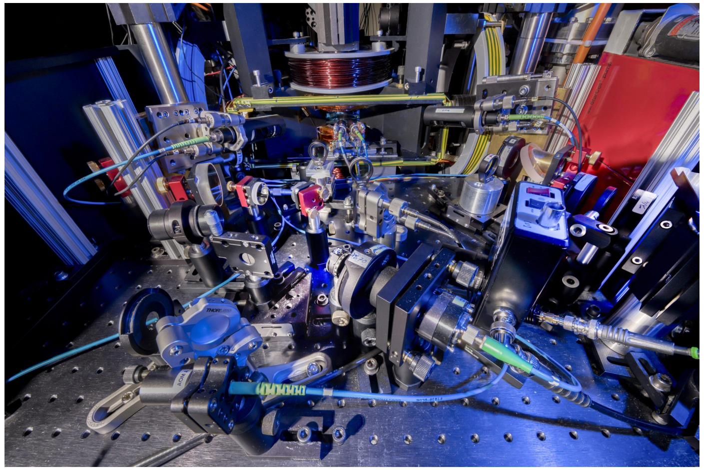
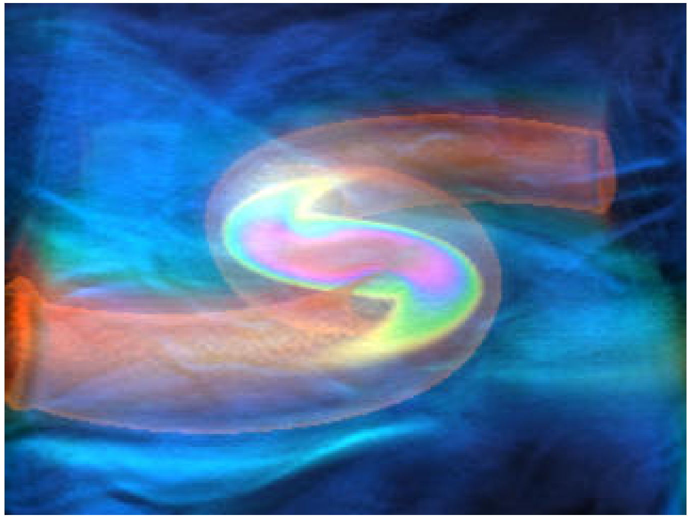
From Cold Atoms to Nuclei and Neutron Stars

Thomas Schaefer, North Carolina State University





Why study the unitary Fermi gas?

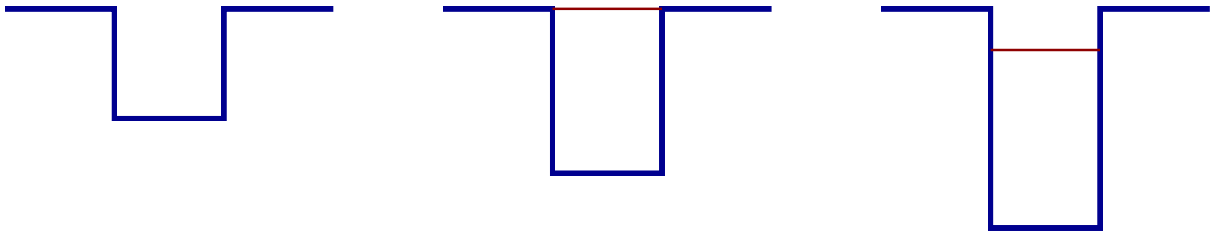
Very simple and clean model system for strong correlations.

Universality connects atomic and nuclear systems.

Impressive progress in experimental control: Tune interactions, temperature, external fields, linear response, etc.

Non-relativistic fermions in unitarity limit

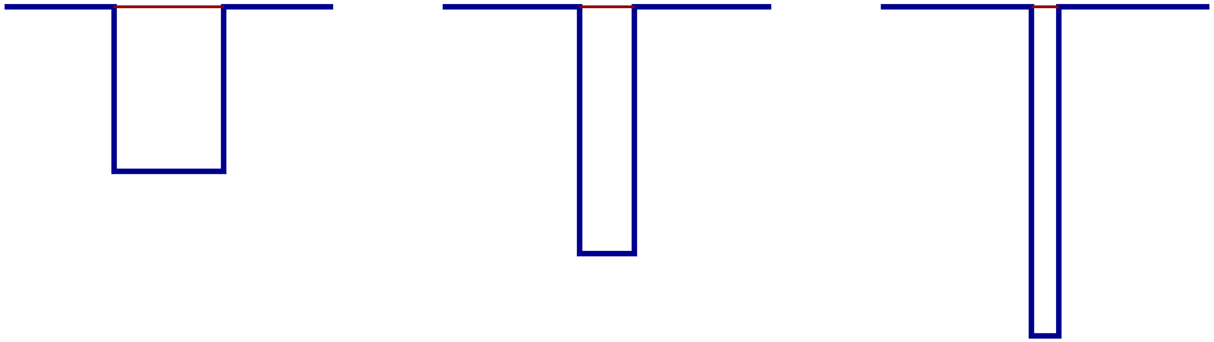
Two body interaction: Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Non-relativistic fermions in unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r} \exp(-r/a)$$

Fermi gas at unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$ (DR: $C_0 \to \infty$)

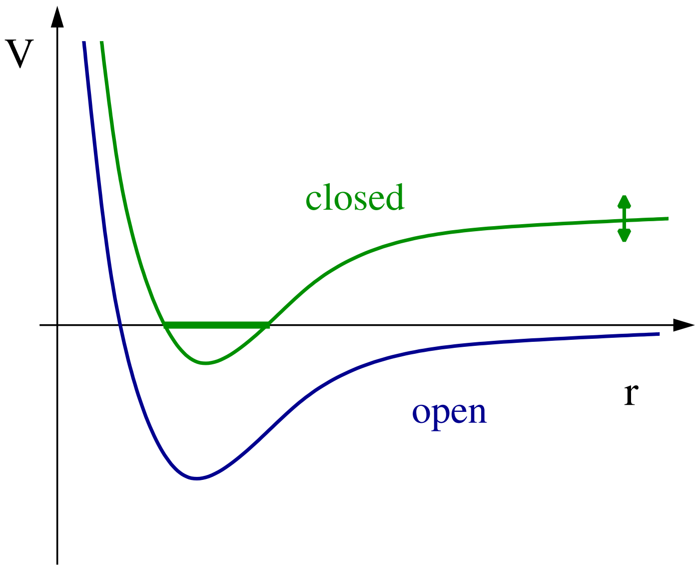
This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

