Nearly Perfect Fluidity in Cold Atomic Gases

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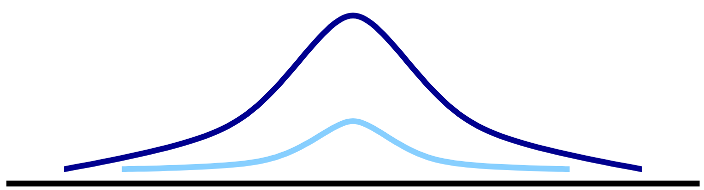
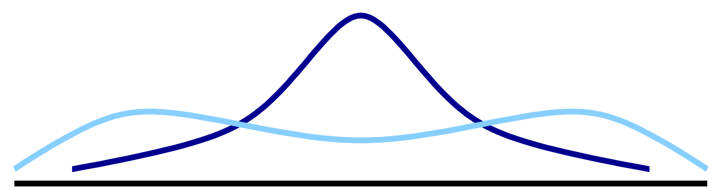
Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



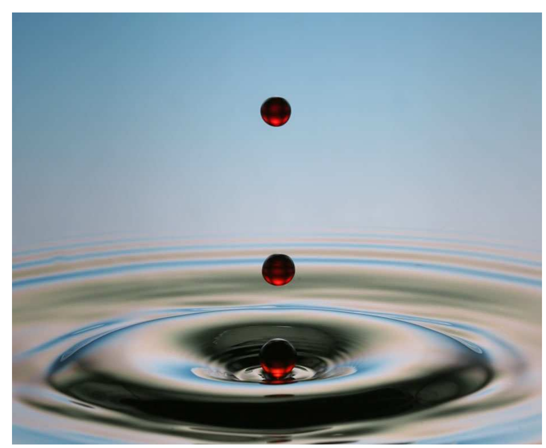
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



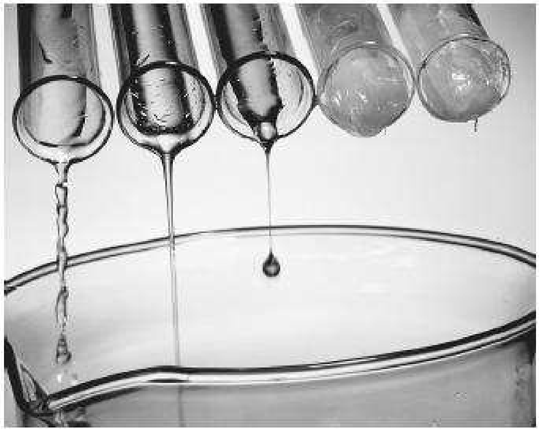
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

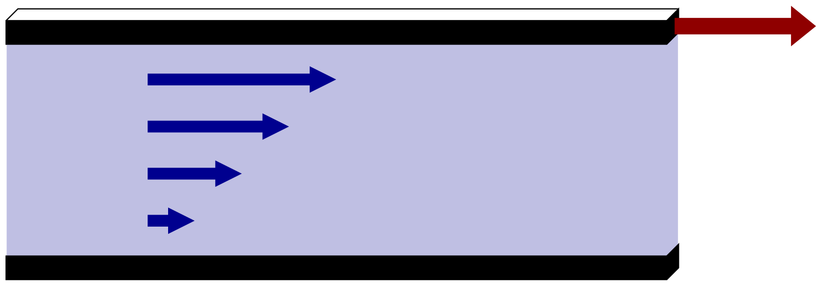
Expansion parameter $Re^{-1} = \dfrac{\eta(\partial v)}{\rho v^2} = \dfrac{\eta}{\rho L v} \ll 1$

$$Re^{-1} = \underbrace{\frac{\eta}{\hbar n}}_{\substack{\text{fluid}\\ \text{property}}} \times \underbrace{\frac{\hbar}{m v L}}_{\substack{\text{flow}\\ \text{property}}}$$

Consider $mvL \sim \hbar$: Hydrodynamics requires $\eta/(\hbar n) < 1$

Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow

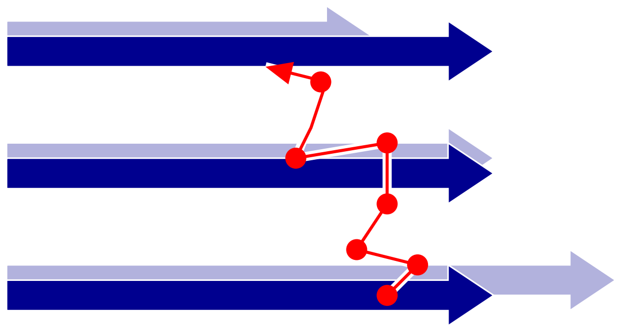


$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

Shear viscosity

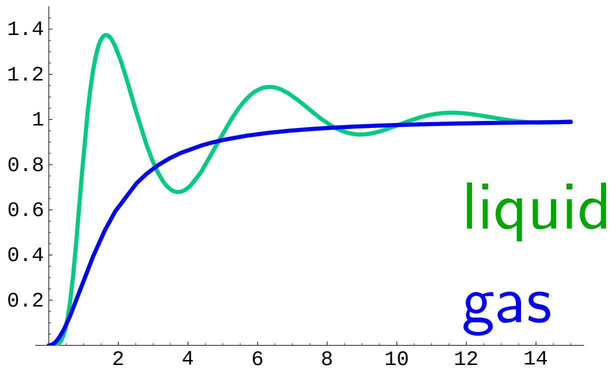
non-interacting gas ($\sigma \to 0$): $\eta \to \infty$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas: $\dfrac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$

but: kinetic theory not reliable!

what happens if the gas condenses into a liquid?



Eyring, Frenkel:

$$\eta \simeq hn \exp(E/T) \geq hn$$

Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

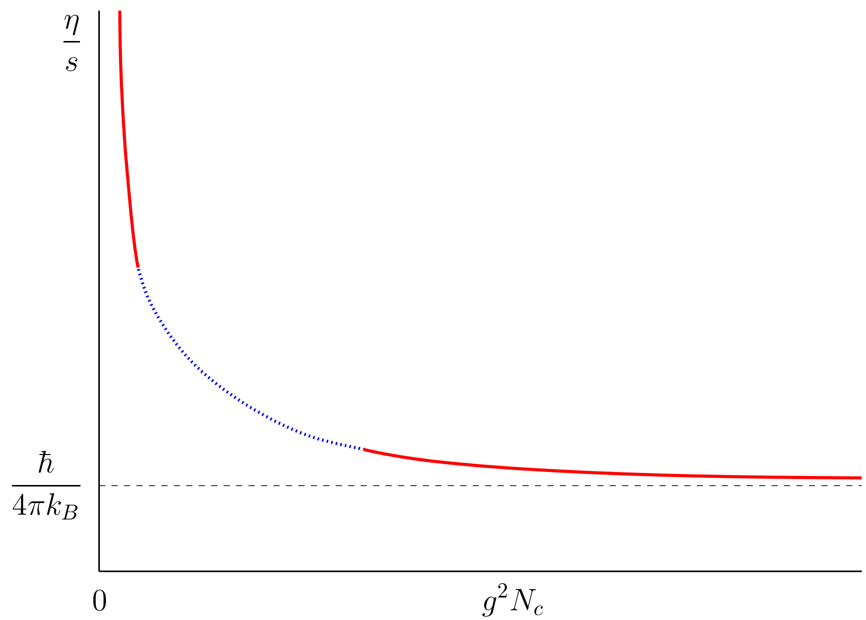
CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon

shear viscosity $\Leftrightarrow$ Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

