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In[1]:= (* Jackson 3.1 *)
(* ----- *)
Clear[a, b]
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In[2]:= Integrate[LegendreP[1, x], {x, 0, 1}]
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$$\text{Out[2]} = \frac{\sqrt{\pi}}{2 \Gamma\left[1 - \frac{1}{2}\right] \Gamma\left[\frac{3+1}{2}\right]}$$

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In[3]:= c[l_] := Sqrt[Pi] / 2 / Gamma[1 - l / 2] / Gamma[(3 + l) / 2]
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In[4]:= Table[c[n], {n, 0, 4}]
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$$\text{Out[4]} = \left\{1, \frac{1}{2}, 0, -\frac{1}{8}, 0\right\}$$

```
In[5]:= A[l_] := (2 * l + 1) / 2 * c[l] * (a^(l + 1) - (-1)^l * b^(l + 1)) / (a^(2 * l + 1) - b^(2 * l + 1))
```

```
In[6]:= Table[A[n], {n, 0, 4}]
```

$$\text{Out[6]} = \left\{\frac{1}{2}, \frac{3(a^2 + b^2)}{4(a^3 - b^3)}, 0, -\frac{7(a^4 + b^4)}{16(a^7 - b^7)}, 0\right\}$$

```
In[7]:= B[l_] := (2 * l + 1) / 2 * c[l] *
(a^(l + 1) * b^(2 * l + 1) - (-1)^l * b^(l + 1) * a^(2 * l + 1)) / (b^(2 * l + 1) - a^(2 * l + 1))
```

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In[8]:= Table[B[n], {n, 0, 4}]
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$$\text{Out[8]} = \left\{0, \frac{3(a^3 b^2 + a^2 b^3)}{4(-a^3 + b^3)}, 0, -\frac{7(a^7 b^4 + a^4 b^7)}{16(-a^7 + b^7)}, 0\right\}$$

```
In[9]:= phi4[r_, x_] := Sum[(A[n] * r^n + B[n] / r^(n + 1)) * LegendreP[n, x], {n, 0, 4}]
phi4[r_, x_] := -0.1 /; r < a
phi4[r_, x_] := 1.1 /; r > b
```

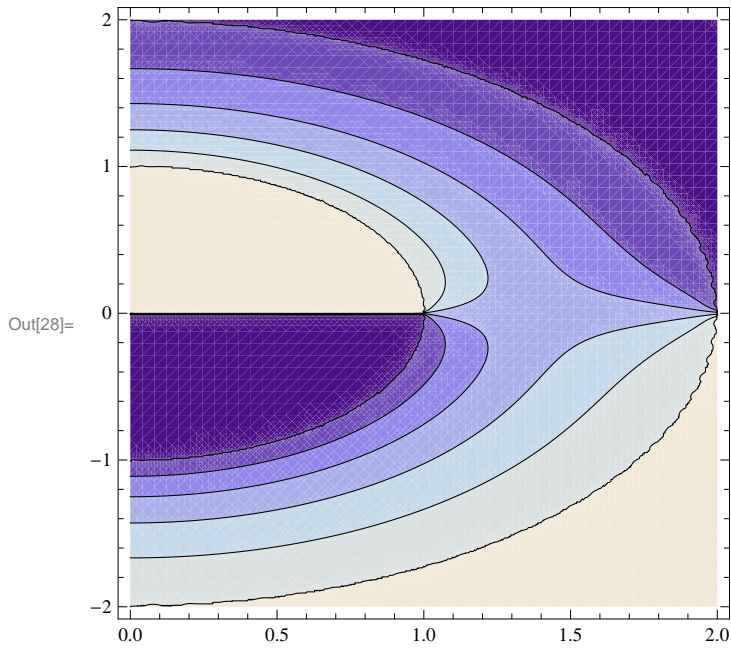
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In[12]:= phi4[rr, Cos[th]]
```

$$\text{Out[12]} = \frac{1}{2} + \left(\frac{3(a^3 b^2 + a^2 b^3)}{4(-a^3 + b^3)} \frac{rr^2}{rr^2} + \frac{3(a^2 + b^2) rr}{4(a^3 - b^3)}\right) \text{Cos[th]} +$$

$$\frac{1}{2} \left(-\frac{7(a^7 b^4 + a^4 b^7)}{16(-a^7 + b^7)} \frac{rr^4}{rr^4} - \frac{7(a^4 + b^4) rr^3}{16(a^7 - b^7)}\right) (-3 \text{Cos[th]} + 5 \text{Cos[th]}^3)$$

```
In[22]:= a = 1; b = 2;
phi[r_, x_] :=
Sum[(A[n] * r^n + B[n] / r^(n + 1)) * LegendreP[n, x], {n, 0, 100}] /; (r > 1.0 && r < 2.0)
phi[r_, x_] := -0.1 /; (r < 1.0 && x < 0)
phi[r_, x_] := -0.1 /; (r > 2.0 && x > 0)
phi[r_, x_] := 1.1 /; (r < 1.0 && x > 0)
phi[r_, x_] := 1.1 /; (r > 2.0 && x < 0)
```

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In[28]:= ContourPlot[phi[Sqrt[x^2+y^2]], y/Sqrt[x^2+y^2]], {x, 0, 2}, {y, -2, 2}, PlotPoints -> 50]
```



```
In[20]:= (* ContourPlot[phi^4[Sqrt[x^2+y^2]], y/Sqrt[x^2+y^2]], {x, 0, 2}, {y, -2, 2}, PlotPoints -> 50] *)
```

```
In[29]:= Plot3D[phi[Sqrt[x^2+y^2]], y/Sqrt[x^2+y^2]], {x, 0, 2}, {y, -2, 2}, PlotPoints -> 50]
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