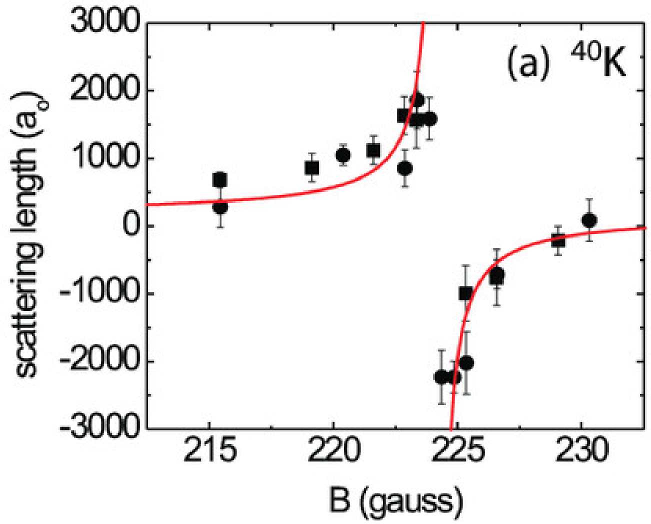
$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

$\phi \sim \psi_\uparrow \psi_\downarrow$ auxiliary "pair" or "dimer" field.

Experimental realization: Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"

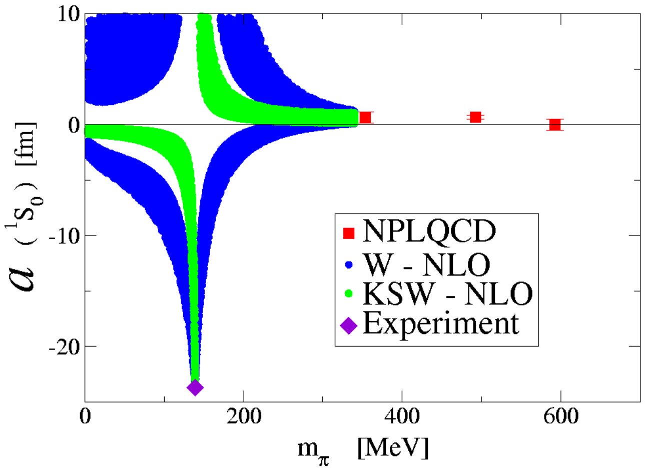




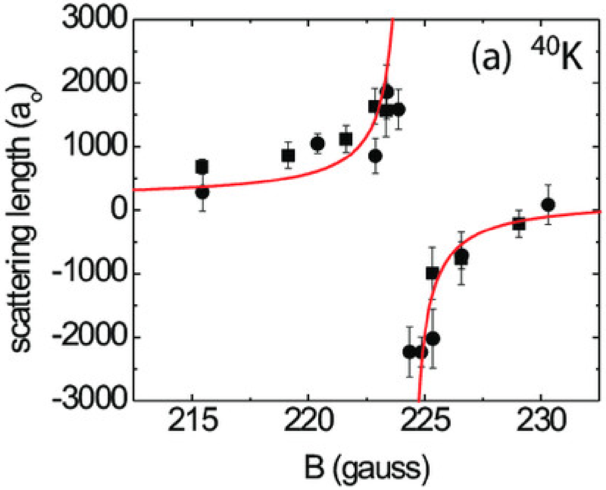
Feshbach resonance $a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$

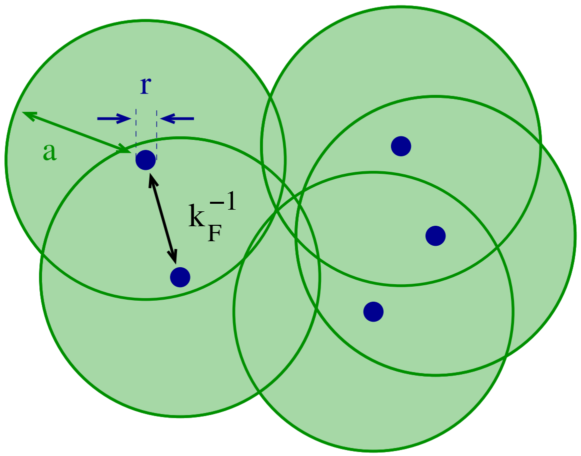
Universality: From neutrons to atoms

Neutron Matter



^{6}Li Feshbach resonance





What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$

Outline

1. Equation of state: From trapped atoms to neutron stars

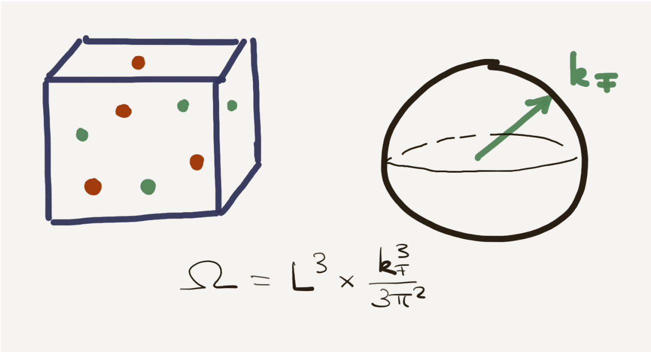
2. The contact: From the tail of the momentum distribution to short range correlations in nuclei.

3. Un-nuclear physics: From trapped few-body systems to the disintegration of halo nuclei

4. Everything flows: Elliptic flow from traps to heavy ions

1. Equation of state

Free fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

$$\mathcal{E} = E/V \sim (N/V)^{5/3}$$

Unitarity limit ($a \to \infty$, $r \to 0$). No expansion parameters.

$$\frac{E}{N} = \xi \, \frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (George Bertsch, 1998): Determine ξ.

Is $\xi > 0$ (is the system stable)?