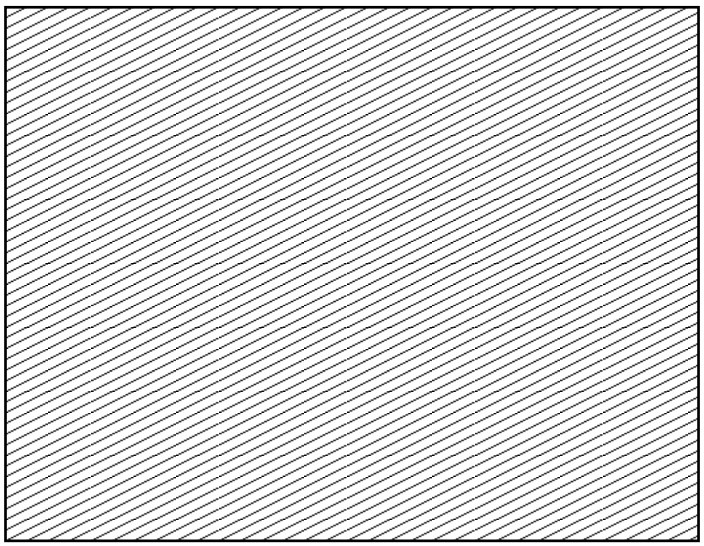
Son and Starinets (2001)



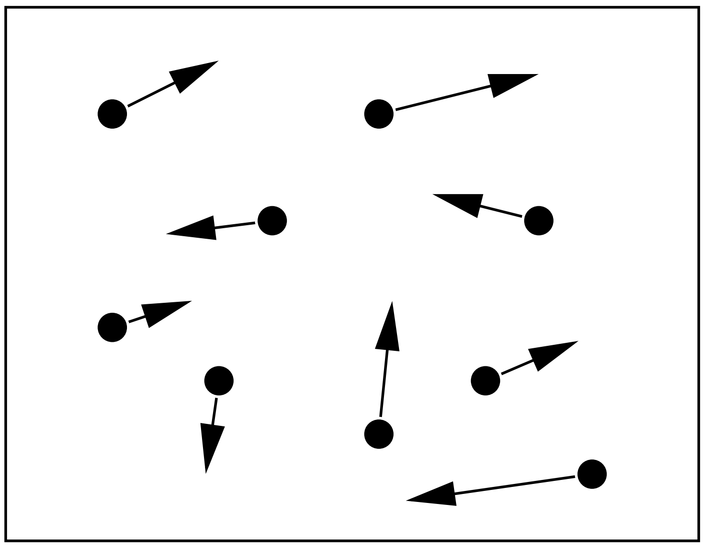
Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

Kinetics vs No-Kinetics



AdS/CFT low viscosity goo



pQCD kinetic plasma

Effective theories for fluids (Here: Weak coupling QCD)

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

Effective theories (Strong coupling)

$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

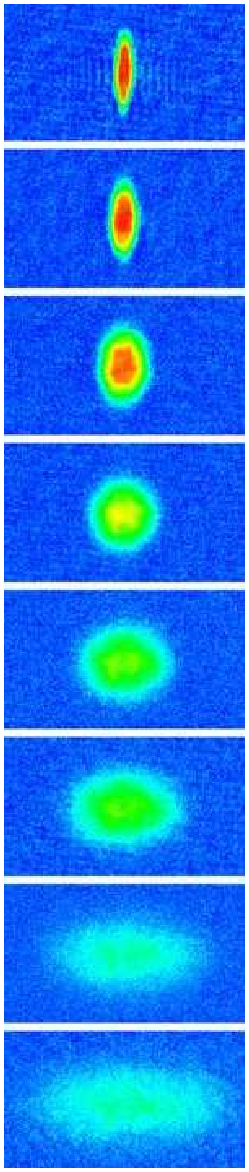
Bound is incompatible with weak coupling and kinetic theory

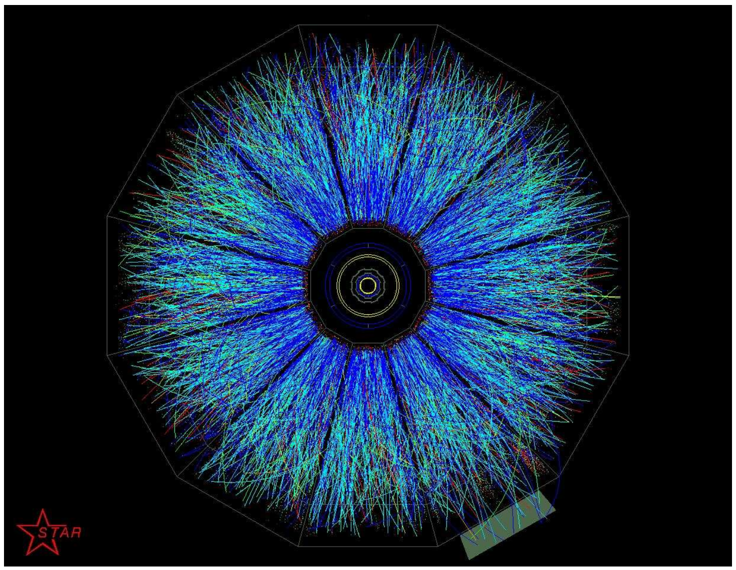
strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

