Ideal Gas

Ideal gas law

$$PV = kNT$$

Equipartition law

$$U = \frac{f}{2} kNT$$

with $f = 3$ for a mono-atomic gas and $f = 5$ for a di-atomic gas. Adiabatic expansion

$$PV^\gamma = const, \qquad \gamma = (f+2)/f$$

Entropy of an ideal mono-atomic gas

$$S = kN \left\{ \log \left(\frac{V}{N v_Q} \right) + \frac{5}{2} \right\}, \qquad v_Q = l_Q^3, \qquad l_Q = \frac{h}{\sqrt{2\pi m k T}}$$

Chemical potential

$$\mu = -kT \log \left(\frac{V}{N v_Q} \right)$$

Entropy and Heat

First law

$$\Delta U = Q + W$$

Thermodynamic Identity

$$dU = TdS - PdV + \mu dN$$

If $W = -PdV$ have $Q = TdS$. Also

$$\frac{1}{T} = \left. \frac{\partial S}{\partial U} \right|_{V,N}, \qquad P = T \left. \frac{\partial S}{\partial V} \right|_{U,N}$$

Specific heat $C = Q/\Delta T$. Have

$$C_V = \left. \frac{\partial U}{\partial T} \right|_{V,N}$$

Efficiency of the Carnot Process operating between two reservoirs at temperatures T_h and T_c

$$\epsilon = \frac{W}{Q_h} = 1 - \frac{T_c}{T_h}$$

Thermodynamic Functions

Enthalpy

$$H = U + PV \qquad \Delta H = Q + W_{other} \ \ (P = const)$$

Free Energy

$$F = U - TS \qquad \Delta F = W \ \ (T = const, \ Q = T\Delta S)$$

Gibbs Free Energy

$$G = U - TS + PV \qquad \Delta G = W_{other} \ \ (P = T = const, \ Q = T\Delta S)$$

Partial derivatives of free energy

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} \quad P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} \quad \mu = \left(\frac{\partial F}{\partial N}\right)_{V,T}$$

Statistical Definition of Entropy

Entropy

$$S = k \log(\Omega)$$

Binomial coefficient

$$\left(\begin{array}{c} N \\ k \end{array} \right) = \frac{N!}{k!(N-k)!}$$

Stirling formula ($N \gg 1$)

$$\log(N!) \simeq N \log(N) - N + \ldots$$

Statistical Mechanics

Partition Function

$$Z = \sum_s \exp(-\beta E_s), \qquad \beta = \frac{1}{kT}$$

A useful sum is the geometric series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Average (internal) energy

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$$

Connection to free energy

$$F = -kT\log(Z)$$

For N not-to-dense, indistinguishable particles

$$Z_{tot} = \frac{1}{N!}(Z_1)^N,$$

where Z_1 is the one-body partition function. Grand partition Function

$$\mathcal{Z} = \sum_s \exp(-\beta(E_s - \mu N_s))$$

Bose and Fermi distribution

$$n_B = \frac{1}{\exp(\beta(\epsilon-\mu))-1}, \quad n_F = \frac{1}{\exp(\beta(\epsilon-\mu))+1}$$

Boltzmann limit $n = \exp(-\beta(E-\mu))$

Numerical Constants

$$\begin{aligned}
k &= 1.381\times 10^{-23}\,J/K = 8.617\times 10^{-5}\,eV/K \\
N_A &= 6.022\times 10^{23} \\
R &= 8.315\,J/mol/K \\
h &= 6.626\times 10^{-34}\,J\cdot s \\
e &= 1.602\times 10^{-19}\,C \\
1\,atm &= 1.013\times 10^{5}\,N/m^2 \\
1\,cal &= 4.186\,J \\
1\,eV &= 1.602\times 10^{-19}\,J \\
1\,u &= 1.661\times 10^{-27}\,kg
\end{aligned} \tag{1}$$