How to measure ξ with trapped atoms

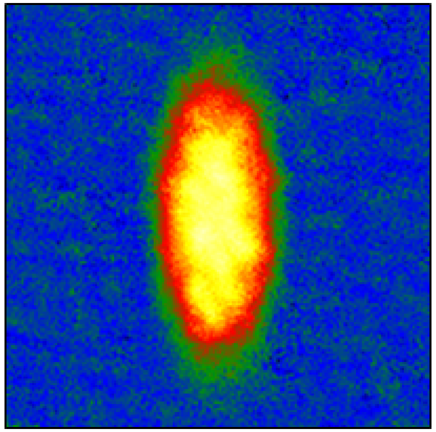
Trapped gas in hydrostatic equilibrium

$$\frac{1}{n}\vec{\nabla}P = -\vec{\nabla}V_{ext} \qquad P = \frac{2}{3}\mathcal{E}$$

Pressure determines size of the cloud ($V_{ext} = \frac{1}{2}m\omega^2 x^2$).

$$r(a=0) = \sqrt{\frac{2E_F}{m\omega^2}} \qquad r(a=\infty) = \xi^{1/4} r(0)$$

Cloud size can be measured with a CCD camera and a ruler

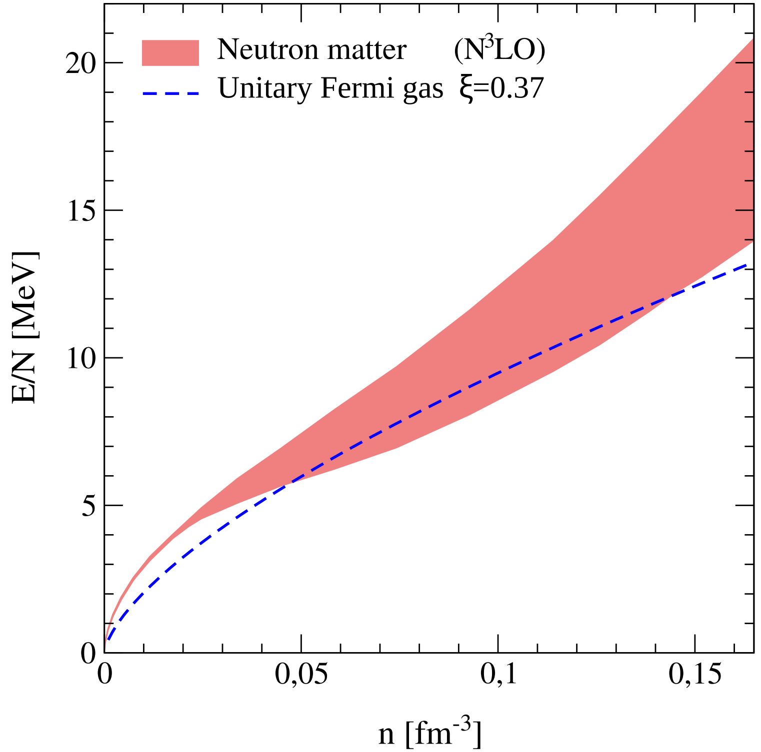


modern value

$$\xi = 0.37(5)$$

(MIT, Sommer et al.)

Neutron matter equation of state

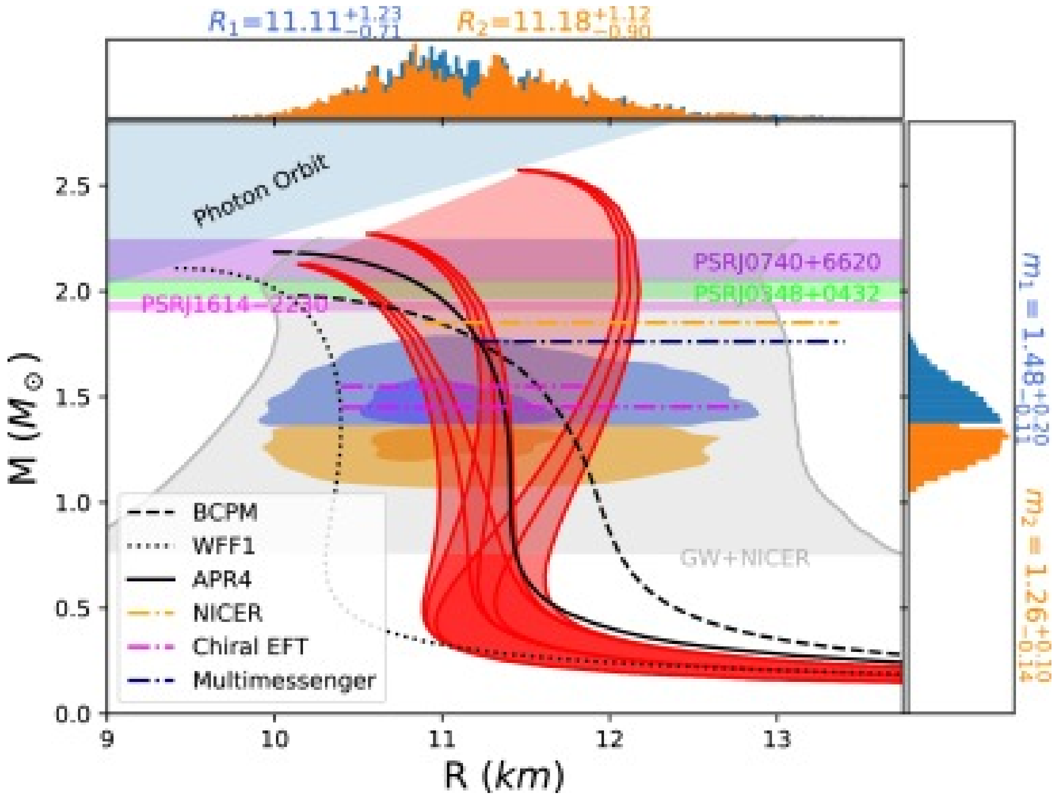


$n \lesssim 0.1\,\mathrm{fm}^{-3}$: Unitary gas with a^{-1}, r corrections.

$n \gtrsim 0.1\,\mathrm{fm}^{-3}$: Repulsive 2-body, 3-body forces.

$n \gtrsim 0.2\,\mathrm{fm}^{-3}$: New degrees of freedom.

Neutron Star Mass-Radius relation



$M < 1.0 M_\odot$: Well constrained neutron matter EOS.

$M \sim (1.4 - 2.0) M_\odot$: Radii constrain high density EOS.

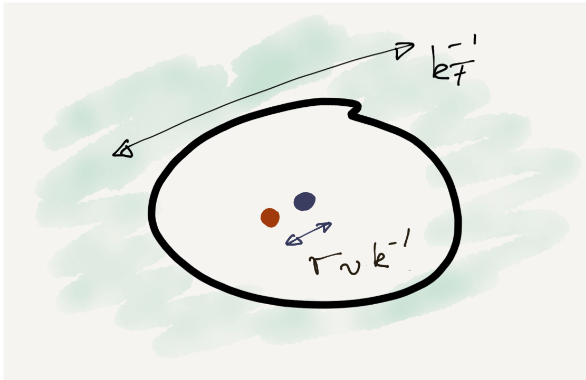
Adam et al. 2006.07983.

