Homework 8, due 11-20

1. The differential rate for the electron capture reaction $e^- + (A, Z) \to (A, Z-1) + \nu_e$ is given by

$$d\lambda = \frac{c}{4\pi^2(\hbar c)^3}\,(2.4G_F)^2\,|M|^2\,|\psi_e(0)|^2\,\delta\left(Q - E_\nu\right)\,d^3p_\nu,$$

   where E_ν, p_ν are the energy and momentum of the neutrino, $\psi_e(0)$ is the wave function of the electron at the origin, and Q is the Q-value (the difference in the binding energies plus the electron mass). The weak coupling constant is $G_F/(\hbar c)^3 = 1.16 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$ and the factor 2.4 takes into account coupling factors for Fermi and Gamov-Teller transitions. The nuclear matrix element $|M|^2$ satisfies the condition $|M|^2 \leq 1$. The electron wave function can be approximated by

$$|\psi_e(0)|^2 \simeq \frac{Z^3}{a_0^3},$$

   where a_0 is the Bohr radius. Integrate over the (unobserved) neutrino momentum to obtain the total rate. You can assume that neutrinos are massless. Estimate the total decay rate for $|M|^2 = 1$ and $Q = 1$ MeV as well as $Q = 10$ MeV.

2. The half lives of uranium 234,235 and 238 are respectively $2.5 \cdot 10^5$ years, $7.1 \cdot 10^8$ years and $4.5 \cdot 10^9$ years. Their relative natural abundances are 0.0057%, 0.72% and 99.27%. Are these data consistent with idea that these nuclei were formed in equal amounts at the same time? Can you explain any discrepancies from the fact that unstable isotopes $^{234}_{90}\mathrm{Th}$ and $^{234}_{91}\mathrm{Pa}$ exits? When was the uranium made?