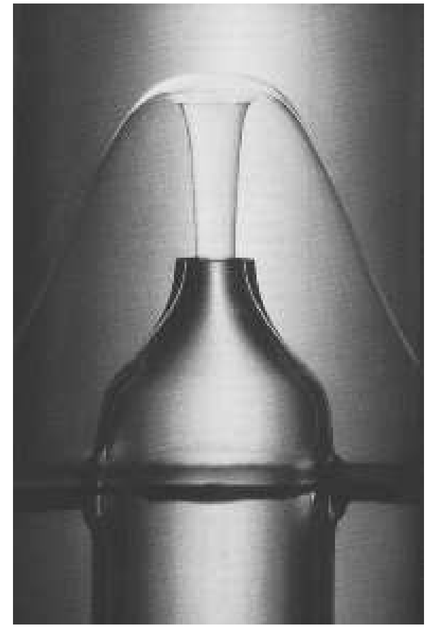
(Almost) scale invariant systems

Perfect Fluids: The contenders





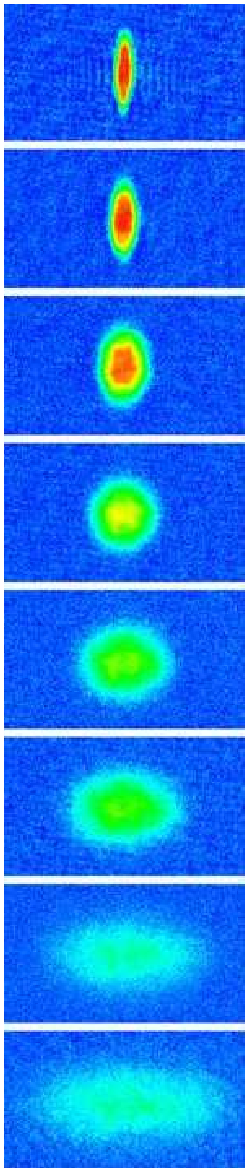
QGP (T=180 MeV)

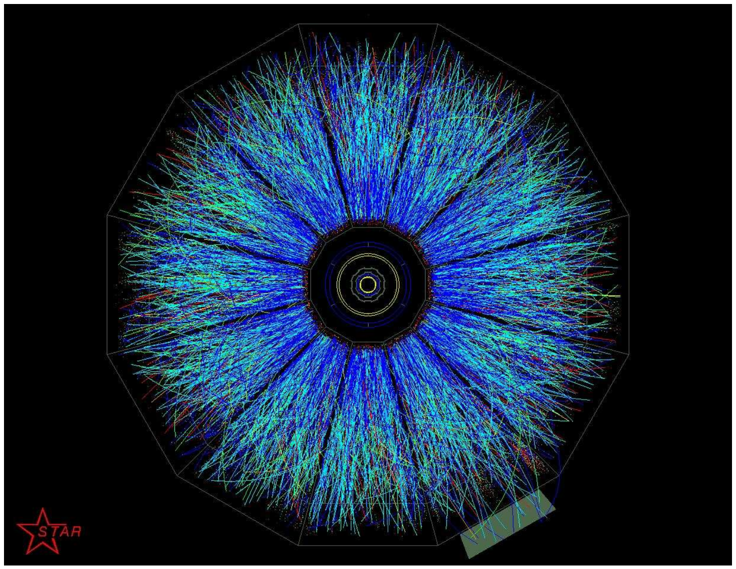


Liquid Helium
(T=0.1 meV)

trapped atoms
(T=0.1 neV)

Perfect Fluids: The contenders





QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

Liquid Helium

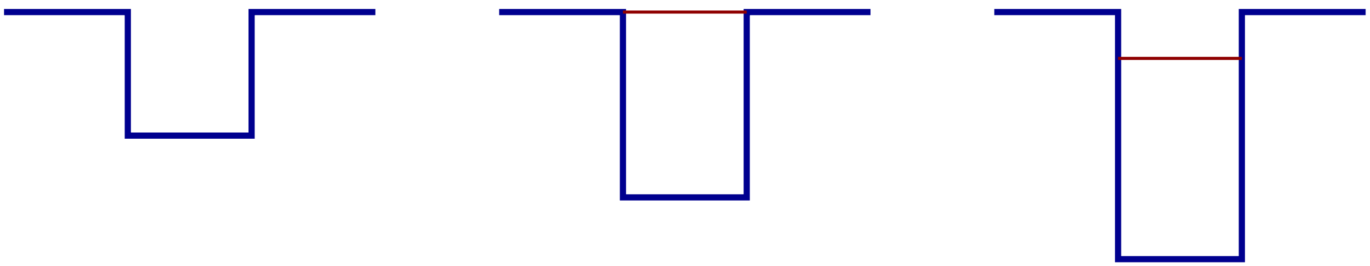
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η / s

Unitarity limit

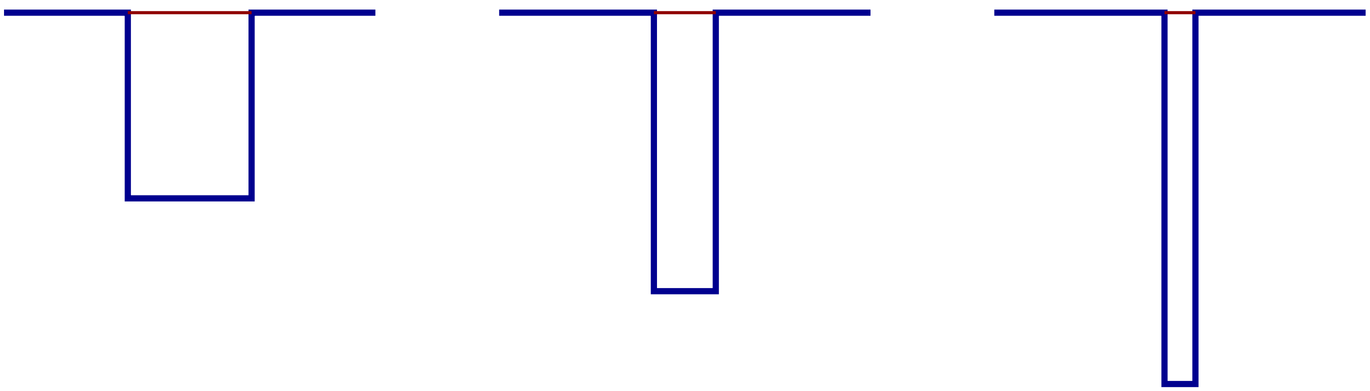
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{ar}} \exp(-r/a)$$

Dilute Fermi gas: field theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth: HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

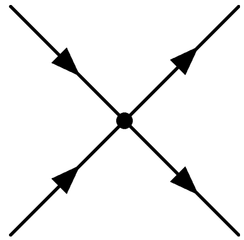
$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \Big(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \Big) - \frac{1}{C_0} \phi^* \phi \ ,$$

Low T ($T < T_c \sim \mu$): Pairing and superfluidity, $\langle \phi \rangle \neq 0$

