Homework 7, due 10-28

1. Consider a particle governed by the Hamiltonian

$$H = \frac{\vec{p}^2}{2m} + V(\vec{x}).$$

   (a) Show that

$$\frac{d}{dt}\langle \vec{x} \cdot \vec{p} \rangle = \frac{1}{m}\langle \vec{p}^2 \rangle - \langle \vec{x} \cdot \vec{\nabla} V(\vec{x}) \rangle.$$

   (b) If the left hand side of this relation vanishes we obtain a simple relation between the average kinetic and potential energies known as the virial theorem. Under what conditions does this happen?

   (c) Consider the Hamiltonian of the 1-dimensional harmonic oscillator. State the virial theorem and check the result by computing the expectations values in the energy eigenstates $|n\rangle$.

2. The (coordinate) correlation function is defined by

$$C(t) = \langle x_H(t) x_H(0) \rangle,$$

   where $x_H(t)$ is the position operator in the Heisenberg representation. Compute $C(t)$ in the ground state of the 1-dimensional harmonic oscillator.

3. The partition function is defined by

$$Z(\beta) = \int dx' \; U(x', t; x', 0)|_{t=-i\hbar\beta} \, ,$$

   where $U(x', t; x, 0)$ is the time evolution operator in the coordinate representation. Compute $Z(\beta)$ for the 1-dimensional harmonic oscillator. Show that

$$E_0 = -\lim_{\beta\to\infty} \frac{1}{Z}\frac{dZ}{d\beta},$$

   where E_0 is the ground state energy.