Formulas and Numerical Constants

Lorentz transformation: The system S' is moving with velocity $(v_x, v_y, v_z) = (v, 0, 0)$ relative to the S system. The Lorentz transformations are

$$x' = \gamma (x - vt), \qquad y' = y, \qquad z' = z, \tag{1}$$
$$t' = \gamma \left(t - \frac{v}{c^2} x \right), \tag{2}$$

where $\gamma = 1/(1-\beta^2)^{1/2}$ and $\beta = v/c$. The inverse Lorentz transformation corresponds to $v \to -v$.

Velocity addition: An object moves with velocity (u_x, u_y, u_z) in the S-system. The components of the velocity in the S'-system are

$$u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}, \tag{3}$$
$$u'_y = \frac{u_y}{\gamma \left(1 - \frac{vu_x}{c^2}\right)}, \qquad u'_z = \frac{u_z}{\gamma \left(1 - \frac{vu_x}{c^2}\right)}. \tag{4}$$

The inverse transformation corresponds to $v \to -v$.

Relativistic kinematics: In the following m always refers to the rest mass of a particle

$$E^2 = p^2 c^2 + m^2 c^4, \tag{5}$$
$$E = \gamma m c^2 \qquad p = \gamma m v, \tag{6}$$

and $\gamma = (1 - v^2/c^2)^{-1}$. The four vector $(E, \vec{p}c)$ transforms under Lorentz transformations like the four vector $(ct, \vec{x})$:

$$p'_x = \gamma \left(p_x - vE/c^2 \right), \qquad p'_y = p_y, \qquad p'_z = p_z, \tag{7}$$
$$E' = \gamma (E - v p_x), \tag{8}$$

Black Body Radiation: Planck's law is

$$R(\lambda) = \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}. \tag{9}$$

The maximum of the distribution occurs at $\lambda T = 2.898 \cdot 10^{-3}$ K m. The total emissivity is given by the Stefan-Boltzmann law

$$R = \sigma T^4, \qquad \sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2} \tag{10}$$

De Broglie relations: De Broglie postulated the following relations between (E, p) and (λ, f)

$$E = hf, \qquad (E = \hbar\omega) \tag{11}$$
$$p = h/\lambda, \qquad (p = \hbar k) \tag{12}$$

where $\hbar = h/(2\pi)$.

Bohr's model: Bohr's model of hydrogen like atoms is based on the quantization condition $L = mvr = n\hbar$. The allowed energies and radii are

$$r_n = \frac{n^2 a_0}{Z}, \qquad a_0 = \frac{\hbar^2}{m_e k e^2}, \tag{13}$$
$$E_n = -\frac{Z^2 E_0}{n^2}, \qquad E_0 = \frac{m_e k^2 e^4}{2\hbar^2}, \tag{14}$$

where $k = 1/(4\pi\epsilon_0)$ is the constant of proportionality in Coulomb's law, e is the charge, and m_e is the mass of the electron. Z is the charge of the nucleus (in units of e). The constant a_0 is called the Bohr radius. The quantity

$$\alpha = \frac{ke^2}{\hbar c} \simeq \frac{1}{137} \tag{15}$$

is called the fine structure constant.

Schrödinger equation: The time-dependent and time-independent Schrödinger equations are

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right\}\psi(x,t), \tag{16}$$
$$E\psi(x) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right\}\psi(x). \tag{17}$$

The wave function is related to the probability

$$P(x,t)\,dx = \psi^*(x,t)\psi(x,t)\,dx. \tag{18}$$

More generally, expectation values are given by

$$\langle O\rangle = \int dx\,\psi^*(x,t)\hat{O}\psi(x,t). \tag{19}$$

An important example is the momentum p

$$\langle p\rangle = \int dx\,\psi^*(x,t)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi(x,t). \tag{20}$$

<u>3d Schrödinger equation:</u> Solutions of the Schrödinger equation for a potential with rotational symmetry have the form

$$\psi(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi), \tag{21}$$

where Y_{lm} are the spherical harmonics, (l,m) label $L^2 = \hbar^2 l(l+1)$ and $L_z = \hbar m$ $(m \leq l)$, and $R_{nl}(r)$ is the radial wave function (labeled by the quantum number n). The ground state wave function of the hydrogen atom is

$$R_{10}(r) = \frac{2}{\sqrt{a_0^3}}\,e^{-r/a_0}, \qquad Y_{00}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}, \tag{22}$$

where a_0 is the Bohr radius defined above.

<u>Numerical Constants:</u>

$$\begin{aligned}
k_B &= 1.381\times 10^{-23}\,J/K = 8.617\times 10^{-5}\,eV/K \\
h &= 6.626\times 10^{-34}\,J\cdot s \\
c &= 2.998\times 10^{8}\,\mathrm{m/sec} \\
hc &= 1240\,eV\cdot nm \\
\hbar c &= 197.33\,\mathrm{MeV}\cdot\mathrm{fm} \\
e &= 1.602\times 10^{-19}\,C \\
1\,cal &= 4.186\,J \\
1\,eV &= 1.602\times 10^{-19}\,J \\
1\,u &= 1.661\times 10^{-27}\,kg = 931.49\,\mathrm{MeV}/c^2 \\
m_e c^2 &= 512\,\mathrm{keV} \\
m_p c^2 &= 935\,\mathrm{MeV}
\end{aligned} \tag{23}$$