Kinetic theory

High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2}$$



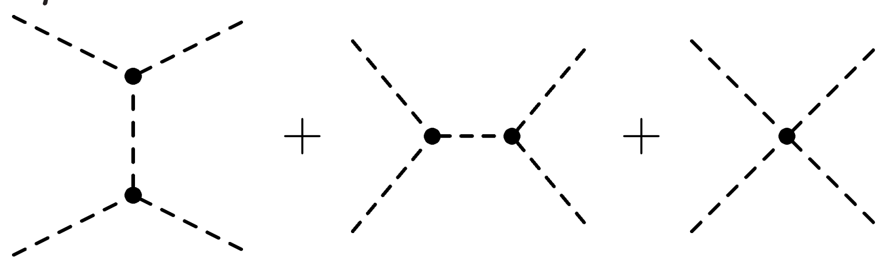
Bruun (2005)

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

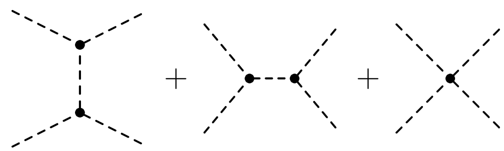
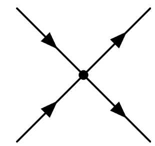
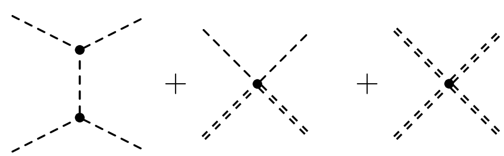
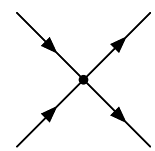
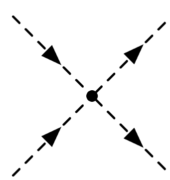
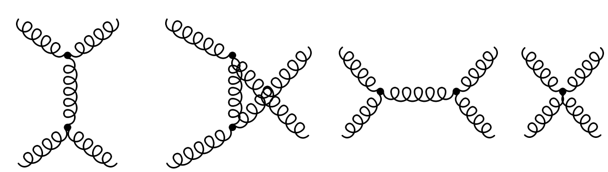
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$

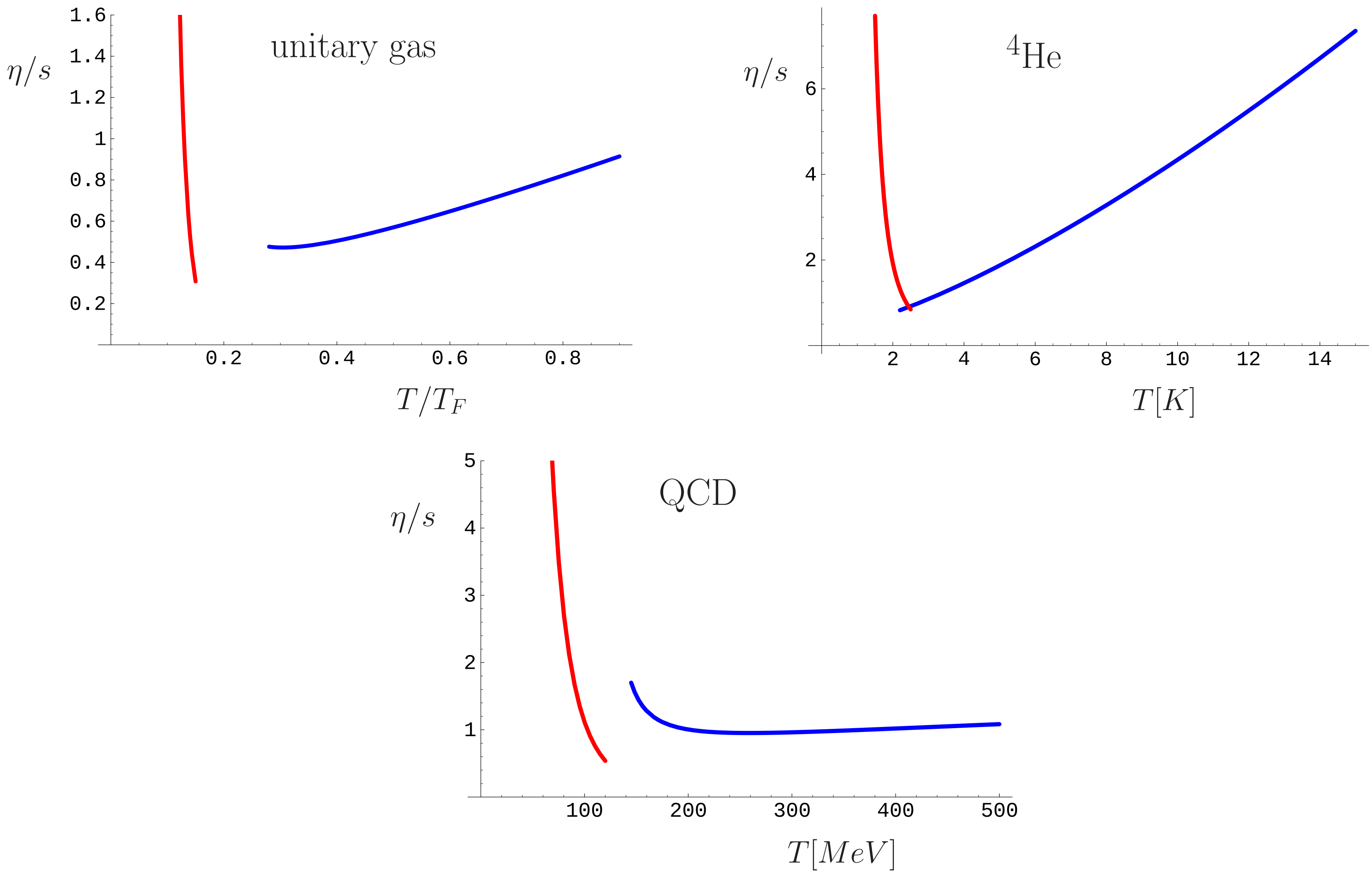


T.S., G.R. (2007)

Kinetic Theory: Quasiparticles

	low temperature	high temperature
unitary gas	phonons	atoms
		
helium	phonons, rotons	atoms
		
QCD	pions	quarks, gluons
		

Kinetic theory summary



Shear viscosity: Sum rules

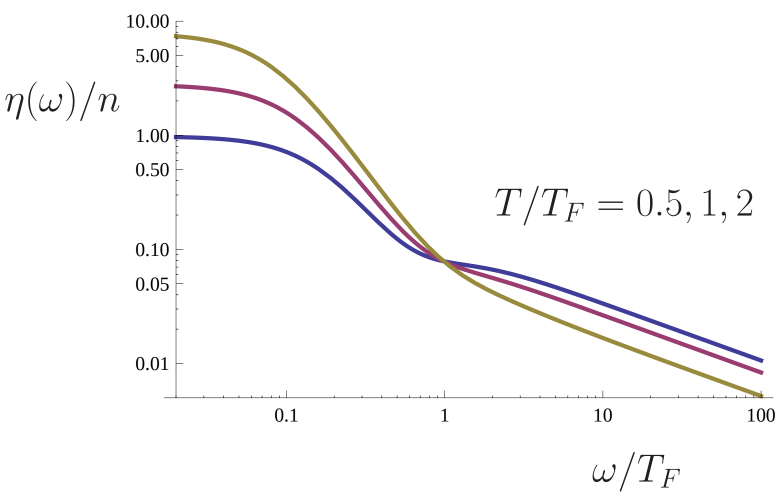
Randeira and Taylor proved the following sum rules