2. Short range correlations and the "Contact"

Consider short distance structure of unitary gas

$$\langle n_\uparrow(R + r/2) n_\downarrow(R - r/2)\rangle \simeq \frac{1}{16\pi^2}\frac{\mathcal{C}}{r^2}$$

$\mathcal{C}$: *Tan's contact density*



Related object: Momentum distribution

$$n_\sigma(k) = \int d^3R \int d^3r\, e^{-ikr}\langle \psi_\sigma^\dagger(R - r/2)\psi_\sigma(R + r/2)\rangle$$

The large momentum (short distance) tail of the distribution is

$$n_\sigma(k) = \frac{C}{k^4} \qquad\qquad C = \int d^3r\, \mathcal{C}(r) \quad \textit{Contact}$$

Shina Tan, cond-mat/0505200.

Short range correlations, continued

The contact is related to the the pair density

$$\mathcal{C} = \langle m^2 \Phi^\dagger \Phi \rangle \qquad \Phi \sim C_0 \psi_\uparrow \psi_\downarrow$$

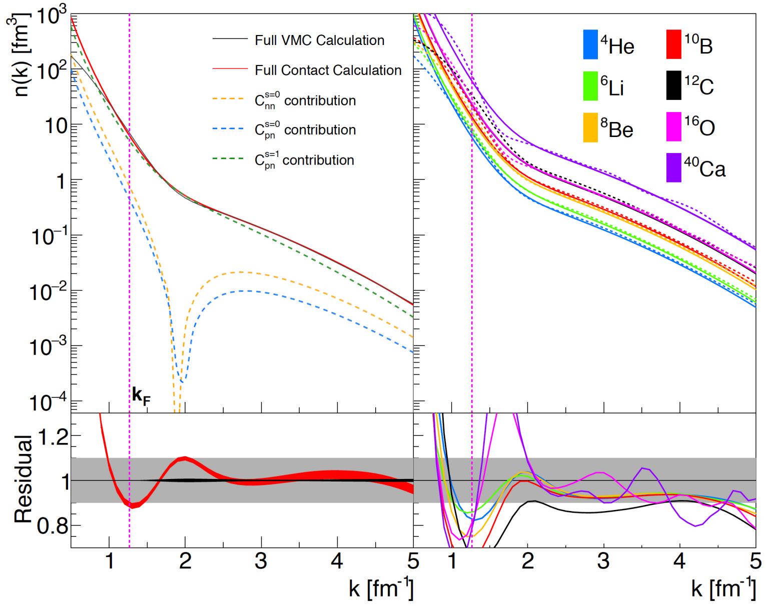
Many universal relations. Example: Thermodynamics

$$\left. \frac{dE}{da^{-1}} \right|_s = -\frac{h^2 C}{4\pi m}$$

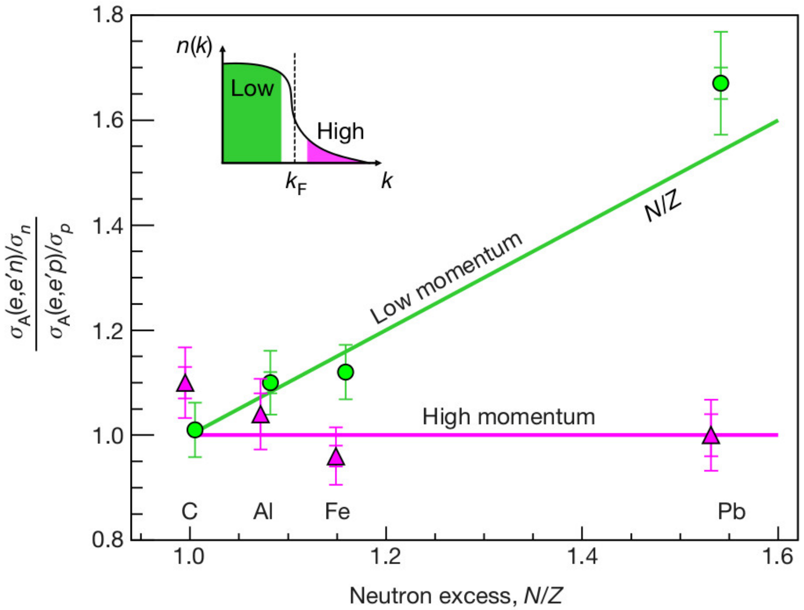
Example: Transport properties

$$\eta(\omega) \sim \frac{\mathcal{C}}{15\pi\sqrt{m\omega}}$$

Short range correlations and the contact in nuclei



Momentum distribution in nuclei (theory)
Weiss et al. 1612.00923



Pair density in nuclei
CLAS collaboration
Nature (2018)

3. Conformal symmetry and Un-nuclear physics

Unitary Fermi gas is invariant under scale

$$x \to sx, \;\; t \to s^2 t \qquad\qquad [D, H] = 2iH$$

and conformal transformations

$$x \to x/(1+ct), \;\; 1/t \to 1/t + c \qquad\qquad [C, H] = iD$$

Constrains correlation functions, e.g. pair propagator

$$G_\Phi(\omega, p) = \frac{1}{\sqrt{p^2/(4m) - \omega}}$$

Conformal symmetry: State operator correspondence

Generalized to operators $\mathcal{U}(t,x)$ with higher mass $M = Nm$

$$i\langle T\mathcal{U}^\dagger \mathcal{U}\rangle_{\omega,p} = \left(\frac{p^2}{2M} - \omega\right)^{\Delta - 5/2} \qquad \textit{conformal dimension } \Delta$$

