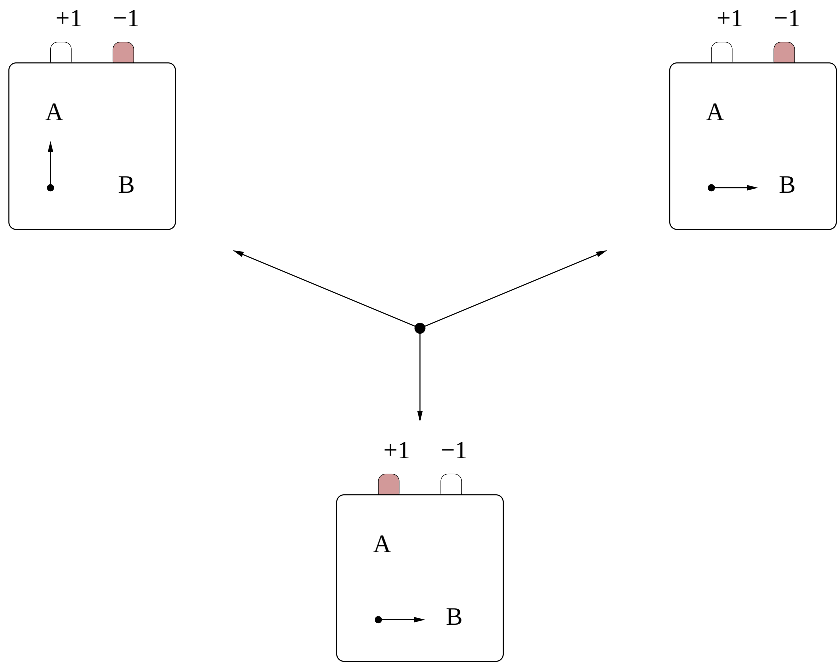

Bell's theorem simplified (GHZ state)

Consider the following entangled state of three spins

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle \right) . \tag{1}$$

We will consider the observables $A_i = \sigma_x^{(i)}$ and $B_i = \sigma_y^{(i)}$, where $i = 1, 2, 3$ refers to the i'th spin. Note that $\sigma_x = \sigma^+ + \sigma^-$ and $\sigma_y = -i(\sigma^+ - \sigma^-)$. This implies

$$A_1 B_2 B_3 |\psi\rangle = -(\sigma^+ + \sigma^-)^{(1)} (\sigma^+ - \sigma^-)^{(2)} (\sigma^+ - \sigma^-)^{(3)} |\psi\rangle = -|\psi\rangle . \tag{2}$$

Analogously, $|\psi\rangle$ is an eigenstate of any product of one A and two B's (with different indices, but in any order) with eigenvalue -1. We also find that $|\psi\rangle$ is an eigenstate of $A_1 A_2 A_3$ with eigenvalue $+1$.
To a "realist" $A_i = \pm 1$ and $B_i = \pm 1$ are real properties of the system. A realist will explain the fact that measurements of A_i and B_i have probabilistic outcomes by appealing to the possibility that the initial state of the system is not uniquely determined, but contains "hidden variables" that are randomly distributed.
When informed that a measurement of a randomly chosen $A_i B_j B_k$ (with i, j, k all different) always yields $+1$ a realist will conclude that

$$A_1 A_2 A_3 = (A_1 B_2 B_3)(A_2 B_1 B_3)(A_3 B_1 B_2) = +1 , \tag{3}$$

because $B_i^2 = +1$. The realist will then conclude that $A_1 A_2 A_3 = +1$ always. But this prediction is always wrong, a measurement of $A_1 A_2 A_3$ yields -1.