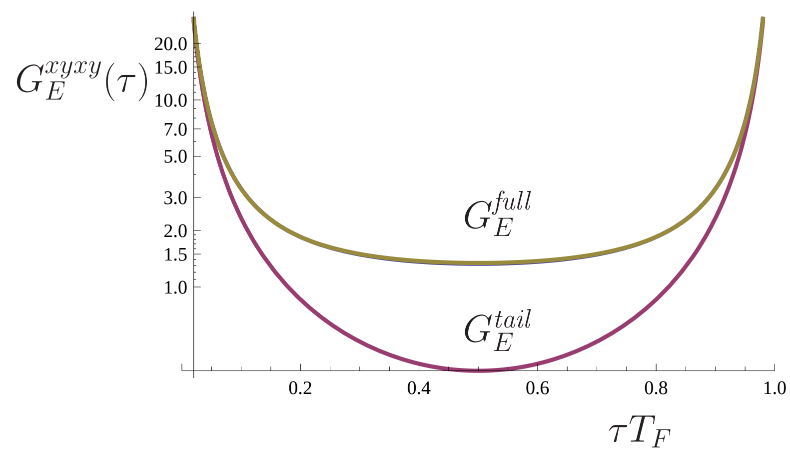
$$\frac{1}{\pi}\int dw\left[\eta(\omega)-\frac{C}{10\pi\sqrt{m\omega}}\right]=\frac{\mathcal{E}}{3}-\frac{C}{10\pi ma}$$

$$\frac{1}{\pi}\int dw\;\zeta(\omega)=\frac{1}{72\pi ma^2}\left(\frac{\partial C}{\partial a^{-1}}\right)$$

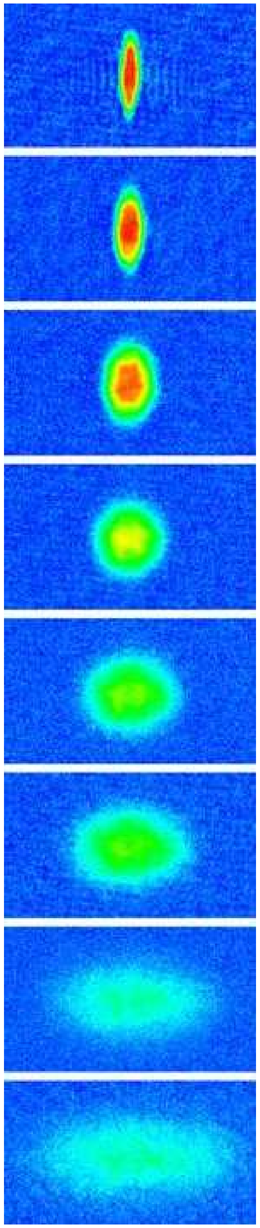
where C is Tan's contact, $\rho(k)\sim C/k^4$.

Sum rules constrain spectral function and euclidean correlator

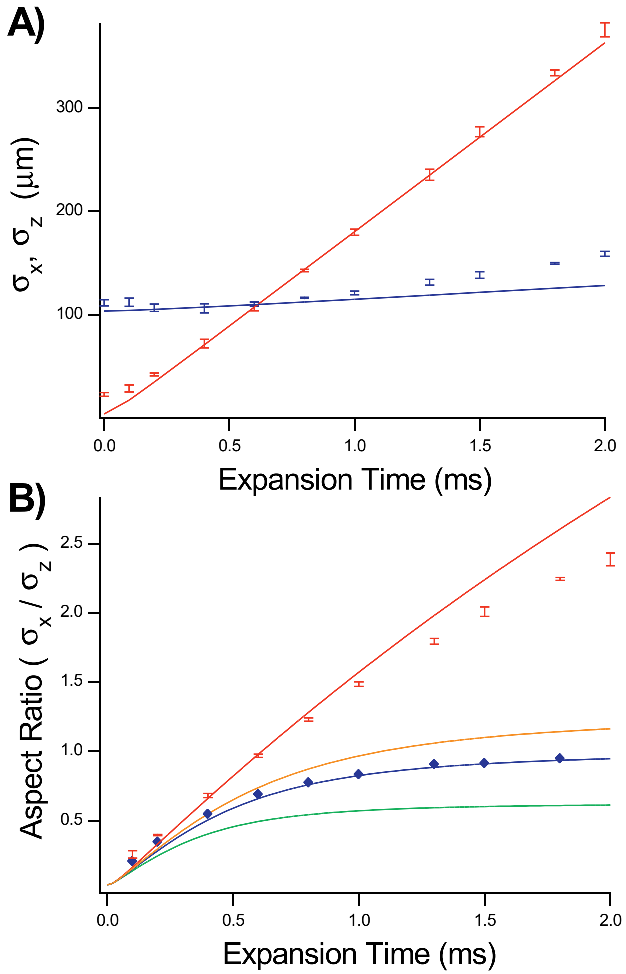




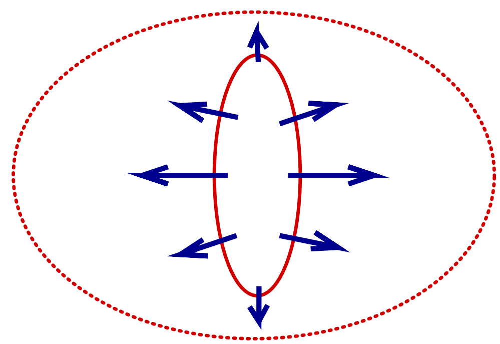
Almost ideal fluid dynamics



O'Hara et al. (2002)

