4)
(01000)45 = (00)(20)(1, 10) + (20)(00)(10, 1) + (01)(01)(5, 5)

<u>Rank 6:</u> $SU_7 \supset SU_6 \times U_1$ (R)
(100000)7 = (00000)1(6) + (10000)6(−1)
(100001)48 = (00000)1(0) + (10001)35(0) + (10000)6(−7) + (00001)$\bar{6}$(7)

$SU_7 \supset SU_2 \times SU_5 \times U_1$ (R)
(100000)7 = (1)(0000)(2, 1)(5) + (0)(1000)(1, 5)(−2)
(100001)48 = (0)(0000)1(0) + (2)(0000)(3, 1)(0) + (0)(1001)(1, 24)(0) + (1)(1000)(2, 5)(−7) + (1)(0001)(2, $\bar{5}$)(7)

$SU_7 \supset SU_3 \times SU_4 \times U_1$ (R)
(100000)7 = (10)(000)(3, 1)(4) + (00)(100)(1, 4)(−3)
(100001)48 = (00)(000)(1, 1)(0) + (11)(000)(8, 1)(0) + (00)(101)(1, 15)(0) + (10)(001)(3, $\bar{4}$)(7) + (01)(100)($\bar{3}$, 4)(−7)

$SU_7 \supset SO_7$ (S)
(100000)7 = (100)7
(100001)48 = (010)21 + (200)27

Table 58 (continued)

$SO_{13} \supset SO_{12}$ (R)
(100000)13 = (000000)1 + (100000)12
(000001)64 = (000001)32 + (000010)32′
(010000)78 = (100000)12 + (010000)66

$SO_{13} \supset SU_2 \times SO_{10}$ (R)
(100000)13 = (2)(00000)(3, 1) + (0)(10000)(1, 10)
(000001)64 = (1)(00001)(2, 16) + (1)(00010)$(2, \overline{16})$
(010000)78 = (2)(00000)(3, 1) + (0)(01000)(1, 45) + (2)(10000)(3, 10)

$SO_{13} \supset Sp_4 \times SO_8$ (R)
(100000)13 = (01)(0000)(5, 1) + (00)(1000)$(8_v, 1)$
(000001)64 = (10)(0001)$(4, 8_s)$ + (10)(0010)$(4, 8_c)$
(010000)78 = (20)(0000)(10, 1) + (00)(0100)(1, 28) + (01)(1000)$(5, 8_v)$

$SO_{13} \supset SU_4 \times SO_7$ (R)
(100000)13 = (010)(000)(6, 1) + (000)(100)(1, 7)
(000001)64 = (100)(001)(4, 8) + (001)(001)$(\bar{4}, 8)$
(010000)78 = (000)(010)(1, 21) + (101)(000)(15, 1) + (010)(100)(6, 7)

$SO_{13} \supset SU_2 \times SU_2 \times SO_9$ (R)
(100000)13 = (1)(1)(0000)(2, 2, 1) + (0)(0)(1000)(1, 1, 9)
(000001)64 = (0)(1)(0001)(1, 2, 16) + (1)(0)(0001)(2, 1, 16)
(010000)78 = (2)(0)(0000)(3, 1, 1) + (0)(2)(0000)(1, 3, 1) + (0)(0)(0100)(1, 1, 36) + (1)(1)(1000)(2, 2, 9)

$SO_{13} \supset SO_{11} \times U_1$ (R)
(100000)13 = (00000)1(2) + (00000)1(−2) + (10000)11(0)
(000001)64 = (00001)32(1) + (00001)32(−1)
(010000)78 = (00000)1(0) + (10000)11(2) + (10000)11(−2) + (01000)55(0)

$SO_{13} \supset SU_2$ (S)
(100000)13 = 13
(000001)64 = 4 + 10 + 12 + 16 + 22
(010000)78 = 3 + 7 + 11 + 15 + 19 + 23