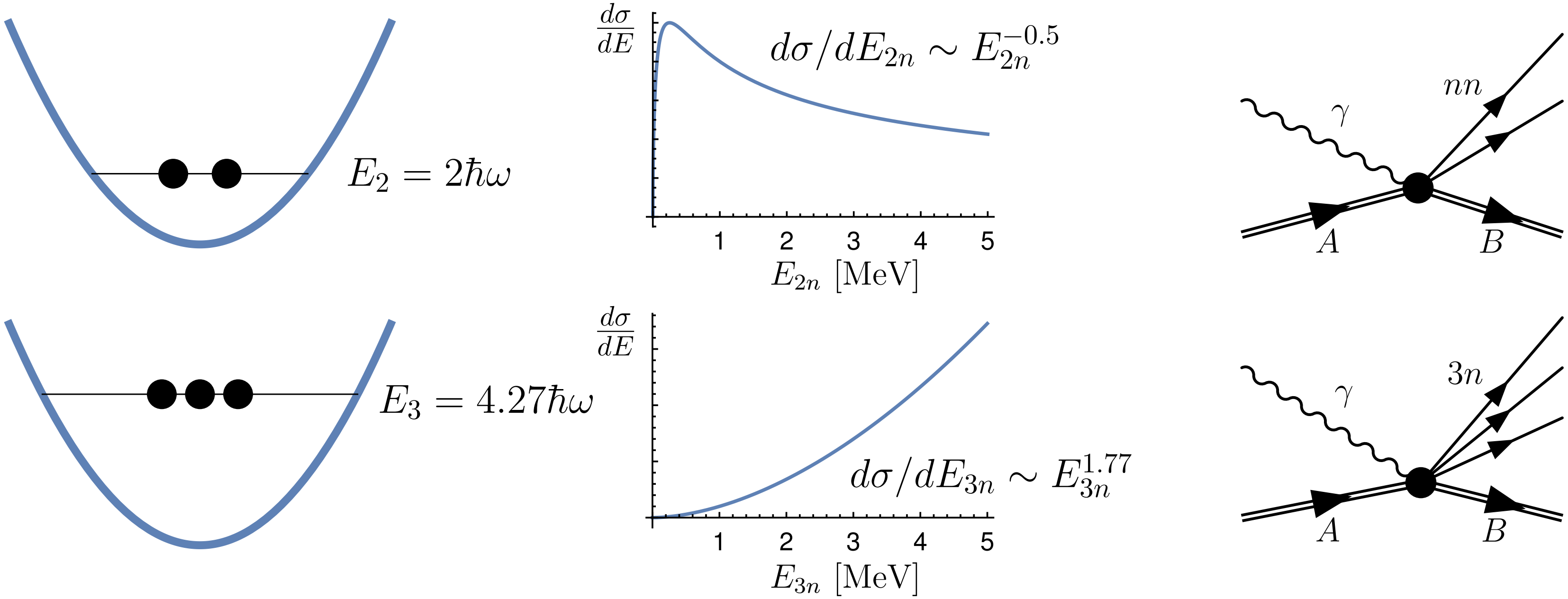
Δ related to ground state in harmonic potential

$$E = \Delta \hbar\omega \qquad \textit{state} - \textit{operator correspondence}$$

E.g single free particle: $\Delta = 3/2$

Hammer, Son, 2103.12610. Baym, Schaefer, 2109.06924.

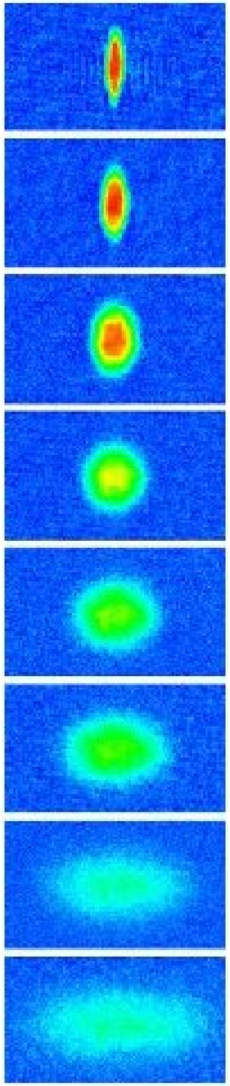
Un-nuclear physics: Nuclear reactions



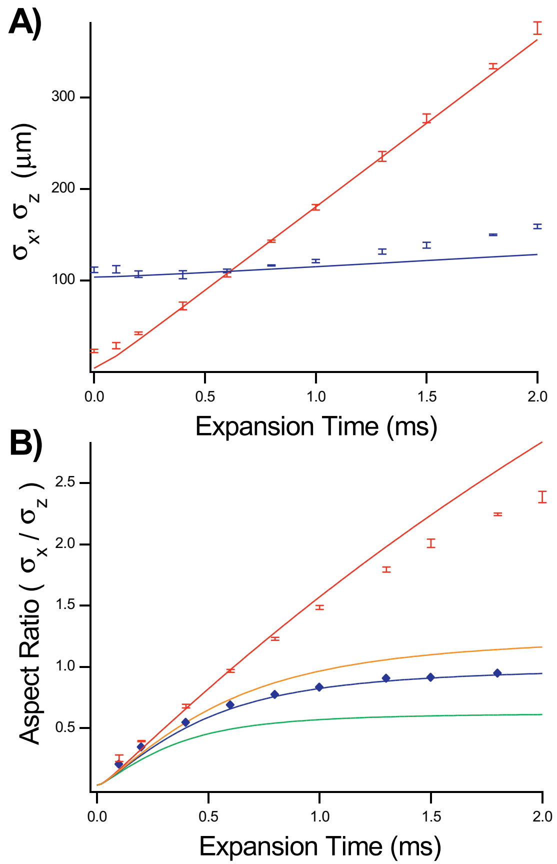
$$x + A \to B + Nn \qquad \frac{d\sigma}{dE} \sim E^{\Delta - 5/2}$$

$$[(ma^2)^{-1} \sim 0.5\, MeV] < E < [(mr^2)^{-1} \sim 5\, MeV]$$

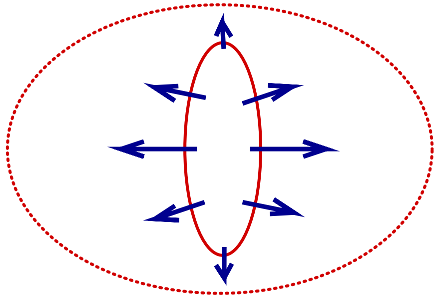
4. Elliptic flow in the unitary Fermi gas



O'Hara et al. (2002)



Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Fluid dynamics

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \vec{\jmath}^{\rho} = 0 \qquad \frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla} \vec{\jmath}^{\epsilon} = 0$$

$$\frac{\partial \pi_i}{\partial t} + \nabla_j \Pi_{ij} = 0 \qquad \vec{\jmath}^{\rho} \equiv \rho \vec{v} = \vec{\pi}$$

Scale invariance: Ideal fluid dynamics

$$\Pi^0_{ij} = P g_{ij} + \rho v_i v_j , \qquad P = \frac{2}{3}\mathcal{E}$$

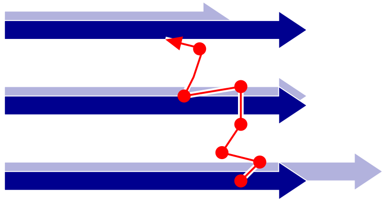
First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta \sigma_{ij} - \zeta g_{ij} \langle \sigma \rangle \qquad \zeta = 0$$

$$\sigma_{ij} = (\nabla_i v_j + \nabla_j v_i - 2/3 \delta_{ij} \nabla \cdot v) \qquad \langle \sigma \rangle = \sigma_{ii}$$

