Practice Problems

1. Consider a system described by the Hamiltonian $H = H_0 + H_1$ where H_0 is the Hamiltonian of the harmonic oscillator

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2,$$

and $H_1 = -fx$.

   (a) Consider H_1 as a pertutbation and compute the eigenvalues and eigenstates of H to first non-trivial order in perturbation theory.

   (b) Compute the eigenvalues and eigenstates exactly. Compare with the results in (a).

2. Consider a system described by the Hamiltonian

$$H = \frac{p^2}{2m} + Ax^p,$$

where p is a positive even integer. Determine the groundstate energy of H approximately by writing H as a perturbed harmonic oscillator, $H = H_0 + H_1$. Use the freedom in choosing H_0 in order to optimize perturbation theory. This can be achieved by requiring the first order perturbation in H_1 vanish, $\langle 0|H_1|0\rangle \equiv 0$. (You don't have to compute the second order correction.)