Homework 9, due 11-12

1. (The method of conformal mappings) Consider a complex function $w(z) = \Phi(x + iy) + i\chi(x + iy)$.
   (a) Show that if $w(z)$ is an analytic function ($w(x, y)$ satisfies the Cauchy-Riemann differential equations) then both Φ and χ satisfy the two-dimensional Laplace equation.
   (b) Show: If Φ is interpreted as the electrostatic potential then lines of constant χ represents the lines of force. Also show that the lines $\Phi = const$ and $\chi = const$ are orthogonal.
   (c) Consider $w(z) = -2\log(z)$. What is the physical situation represented by $\Phi(x, y) = \text{Re}w(z)$?
   (d) Find a complex function $w(z)$ that represents two parallel line charges $\pm\lambda$ in three dimensions. Calculate the surface charge density induced by a line charge located at $x = a$, $y = z = 0$ on a conducting plane at $x = 0$.
2. In an anisotropic medium we have $D_i = \epsilon_{ij} E_j$ where $\epsilon_{ij} = \epsilon_{ji}$ is the dielectric tensor. Determine the field of a point charge at the origin in an anisotropic medium. (Note: ϵ_{ij} can be diagonalized. This implies that it is sufficient to consider the case $\epsilon_{ij} = \text{diag}(\epsilon_x, \epsilon_y, \epsilon_z)$.)