$Sp_{12} \supset SU_6 \times U_1$ (R)
(100000)12 = (10000)6(1) + (00001)$\bar{6}$(−1)
(200000)78 = (00000)1(0) + (10001)35(0) + (20000)21(2) + (00002)$\overline{21}$(−2)

$Sp_{12} \supset SU_2 \times Sp_{10}$ (R)
(100000)12 = (1)(00000)(2, 1) + (0)(10000)(1, 10)
(200000)78 = (2)(00000)(3, 1) + (0)(20000)(1, 55) + (1)(10000)(2, 10)

$Sp_{12} \supset Sp_4 \times Sp_8$ (R)
(100000)12 = (10)(0000)(4, 1) + (00)(1000)(1, 8)
(200000)78 = (20)(0000)(10, 1) + (00)(2000)(1, 36) + (10)(1000)(4, 8)

$Sp_{12} \supset Sp_6 \times Sp_6$ (R)
(100000)12 = (100)(000)(6, 1) + (000)(100)(1, 6)
(200000)78 = (200)(000)(21, 1) + (000)(200)(1, 21) + (100)(100)(6, 6)

$Sp_{12} \supset SU_2$ (S)
(100000)12 = 12
(200000)78 = 3 + 7 + 11 + 15 + 19 + 23

Table 58 (continued)

$Sp_{12} \supset SU_2 \times SU_4$ (S)
(100000)12 = (1)(010)(2, 6)
(200000)78 = (2)(000)(3, 1) + (0)(101)(1, 15) + (2)(020)(3, 20′)

$Sp_{12} \supset SU_2 \times Sp_4$ (S)
(100000)12 = (2)(10)(3, 4)
(200000)78 = (2)(00)(3, 1) + (0)(20)(1, 10) + (2)(01)(3, 5) + (4)(20)(5, 10)

$SO_{12} \supset SU_6 \times U_1$ (R)
(100000)12 = (10000)6(1) + (00001)$\bar{6}$(−1)
(000001)32 = (10000)6(−2) + (00001)$\bar{6}$(2) + (00100)20(0)
(000010)32′ = (00000)1(3) + (00000)1(−3) + (01000)15(−1) + (00010)$\overline{15}$(1)
(010000)66 = (00000)1(0) + (01000)15(2) + (00010)$\overline{15}$(−2) + (10001)35(0)

$SO_{12} \supset SU_2 \times SU_2 \times SO_8$ (R)
(100000)12 = (1)(1)(0000)(2, 2, 1) + (0)(0)(1000)(1, 1, 8_v)
(000001)32 = (0)(1)(0001)(1, 2, 8_s) + (1)(0)(0010)(2, 1, 8_c)
(000010)32′ = (0)(1)(0010)(1, 2, 8_c) + (1)(0)(0001)(2, 1, 8_s)
(010000)66 = (2)(0)(0000)(3, 1, 1) + (0)(2)(0000)(1, 3, 1) + (0)(0)(0100)(1, 1, 28) + (1)(1)(1000)(2, 2, 8_v)

$SO_{12} \supset SU_4 \times SU_4$ (R)
(100000)12 = (010)(000)(6, 1) + (000)(010)(1, 6)
(000001)32 = (100)(100)(4, 4) + (001)(001)($\bar{4}$, $\bar{4}$)
(000010)32′ = (100)(001)(4, $\bar{4}$) + (001)(100)($\bar{4}$, 4)
(010000)66 = (101)(000)(15, 1) + (000)(101)(1, 15) + (010)(010)(6, 6)

$SO_{12} \supset SO_{10} \times U_1$ (R)
(100000)12 = (00000)1(2) + (00000)1(−2) + (10000)10(0)
(000001)32 = (00001)16(1) + (00010)$\overline{16}$(−1)
(000010)32′ = (00001)16(−1) + (00010)$\overline{16}$(1)
(010000)66 = (00000)1(0) + (10000)10(2) + (10000)10(−2) + (01000)45(0)

