Homework 4, due 10-6

1. A particle of mass m moves in the potential

$$V(x) = \begin{cases} V_1 & x < 0 \\ 0 & 0 < x < a \\ V_2 & a < x \end{cases} ,$$

where $V_1, V_2 > 0$.

   (a) Find the equation that determines the energy levels.

   (b) Consider the case $V_1 = V_2$. Draw a graph that shows the eigenvalue equation.

   (c) Is there always at least one bound state? How does the number of bound states scale with V?

   (d) Derive an approximate analytical expression for the energy level in a shallow potential, that means in the case that the potential $V = V_1 = V_2$ allows just one bound state with energy $E \simeq V$. Compute the first correction to to $E \simeq V$.

2. A particle of mass m moves in the potential $V = -V_0\delta(x)$ $(V_0 > 0)$.

   (a) Find the energy and wave function of the groundstate.

   (b) Are there any excited bound states?

   Hint for part a: First consider the Schrödinger equation in the regime $x > 0$ and $x < 0$. Then determine the boundary condition at $x = 0$.