Homework 2, due 9-24

1. Prove (or find) the following relations

   (a) $[A, BC] = B[A, C] + [A, B]C$

   (b) $[AB, C] =?$

   (c) $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$

   (d) $[A, BC] = \ldots \{A, B\} \ldots + \ldots$

   where $\{A, B\} = AB + BA$.

2. The operators X and K satisfy $[K, X] = -i\mathbb{1}$. Compute $[K, X^2]$ and $[K^2, X^2]$. Compare your results to the Poisson brackets $\{p, x^2\}_P$ and $\{p^2, x^2\}_P$. Here, $\{f, g\}_P = (\partial f/\partial x)(\partial g/\partial p) - (\partial g/\partial x)(\partial f/\partial p)$.

3. Consider the operator $f(X)$ where $f(x) = \exp(-x^2)$. Compute

   $$\langle k|f(X)|k'\rangle,$$

   where $K|k\rangle = |k\rangle k$.

4. Show (or compute)

   (a) $\delta(ax) = \delta(x)/|a|$

   (b) $\delta(f(x)) =$

   (c) $\delta(x^2 - a^2) =$

5. Consider the following argument: We showed in class that the eigenvalue problem for K can be written (in coordinate space) as

   $$-i\frac{d}{dx}\psi_k(x) = k\psi_k(x).$$

   This equation is solved by $\psi_k(x) = \exp(-px)$ where p is real and the eigenvalue $k = ip$ is imaginary. Does this contradict the statement that hermitean operators have real eigenvalues?
   You probably noticed that this eigenfunction is not acceptable because it cannot be normalized (not even to the delta function). But what if we restrict our Hilbertspace to complex functions on $[0, \infty)$?