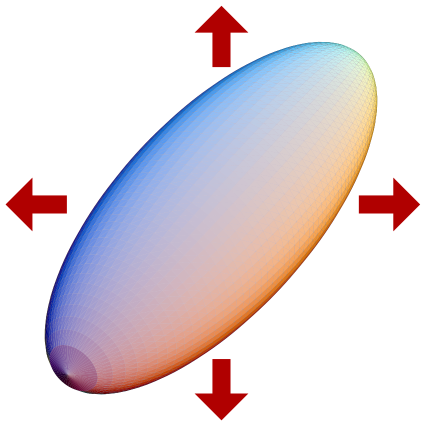


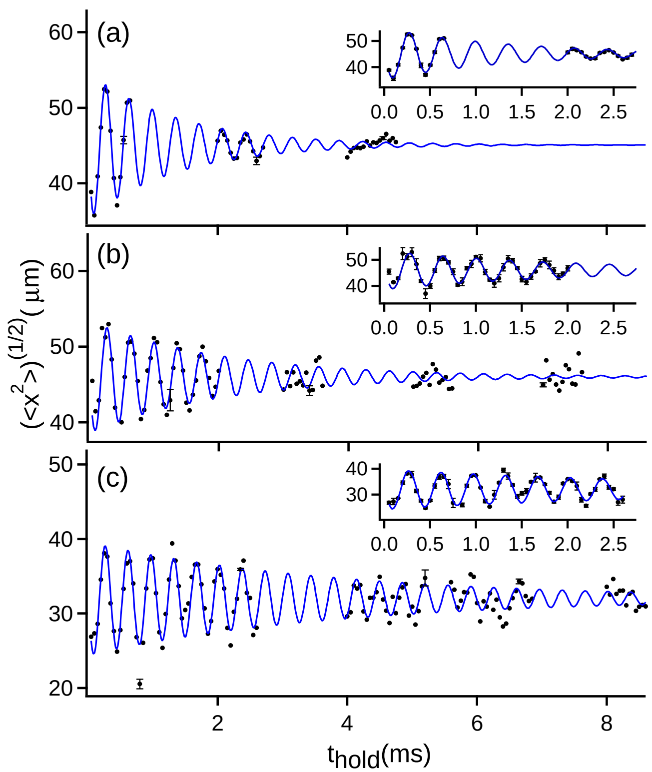
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Hydrodynamics: Collective modes

Radial breathing mode





Ideal fluid hydrodynamics ($P = \frac{2}{3}\mathcal{E}$)

$$\begin{aligned}
\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) &= 0 \\
\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} &= -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}
\end{aligned}$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

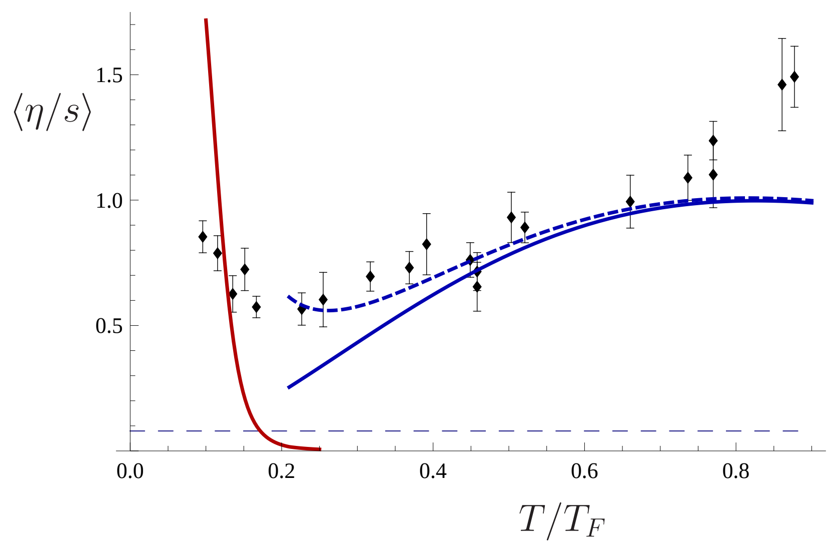
Damping of collective mode

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio (assuming $\zeta = \kappa = 0$)

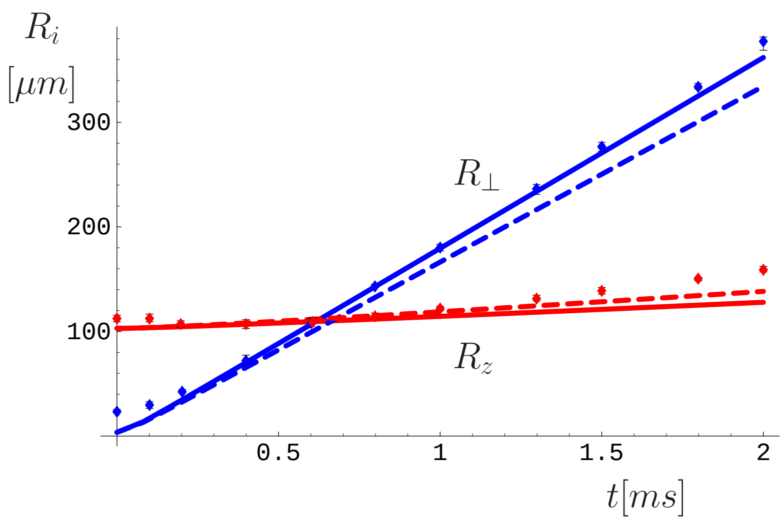
$$\boxed{\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\,\frac{\Gamma}{\omega_\perp}\,\frac{E_0}{E_F}\,\frac{N}{S}}$$

