$$\therefore \langle x^2\rangle - \langle x\rangle^2 = L^2\left\{\left(\frac{1}{3}-\frac{1}{2\pi^2}\right)-\frac{1}{4}\right\}$$

$$= L^2\left\{\frac{1}{12}-\frac{1}{2\pi^2}\right\}$$

$$\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = L\cdot\sqrt{\frac{1}{12}-\frac{1}{2\pi^2}} \simeq 0.18\,L$$

ALSO $\langle p^2\rangle = \frac{\pi^2\hbar^2}{L^2}$ $\langle p\rangle = 0$

$$\therefore \sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \frac{\pi\hbar}{L}$$

$$\therefore \sigma_p\cdot\sigma_x = 0.18\,\pi\hbar \simeq \underline{0.57\,\hbar}$$

6-35) a) $\psi_0(x) = C_0\, e^{-m\omega x^2/2\hbar}$

$$1 \stackrel{!}{=} \int dx\,(\psi_0(x))^2 = C_0^2\int_{-\infty}^{\infty} dx\, e^{-\frac{m\omega x^2}{\hbar}}$$

$$= C_0^2\sqrt{\frac{\hbar}{m\omega}}\int du\, e^{-u^2}$$

$$= C_0^2\sqrt{\frac{\hbar}{m\omega}}\sqrt{\pi} \quad \therefore \quad \underline{C_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4}}$$

b) $\langle x^2\rangle = \int dx\, x^2\,(\psi_0(x))^2$

$$= C_0^2\int dx\, x^2\, e^{-\frac{m\omega x^2}{\hbar}}$$

$$= \left(\frac{m\omega}{\hbar\pi}\right)^{1/2}\left(\frac{\hbar}{m\omega}\right)^{3/2}\int du\, u^2 e^{-u^2}$$

$$= \left(\frac{m\omega}{\hbar\pi}\right)^{1/2}\left(\frac{\hbar}{m\omega}\right)^{3/2}\frac{\sqrt{\pi}}{2} = \frac{\hbar}{2m\omega}$$