Shear viscosity: Theory

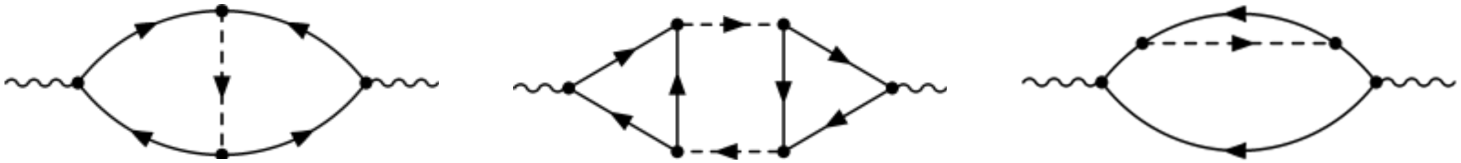
Kinetic theory: Momentum transport by diffusion of atoms

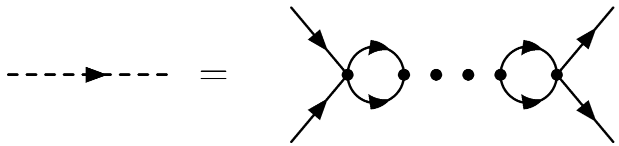


$$\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \quad (T \gtrsim T_F)$$

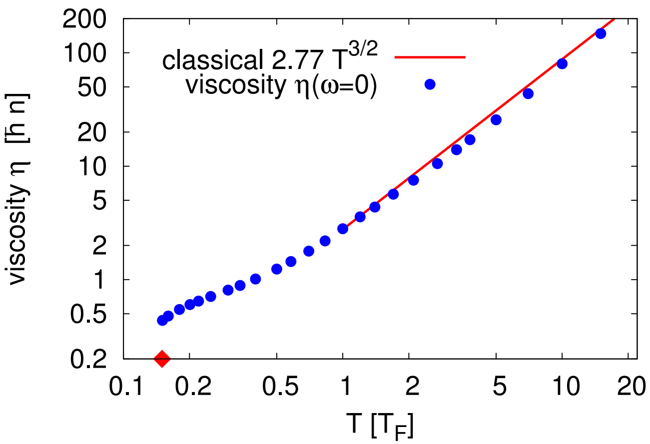
QFT: Diagrammatic content of Boltzmann equation. Kubo formula with Maki-Thompson + Azlamov-Larkin + Self-energy

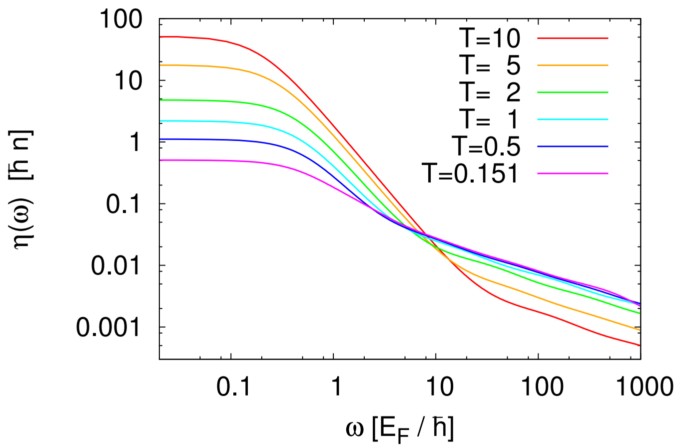
$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\int dt d^3x\, e^{-i(\omega t - kx)}\,\Theta(t)\langle[\Pi_{xy}(0),\Pi_{xy}(t,x)]\rangle$$





Can be used to extrapolate kinetic theory to $T \sim T_F$



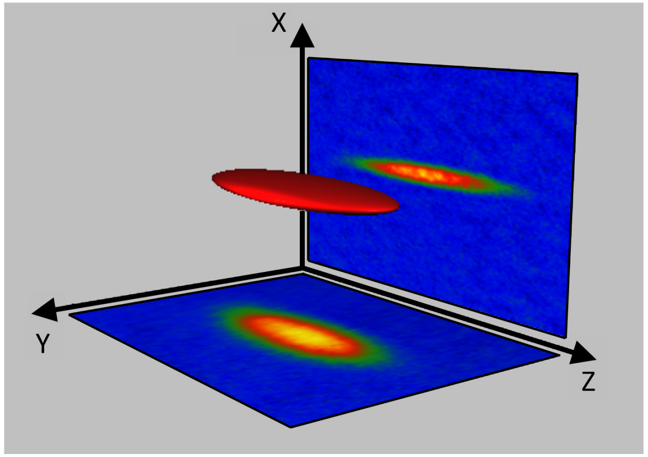


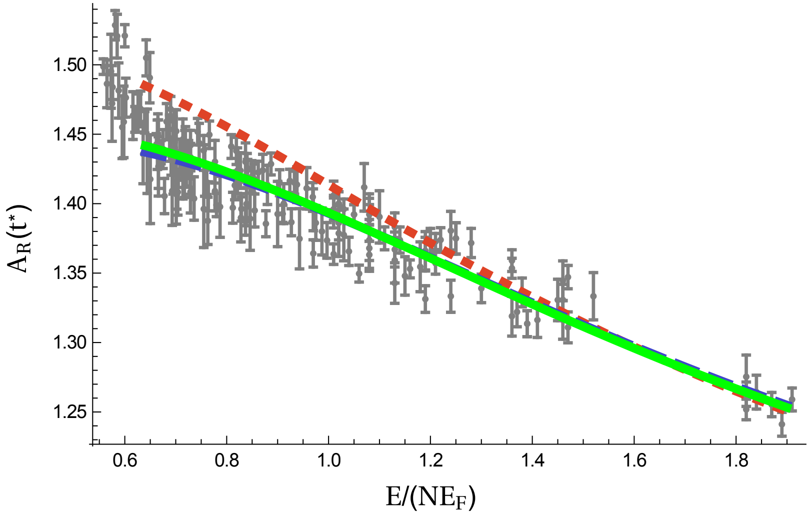
$$\eta(T \sim T_c) \sim \hbar n$$

Drude peak, universal tail.

