6-30) $\langle p^2 \rangle \equiv \int dx\, \psi^*(x) \left(-i\hbar \frac{d}{dx}\right)^2 \psi(x)$

$$= \int dx\, \psi^*(x) (-\hbar^2) \frac{d^2}{dx^2} \psi(x)$$

SCHRÖDINGER $\left\{ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right\} \psi(x) = E\, \psi(x)$

$$\Rightarrow \quad -\hbar^2 \frac{d^2}{dx^2} \psi(x) = 2m \left(E - V(x)\right) \psi(x)$$

THEN $\langle p^2 \rangle = \int dx\, \psi^*(x)\, 2m\left(E - V(x)\right) \psi(x)$

$$= 2m \langle E - V(x) \rangle$$

FOR $V(x) = 0$ GET $\langle p^2 \rangle = 2m \langle E \rangle$

$$\Rightarrow \quad \langle p^2 \rangle = 2m \frac{n^2 \pi^2 \hbar^2}{2m L^2} = \frac{n^2 \pi^2 \hbar^2}{L^2}$$

6-31) $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$

$$\langle x \rangle = \frac{2}{L} \int_0^L dx\, x \sin^2\left(\frac{\pi x}{L}\right) = \frac{L}{2} \quad \text{(BY SYMMETRY)}$$

$$\langle x^2 \rangle = \frac{2}{L} \int_0^L dx\, x^2 \sin^2\left(\frac{\pi x}{L}\right)$$

$$= \frac{2}{L} \left(\frac{L}{\pi}\right)^3 \int_0^\pi du\, u^2 \sin^2(u)$$

$$= \frac{2L^2}{\pi^3} \left\{ \frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right) \sin(2u) - \frac{u \cos(2u)}{4} \right\}_0^\pi$$

$$= \frac{2L^2}{\pi^3} \frac{\pi}{12} \left(2\pi^2 - 3\right) = L^2 \left(\frac{1}{3} - \frac{1}{2\pi^2}\right)$$