PHY 125, useful formulas

1. velocity and acceleration:

$$\vec{v} = \frac{d\vec{r}}{dt}, \qquad \vec{a} = \frac{d\vec{v}}{dt} \tag{1}$$

   linear motion with constant acceleration

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2}$$

$$v = v_0 + at \tag{3}$$

   also: $v^2 = v_0^2 + 2a(x - x_0)$

2. Newton's laws

$$\sum_i \vec{F}_i = 0 \;\Rightarrow \vec{v} = \text{const} \tag{4}$$

$$\sum_i \vec{F}_i = m\vec{a} \tag{5}$$

$$\vec{F}_{12} = -\vec{F}_{21} \tag{6}$$

3. static friction (N normal force)

$$F_s \leq \mu_s N \tag{7}$$

   kinetic friction

$$F_k = \mu_k N \tag{8}$$

4. Newton's law of gravity

$$\vec{F} = -\frac{GmM}{r^2}\hat{r} \tag{9}$$

5. centripetal acceleration

$$a_c = \frac{v^2}{r} \tag{10}$$

6. Work

$$W = \int \vec{F} \cdot d\vec{r} \tag{11}$$

   constant force $W = \vec{F} \cdot \vec{d}$

7. kinetic energy $K = \frac{1}{2}mv^2$. Energy conservation for conservative forces

$$K_1 + U_1 = K_2 + U_2, \tag{12}$$

   where $U = -\int \vec{F} \cdot d\vec{r} + \text{const}$ is the potential energy associated with the conservative force.

8. Hooke's law (spring constant k)

$$F_s = -kx \tag{13}$$

   elastic potential energy $E = \frac{1}{2}kx^2$.

9. Gravitational potential energy

$$U(r) = -\frac{GmM}{r} \tag{14}$$

   Approximate result near the surface of the earth: $U = mgh$.

10. Power $P = \frac{dW}{dt}$.

11. Linear momentum $\vec{p} = m\vec{v}$. Newton's equation of motion

$$\sum_i \vec{F}_i = \frac{d\vec{p}}{dt} \tag{15}$$

   Momentum Conservation: $\vec{P} = \sum_i \vec{p}_i$

$$\frac{d\vec{P}}{dt} = \sum_i \vec{F}_i^{ext}. \tag{16}$$

   Then: $\vec{F}_i^{ext} = 0 \;\Rightarrow\; \vec{P} = \text{const}.$

12. rotation around a fixed axis

$$\omega = \frac{d\theta}{dt}, \qquad \alpha = \frac{d\omega}{dt}. \tag{17}$$

constant angular acceleration

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2. \tag{18}$$

13. A particle in a rigid body rotating with angular velocity ω has tangential velocity given by

$$v = r\omega \tag{19}$$

The tangential and radial acceleration are

$$a_{tan} = r\alpha, \qquad a_{rad} = \frac{v^2}{r} = \omega^2 r. \tag{20}$$

14. The moment of inertia of a rigid body about a given axis is defined by

$$I = \sum_i m_i r_i^2. \tag{21}$$

The angular momentum is

$$L = I\omega \tag{22}$$

and the rotational kintic energy

$$K = \frac{1}{2} I\omega^2. \tag{23}$$

15. When a force $\vec{F}$ acts on a body, the torque of that force with respect to a point O is given by

$$\tau = Fl \tag{24}$$

where l is the lever arm. A definition involving vectors is $\vec{\tau} = \vec{r} \times \vec{F}$. The angular acceleration is related to the torque by

$$\tau = I\alpha. \tag{25}$$

16. The angular momentum of a point particle is given by

$$\vec{L} = \vec{r} \times \vec{p} \tag{26}$$

In relation between torque and the change of angular momentum is

$$\vec{\tau} = \frac{d\vec{L}}{dt}. \tag{27}$$

17. Useful numbers

$$g = 9.81 \frac{m}{s^2} \tag{28}$$

$$G = 6.67 \cdot 10^{-11} \frac{Nm^2}{s^2} \tag{29}$$

$$M_E = 5.97 \cdot 10^{24} kg \tag{30}$$

$$r_E = 6.38 \cdot 10^6 m. \tag{31}$$

Simple math

1. Circle of radius r

$$A(\text{rea}) = \pi r^2, \qquad C(\text{ircumference}) = 2\pi r \tag{32}$$

2. two-dimensional vector $\vec{A} = A_x \vec{i} + A_y \vec{j}$

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} \tag{33}$$

$$A_x = A\cos(\theta) \tag{34}$$

$$A_y = A\sin(\theta) \tag{35}$$

$$\tan(\theta) = \frac{A_y}{A_x} \tag{36}$$

where θ is the angle between $\vec{A}$ and $\vec{i}$.

3. scalar product: $\vec{A} = A_x\vec{i} + A_y\vec{j}$, $\vec{B} = B_x\vec{i} + B_y\vec{j}$

$$\vec{A} \cdot \vec{B} = AB\cos(\theta) = A_xB_x + A_xB_y. \tag{37}$$

4. quadratic equation $ax^2 + bx + c = 0$ has solutions

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{38}$$

5. If $y = ax^n$ then

$$\frac{dy}{dx} = anx^{n-1}. \tag{39}$$