Transformation laws in special relativity

Lorentz transformation: The system S' is moving with velocity $(v_x, v_y, v_z) = (v, 0, 0)$ relative to the S system. The Lorentz transformations are

$$
\begin{aligned}
x' &= \gamma\left(x - vt\right), \qquad y' = y, \qquad z' = z, && (1)\\
t' &= \gamma\left(t - \frac{v}{c^2}x\right), && (2)
\end{aligned}
$$

where $\gamma = 1/(1-\beta^2)^{1/2}$ and $\beta = v/x$. The inverse Lorentz transformation corresponds to $v \to -v$.

Velocity addition: An object moves with velocity (u_x, u_y, u_z) in the S-system. The components of the velocity in the S'-system are

$$
\begin{aligned}
u'_x &= \frac{u_x - v}{1 - \frac{vu_x}{c^2}}, && (3)\\
u'_y &= \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}, \qquad u'_z = \frac{u_z}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}. && (4)
\end{aligned}
$$

The inverse transformation corresponds to $v \to -v$.

Doppler effect: A source in the S'-system emits light waves with a frequency f'. The frequency measured in S is

$$f = \frac{f'}{\gamma(1 + \beta\cos(\alpha))}, \tag{5}$$

where α is the angle between the line of sight and the x-axis. If the source is moving along the x-axis then $\alpha = 0$ and

$$f = f'\sqrt{\frac{1-\beta}{1+\beta}}. \tag{6}$$

If S' is approaching rather than receding $\beta \to -\beta$.

Aberration of starlight: If the light is emitted at an angle α' with respect to the x'-axis, then the angle α observed by S is

$$\cos(\alpha) = \frac{\cos(\alpha') + \beta}{1 + \beta\cos(\alpha')}. \tag{7}$$