Enss, Zwerger (2011), see also Levin (2014)

Fluid dynamics analysis



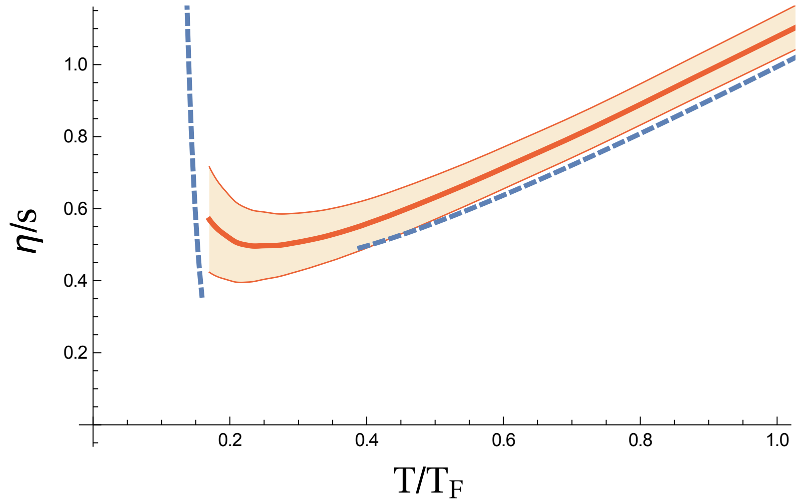


$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

Grey, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0 (mT)^{3/2} \left\{ 1 + \eta_2 n\lambda^3 + \eta_3 (n\lambda^3)^2 + \ldots \right\}$$

Reconstruct η/s (normal fluid)



$T_c \sim 0.17 T_F$. Kinetic theory at low and high T (blue dashed)

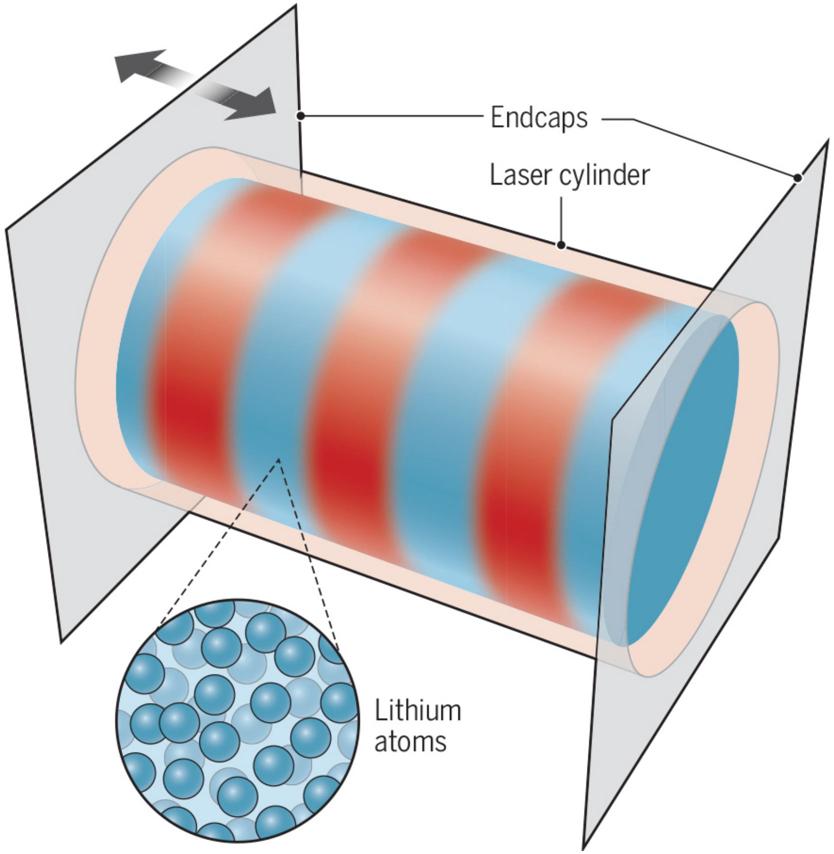
$$\eta|_{T \gg T_c} = (0.265 \pm 0.02)(mT)^{3/2}$$

$$\eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

Phenomenology: Two component model works well, $\eta \sim \eta_0 (mT)^{3/2} + \eta_1 \hbar n$

$$\eta/s|_{T_c} = 0.56 \pm 0.20$$

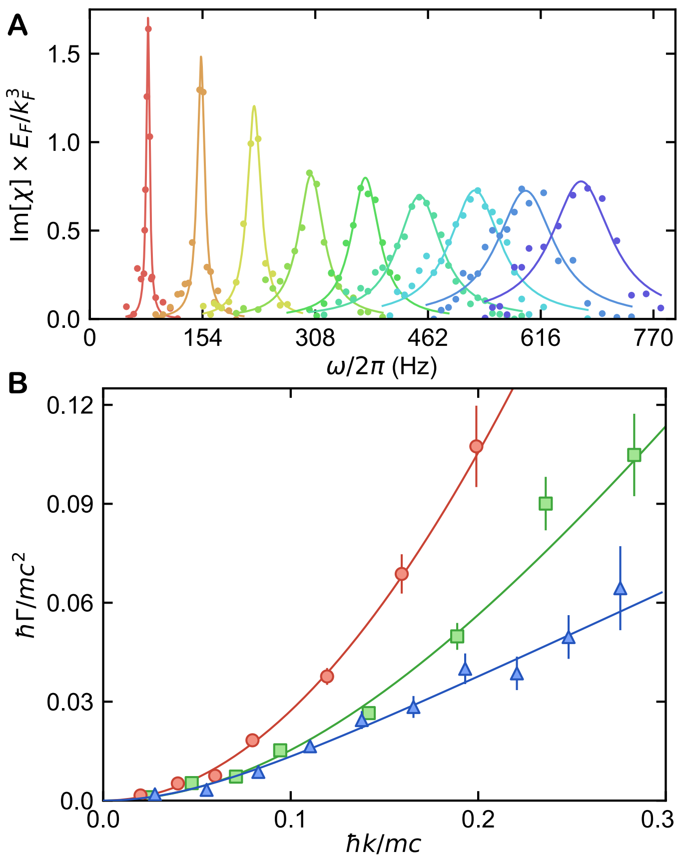
Sound attenuation (MIT)