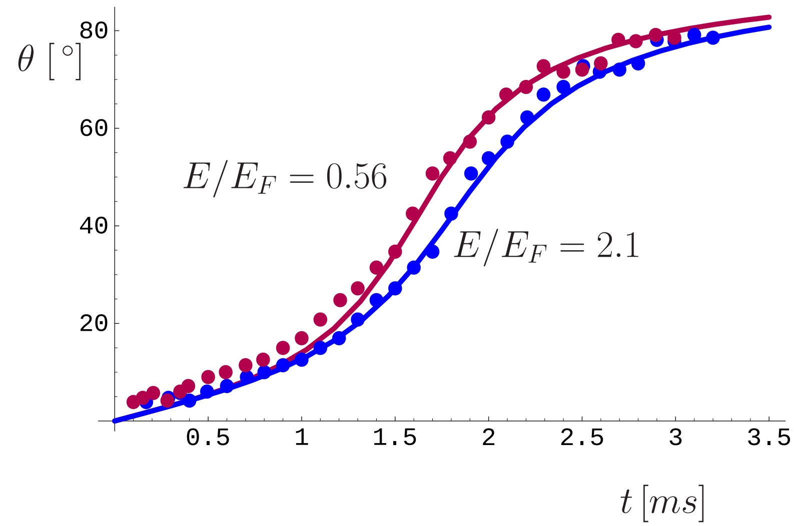


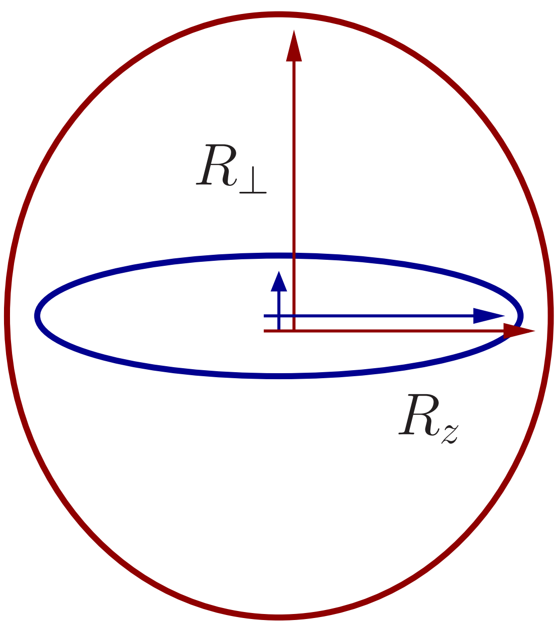
Schaefer (2007), see also Bruun, Smith

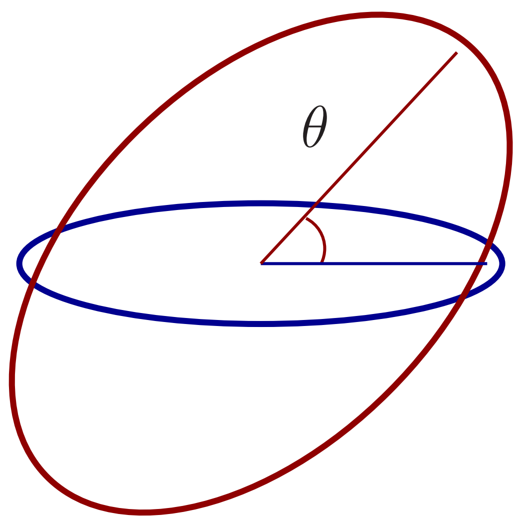
$T \ll T_F$ $\quad$ $T \gg T_F$, $\tau_R \simeq \eta/P$

Hydrodynamics: Free expansion and rotation









O'Hara et al (2002), Clancy et al (2007)

Scaling Flows

Universal equation of state $$P = \frac{2}{3}\mathcal{E}$$

Equilibrium density profile

$$n_0(x) = n(\mu(x), T) \qquad \mu(x) = \mu_0\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

Scaling Flow: Stretch and rotate profile

$$\mu_0 \to \mu_0(t), \qquad T \to T_0(\mu_0(t)/\mu_0), \qquad R_x \to R_x(t), \ \ldots$$

Linear velocity profile

$$\boxed{\vec{v}(x,t) = (\alpha_x x, \alpha_y y, \alpha_z z) + \alpha\vec{\nabla}(xy) + \vec{\omega}\times\vec{x}}$$

"Hubble flow"

Scaling hydrodynamics

Write $R_i(t) = b_i(t) R_i(0)$. Euler equation

$$\ddot{b}_\perp = \frac{\omega_\perp^2}{(b_\perp^2 b_{||})^{2/3}} \frac{1}{b_\perp} \qquad\qquad b_\perp(\omega_\perp t \gg 1) \sim \sqrt{\frac{3}{2}}\, \omega_\perp t$$

Dissipation breaks scaling behavior. Consider moments of Navier-Stokes equation

$$\int d^3x\, x_k\, (\rho \dot{v}_i + \ldots) = \int d^3x\, x_k\, (-\nabla_i P - \nabla_j \delta\Pi_{ij})$$

Integration by parts: only sensitive to $\langle \eta \rangle / E_0$.

$$\ddot{b}_\perp = \frac{\omega_\perp^2}{(b_\perp^2 b_{||})^{2/3} b_\perp} - \frac{2\beta\omega_\perp}{b_\perp} \left(\frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_{||}}{b_{||}} \right)$$

$$\beta = \frac{\langle \eta \rangle}{N} \frac{E_F}{E_0} \frac{1}{(3N\lambda)^{1/3}}$$

Scaling hydrodynamics, continued

Friction term leads to delayed expansion

$$\frac{\delta t_0}{t_0} = 0.008 \left(\frac{\langle \eta/s \rangle}{1/(4\pi)}\right) \left(\frac{2 \cdot 10^5}{N}\right)^{1/3} \left(\frac{S/N}{2.3}\right) \left(\frac{0.85}{E_0/E_F}\right)$$

Effect does not exponentiate, harder to observe.

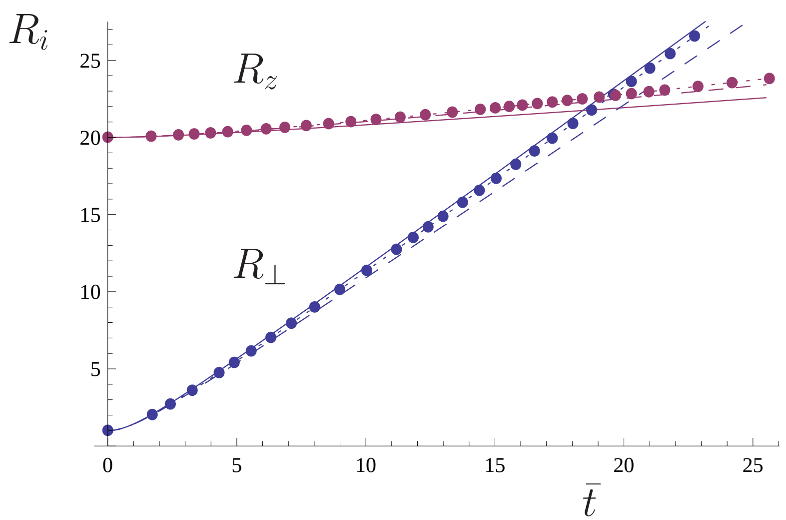
<u>Issues:</u> Reheating not taken into acount.

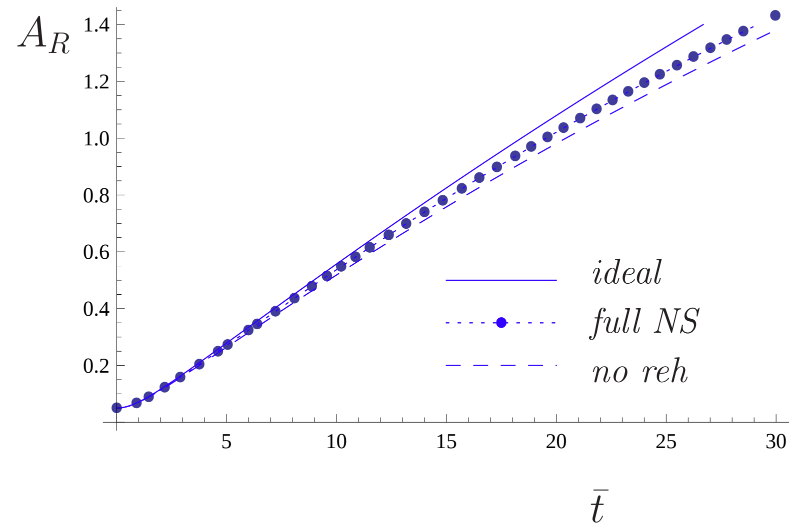
Dilute corona $\eta \sim T^{3/2} \longrightarrow \nabla_i \delta \Pi_{ij} = 0$. No force (?)