$SO_{12} \supset SU_2 \times Sp_6$ (S)
(100000)12 = (1)(100)(2, 6)
(000001)32 = (3)(000)(4, 1) + (1)(010)(2, 14)
(000010)32′ = (2)(100)(3, 6) + (0)(001)(1, 14′)
(010000)66 = (2)(000)(3, 1) + (0)(200)(1, 21) + (2)(010)(3, 14)

$SO_{12} \supset SU_2 \times SU_2 \times SU_2$ (S)
(100000)12 = (3, 2, 2)
(000001)32 = (1, 4, 1) + (3, 2, 3) + (5, 2, 1)
(000010)32′ = (1, 1, 4) + (3, 3, 2) + (5, 1, 2)
(010000)66 = (3, 1, 1) + (1, 3, 1) + (1, 1, 3) + (3, 3, 3) + (5, 3, 1) + (5, 1, 3)

$SO_{12} \supset SO_{11}$ (S)
(100000)12 = (00000)1 + (10000)11
(000001)32 = (00001)32
(000010)32′ = (00001)32
(010000)66 = (100000)11 + (01000)55

$SO_{12} \supset SU_2 \times SO_9$ (S)
(100000)12 = (2)(0000)(3, 1) + (0)(1000)(1, 9)
(000001)32 = (1)(0001)(2, 16)
(000010)32′ = (1)(0001)(2, 16)
(010000)66 = (2)(0000)(3, 1) + (0)(0100)(1, 36) + (2)(1000)(3, 9)

Table 58 (continued)

$SO_{12} \supset Sp_4 \times SO_7$ (S)
(100000)12 = (01)(000)(5, 1) + (00)(100)(1, 7)
(000001)32 = (10)(001)(4, 8)
(000010)32′ = (10)(001)(4, 8)
(010000)66 = (20)(000)(10, 1) + (00)(010)(1, 21) + (01)(100)(5, 7)

$E_6 \supset SO_{10} \times U_1$ (R)
(100000)27 = (00000)1(4) + (10000)10(−2) + (00001)16(1)
(000001)78 = (00000)1(0) + (00001)16(−3) + (00010)$\overline{16}$(3) + (01000)45(0)

$E_6 \supset SU_2 \times SU_6$ (R)
(100000)27 = (1)(00001)(2, $\bar{6}$) + (0)(01000)(1, 15)
(000001)78 = (2)(00000)(3, 1) + (0)(10001)(1, 35) + (1)(00100)(2, 20)

$E_6 \supset SU_3 \times SU_3 \times SU_3$ (R)
(100000)27 = (01)(10)(00)($\bar{3}$, 3, 1) + (10)(00)(10)(3, 1, 3) + (00)(01)(01)(1, $\bar{3}$, $\bar{3}$)
(000001)78 = (11)(00)(00)(8, 1, 1) + (00)(11)(00)(1, 8, 1) + (00)(00)(11)(1, 1, 8) + (10)(10)(01)(3, 3, $\bar{3}$) + (01)(01)(10)($\bar{3}$, $\bar{3}$, 3)

$E_6 \supset SU_3$ (S)
(100000)27 = (22)27
(000001)78 = (11)8 + (41)35 + (14)$\overline{35}$

$E_6 \supset G_2$ (S)
(100000)27 = (02)27
(000001)78 = (10)14 + (11)64

$E_6 \supset Sp_8$(S)
(100000)27 = (0100)27
(000001)78 = (2000)36 + (0001)42

$E_6 \supset F_4$ (S)
(100000)27 = (0000)1 + (0001)26
(000001)78 = (0001)26 + (1000)52

$E_6 \supset SU_3 \times G_2$ (S)
(100000)27 = (02)(00)($\bar{6}$, 1) + (10)(01)(3, 7)
(000001)78 = (11)(00)(8, 1) + (00)(10)(1, 14) + (11)(01)(8, 7)

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