

Q1122

$$a) H = \frac{\hbar}{2} \begin{pmatrix} A & b \\ b & -A \end{pmatrix}$$

EIGENVALUES $(A-\lambda)(-A-\lambda) - b^2 = 0$

$$-A^2 + \lambda^2 - b^2 = 0 \quad \Rightarrow \lambda = \pm \sqrt{A^2 + b^2}$$

$$E_{1,2} = \pm \frac{\hbar}{2} \sqrt{A^2 + b^2}$$

b) $H_0 = \frac{\hbar}{2} \begin{pmatrix} A & \\ & -A \end{pmatrix} \quad V = \frac{\hbar}{2} \begin{pmatrix} & b \\ b & \end{pmatrix}$

$$E_1^0 = \frac{\hbar}{2} A \quad E_2^0 = -\frac{\hbar}{2} A \quad |1^0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |2^0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

FIRST ORDER $\delta E_1^0 = \langle 1^0 | V | 1^0 \rangle = 0 \quad \delta E_2^0 = 0$

SECOND ORDER $\delta E_1^0 = \frac{|\langle 1^0 | V | 2^0 \rangle|^2}{E_1^0 - E_2^0}$

USE $\langle 1^0 | V | 2^0 \rangle = (1 \ 0) \frac{\hbar}{2} \begin{pmatrix} 0 & b \\ b & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{b\hbar}{2}$

$$\delta E_1^0 = + \frac{\left(\frac{b\hbar}{2}\right)^2}{A\hbar} = + \hbar \frac{b^2}{4A} //$$

$$\delta E_2^0 = - \delta E_1^0 = - \hbar \frac{b^2}{4A} //$$

CHECK $E_{\pm} = \frac{\hbar}{2} \sqrt{A^2 + b^2} = A \frac{\hbar}{2} \sqrt{1 + \frac{b^2}{A^2}}$

$$\approx A \frac{\hbar}{2} \left\{ 1 + \frac{b^2}{2A^2} + \dots \right\} = A \frac{\hbar}{2} + \frac{b^2}{2A} \frac{\hbar}{2} + \dots$$

$$= \underbrace{A \frac{\hbar}{2}}_{E_1^0} + \underbrace{\frac{b^2}{2A} \frac{\hbar}{2}}_{\delta E_1}$$