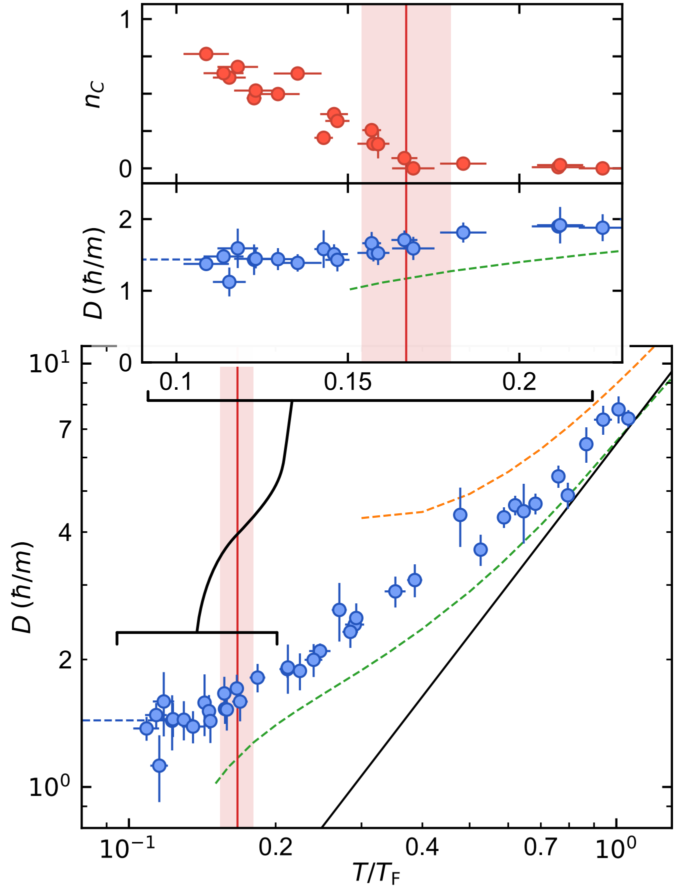


Sound attenuation (MIT)

Spectral response $\rho_k(\omega)$.

Sound diffusivity $D_s(T)$





Damping rate $\Gamma(k)$

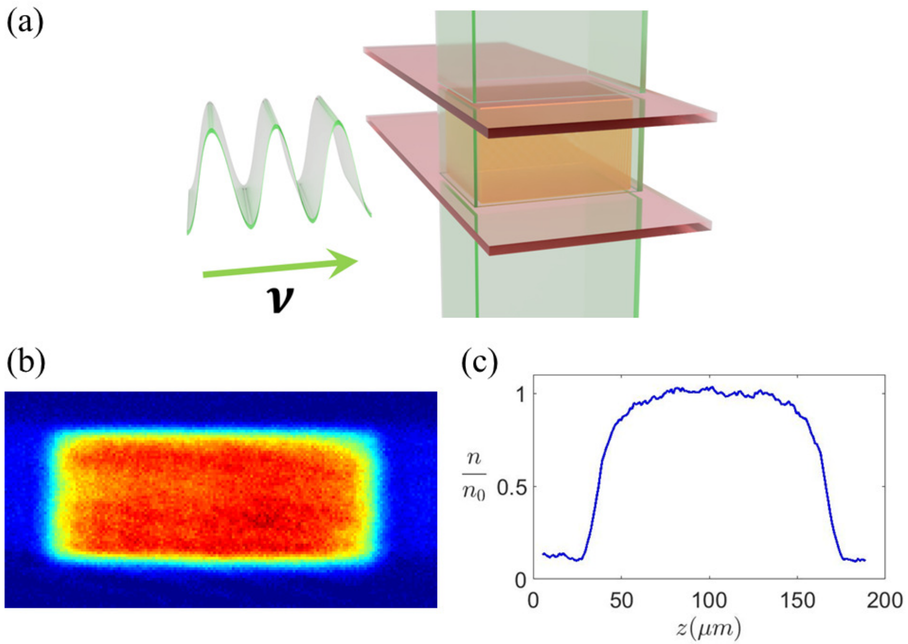
$$D_s = \frac{4\eta}{3\rho} + \frac{4\kappa T}{15P}.$$

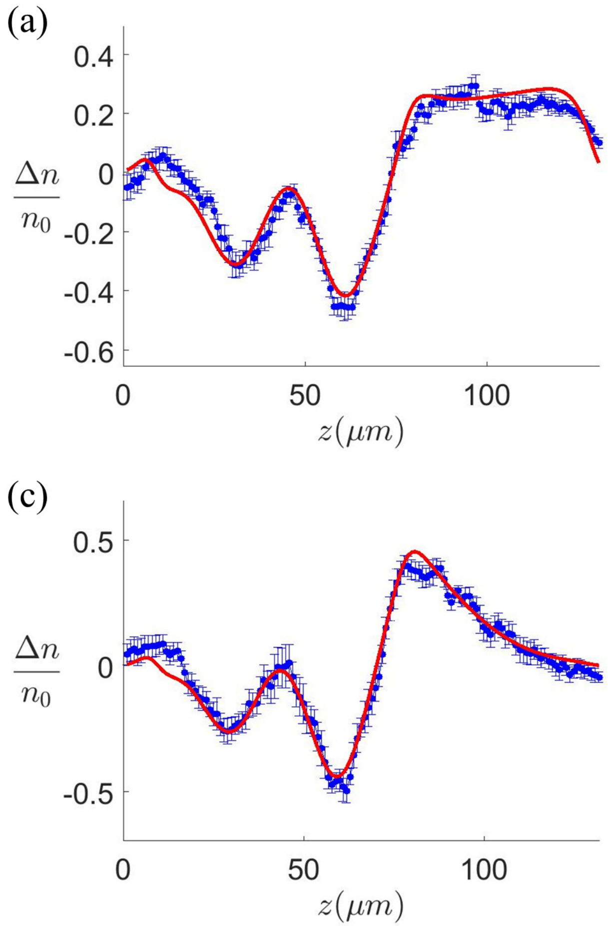
$(T/T_F = 0.36, 0.21, 0.13)$.

Patel et al., Science (2021)

Linear Response (NC State)

Baird et al., PRL 2019





$$(\kappa/\eta)(T \gg T_c) = 0.93(14)(15/4)(k_B/m)$$

Final thoughts

The unitary Fermi gas has become a paradigm for strongly correlated quantum liquids.

Universality relates the cold atomic gas to dilute neutron matter. Important for understanding analytic aspects, and as a benchmark for quantum Monte Carlo calculations.

Range of ideas continues to expand: From thermodynamics to transport, short range correlations and un-nuclear reactions.

Continued role for ultracold gases as quantum simulators, not just universal gate based quantum computers.