Homework 6, due 4-9

1. The energy flux in an electromagnetic field is given by the Poynting vector
$$\vec{S} = \frac{c}{8\pi}\mathrm{Re}(\vec{E} \times \vec{H}^*).$$
   (a) Calculate the energy flux in a plane electromagnetic wave.
   (b) Show that in the case of total internal reflection there is no energy flux through the interface between the two media.

2. Consider the propagation of an electromagnetic wave in a rectangular wave guide. The wave guide has walls made of an ideal conductor. The wave guide has dimensions $a \times b$ in the $x - y$ plane and is infinitely long in the z direction.
   (a) Write down an appropriate ansatz for a monochromatic wave $\vec{E}, \vec{B} \sim \exp(ikz - i\omega t)$. Make sure that the boundary conditions are satisfied.
   (b) Determine the dispersion relation $\omega(k)$. Discuss the physical significance of your result.