Homework 2, due 9-18

In class we solved the Schrödinger equation for the s-wave bound state of the deuteron under the assumption that the bound state is shallow. Repeat the calculation without using this approximation. Use the potential

$$V(r) = \begin{cases} -V_0 & r < a \\ 0 & r > a \end{cases}$$

with $a = 1.4$ fm. Fix V_0 from the measured binding energy $B = -2.2$ MeV. Determine the rms radius $\sqrt{\langle r^2 \rangle}$ of the deuteron.

Hints: You will have to solve the eigenvalue equation numerically. You can do that graphically, using your pocket calculator, or using a numerical math package such as mathematica/maple/matlab. Note that the rms radius is defined as $\sqrt{\langle r^2 \rangle}$, where $\langle r^2 \rangle$ is the expectation value of r^2 in the ground state. Remember that

$$\langle r^2 \rangle = \frac{\int d^3r\, r^2 |\psi(r)|^2}{\int d^3r\, |\psi(r)|^2}$$

and

$$\int d^3r\, r^2 |\psi(r)|^2 = 4\pi \int r^2 dr\, r^2 |\psi(r)|^2 = 4\pi \int dr\, r^2 u(r)^2.$$