Homework 3, due 9-30

1. Consider the time evolution operator $U(t, t_0)$ defined by $|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle$. The time evolution operator in the coordinate basis

$$U(x, t; x', t_0) = \langle x|U(t, t_0)|x'\rangle$$

is often called the (coordinate space) propagator. What is the physical meaning of $U(x, t; x', t_0)$? Convince yourself that

$$\psi(x, t) = \int dx'\, U(x, t, x', t_0)\psi(x', t_0).$$

Consider a free particle governed by the Hamilton operator $H = P^2/(2m)$.

(a) Compute $U(x, t; x', t_0)$.

(b) The initial wave function is given by the wave packet

$$\psi(x', t_0 = 0) = \frac{1}{(\pi\Delta^2)^{1/4}} e^{ip_0x'/\hbar} e^{-x'^2/(2\Delta^2)}.$$

Compute $\psi(x, t)$.

(c) Study the averages $\langle X\rangle$ and $\langle \Delta X\rangle$.

2. Consider the spin operators $S_{x,y,z} = \frac{\hbar}{2}\sigma_{1,2,3}$, where σ_i are the Pauli matrices introduced in problem set 1. The associated uncertainties are

$$\langle(\Delta S_x)^2\rangle = \langle S_x^2\rangle - \langle S_x\rangle^2, \ \ldots$$

Consider the state $|S_z \uparrow\rangle$ which is the eigenstate of S_z with eigenvalue $\hbar/2$.

(a) Compute $\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle$ in this state. Compare your result to the uncertainty relation for S_x, S_y.

(b) Repeat the calculation in the state $|S_x \uparrow\rangle$.

(c) Find the state that minimizes the the uncertainty $\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle$. Again, check that the uncertainty relation is satisfied.