Homework 7, due 11-6

The differential decay rate for the β decay reaction $(A, Z) \to (A, Z+1) + e^- + \bar{\nu}_e$ is given by

$$d\lambda = \frac{c^6}{8\pi^5\hbar}\left(\frac{2.4G_F}{(\hbar c)^3}\right)^2 |M|^2\,\delta\left(\Delta B - E_\nu - E_e\right)\,d^3p_e\,d^3p_\nu,$$

where E_e, p_e are the energy and momentum of the electron, E_ν, p_ν are the energy and momentum of the neutrino, and ΔB is the difference in the binding energies. The weak coupling constant is $G_F/(\hbar c)^3 = 1.16 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$ and the factor 2.4 takes into account coupling factors for Fermi and Gamov-Teller transitions. The nuclear matrix element $|M|^2$ satisfies the condition $|M|^2 \leq 1$.

1. Integrate over the (unobserved) neutrino momentum to obtain the differential rate as a function of the electron energy. You can assume that neutrinos are massless.
2. Also integrate over the electron momentum in order to obtain the total decay rate. How does the rate scale with $Q \equiv \Delta B - m_e c^2$ in the limit $Q \gg m_e c^2$?
3. Estimate the total decay rate for $|M|^2 = 1$ and $Q = 1$ MeV as well as $Q = 10$ MeV.

Note that the Dirac δ-function is defined by

$$\int dx\, f(x)\delta(x-a) = f(a).$$

You can easily show that

$$\int dx\, f(x)\delta(g(x)) = \frac{1}{|g'(a)|}f(a),$$

where $g(a) = 0$.