Homework 9, due 11-11

Consider a particle governed by the Hamiltonian

$$H = \frac{p^2}{2m} + g(x^2 - f^2)^2.$$

Find an approximate result for the groundstate energy of H by using a variational wave function based on the eigenstates of the harmonic oscillator

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

We will restrict ourselves to a wave function which is a linear superposition of the first two parity even states, $|\psi\rangle = \alpha|0\rangle + \beta|2\rangle$, where $H_0|n\rangle = |n\rangle(\hbar\omega)(n+1/2)$. For simplicity, we will choose $2m = g = \hbar = 1$ and $f = 1.5$.

1. Begin by using $|\psi\rangle = |0\rangle$. Compute the groundstate energy as a function of ω and optimize the choice of the parameter ω. What is the bound for the ground state energy of H?

2. Repeat the exercise with $|\psi\rangle = |2\rangle$. Is the resulting bound better than the one obtained in a?

3. Finally, use $|\psi\rangle = \alpha|0\rangle + \beta|2\rangle$. What is your final estimate?

Hint: Finding the optimal value of ω in parts 1) and 2) involves solving a cubic equation. You can plot the function using you favorite software package and determine the bound graphically. Part 3) is equivalent to diagonalizing H in the basis spanned by $|0\rangle, |2\rangle$ (why?). In this case the equation for ω is definitely too hard to be solved analytically.