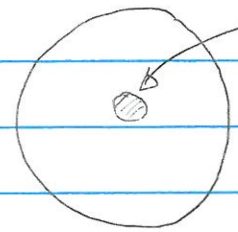
DIGRESSION: δ-FCT CONTRIBUTION



SMALL SPHERE, SOME CHARGE DISTRIBUTION INSIDE

CONSIDER COMPUTING INTEGRAL OVER $\vec{E}$ IN THE LARGE SPHERE

IDEA: USE MULTIPOLE FORMULA FOR $\vec{E}$ EVERYWHERE

SUBTLETY: SMALL SPHERE GIVES δ-CONTR

FIND: $$E_{DIP} = \frac{1}{4\pi\epsilon_0}\left\{\frac{3\hat{n}(\vec{p}\cdot\hat{n})-\vec{p}}{|\vec{x}-\vec{x}_0|^3} - \frac{4\pi}{3}\vec{p}\,\delta(\vec{x}-\vec{x}_0)\right\}$$

PROOF: WRITE

$$\int_{r<R} \vec{E}\,d^3x = \int_{r<R} (-\vec{\nabla}\phi)\,d^3x$$

$$= -\int_{r=R} \hat{n}\,\phi\,d\Omega\,R^2$$

MULTIPOLE EXPANSION

$$\phi = \frac{1}{\epsilon_0}\sum_{\ell m}\frac{q_{\ell m}}{r^{\ell+1}}Y_{\ell m}(\theta,\varphi)$$

ALSO NOTE: $\hat{n} = (\sin\theta\sin\varphi, \sin\theta\cos\varphi, \cos\theta)$
LINEAR COMBINATION OF Y_{1m}

$$\hat{n}_z \sim Y_{10} \quad \text{ETC}$$

ESG

↳ ONLY $\ell = 1$ CONTRIBUTES

<u>NOTE</u>: $\ell = 1$ TERM EASIER TO COMPUTE IN CARTESIAN COORDS

$$\int \vec{E}\, d^3x = -\int \hat{n}\, \Phi\, d\Omega\, R^2$$

$$= -R^2 \frac{1}{4\pi\epsilon_0} \frac{1}{R^2} \int \hat{n}\, \vec{p}\cdot\hat{n}\, d\Omega$$

NEED $\displaystyle\int \hat{n}_i \hat{n}_j\, d\Omega = A\,\delta_{ij}$

$$\int \hat{n}^2\, d\Omega = \int d\Omega = 4\pi = 3A \qquad A = \frac{4\pi}{3}$$

$$↳ \int \hat{n}\, \vec{p}\cdot\hat{n}\, d\Omega = \frac{4\pi}{3}\vec{p}$$

<u>GET</u> $\displaystyle\int \vec{E}\, d^3x = -\frac{\vec{p}}{3\epsilon_0}$

<u>BUT</u> DIRECT INTEGRATION OF $\vec{E}_{DIP}$ GIVES ZERO

$$\int d^3x\, \vec{E}_{DIP} = \frac{1}{4\pi\epsilon_0}\int r^2 dr \int d\Omega\, \frac{3\hat{n}(\vec{p}\cdot\hat{n}) - \vec{p}}{r^3}$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{dr}{r} \underbrace{\int d\Omega\,(3\hat{n}(\vec{p}\cdot\hat{n}) - \vec{p})}_{=0} = 0$$

SUBTLETY ARISES BECAUSE THIS IS DIVERGENT

MUST INCLUDE δ-FCT TO GET CORRECT INTEGRAL