1) $\psi_0(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$

$$E_0 = \frac{\hbar^2}{2m} \frac{\pi^2}{L^2}$$

2) $H = cS_z \qquad H|\uparrow\rangle = E_\uparrow |\uparrow\rangle$

$$H|\downarrow\rangle = E_\downarrow |\downarrow\rangle$$

$$E_{\uparrow,\downarrow} = \pm \frac{\hbar}{2} c$$

FIND $|S_x = +\frac{\hbar}{2}\rangle$: $\qquad |S_x = \frac{\hbar}{2}\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$\Rightarrow \quad \beta = \alpha \qquad \Rightarrow |S_x = \frac{\hbar}{2}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

TIME EVOLUTION

$$|\psi(t)\rangle = e^{-iHt/\hbar} |S_x = +\frac{\hbar}{2}\rangle$$

$$= \frac{1}{\sqrt{2}} \left(e^{-iE_\uparrow t/\hbar} |\uparrow\rangle + e^{-iE_\downarrow t/\hbar} |\downarrow\rangle \right)$$

$$= \frac{1}{\sqrt{2}} \left(e^{-ict/2} |\uparrow\rangle + e^{+i\frac{ct}{2}} |\downarrow\rangle \right)$$

$$P = |\langle S_x = \tfrac{\hbar}{2} | \psi(t)\rangle|^2 = \left| \frac{1}{2} \left(e^{-ict/2} + e^{ict/2} \right) \right|^2$$

$$= \cos^2\left(\frac{ct}{2}\right)$$