$Kn \sim (b_{||}/b_{\perp})^{1/3}$ drops $\longrightarrow$ No freezeout (?)

Navier-Stokes: Numerical results

Consider $\eta = \alpha_n n$ with $\alpha_n = const.$





Reheating leads to reacceleration.

Characteristic hydro effect: curvature of $A_R(t)$.

Scaling solution overestimates viscous effects by factor ~ 2.

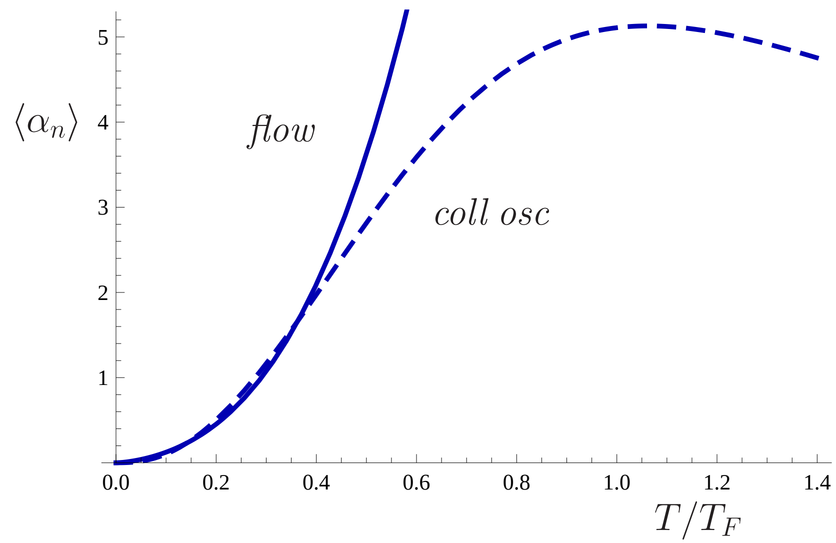
Relaxation time model

In real systems stress tensor does not relax to Navier-Stokes form instantaneously. Consider

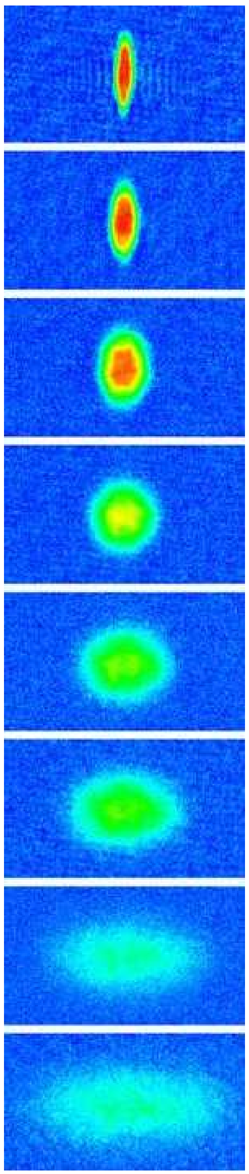
$$\tau_R\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\delta\Pi_{ij}=\delta\Pi_{ij}-\eta\left(\partial_i v_j+\partial_j v_i-\frac{2}{3}\delta_{ij}\partial\cdot v\right)$$

In kinetic theory $\tau_R \simeq (\eta/n)\, T^{-1}$

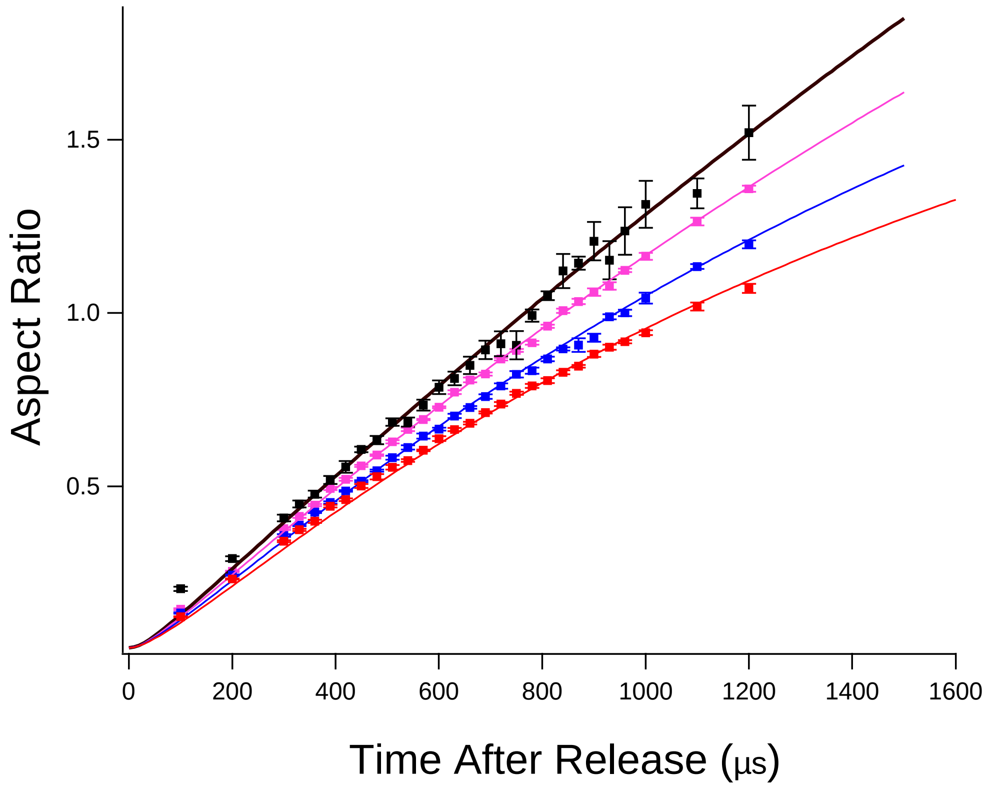
- disspiation from $\eta \sim (mT)^{3/2}$: corona excerts drag force.
- find $\langle\alpha_n\rangle \sim T^3$
- system dependence



Elliptic flow: High T limit



Quantum viscosity $\eta = \eta_0 \dfrac{(mT)^{3/2}}{\hbar^2}$



$$\eta = \eta_0 (mT)^{3/2}$$

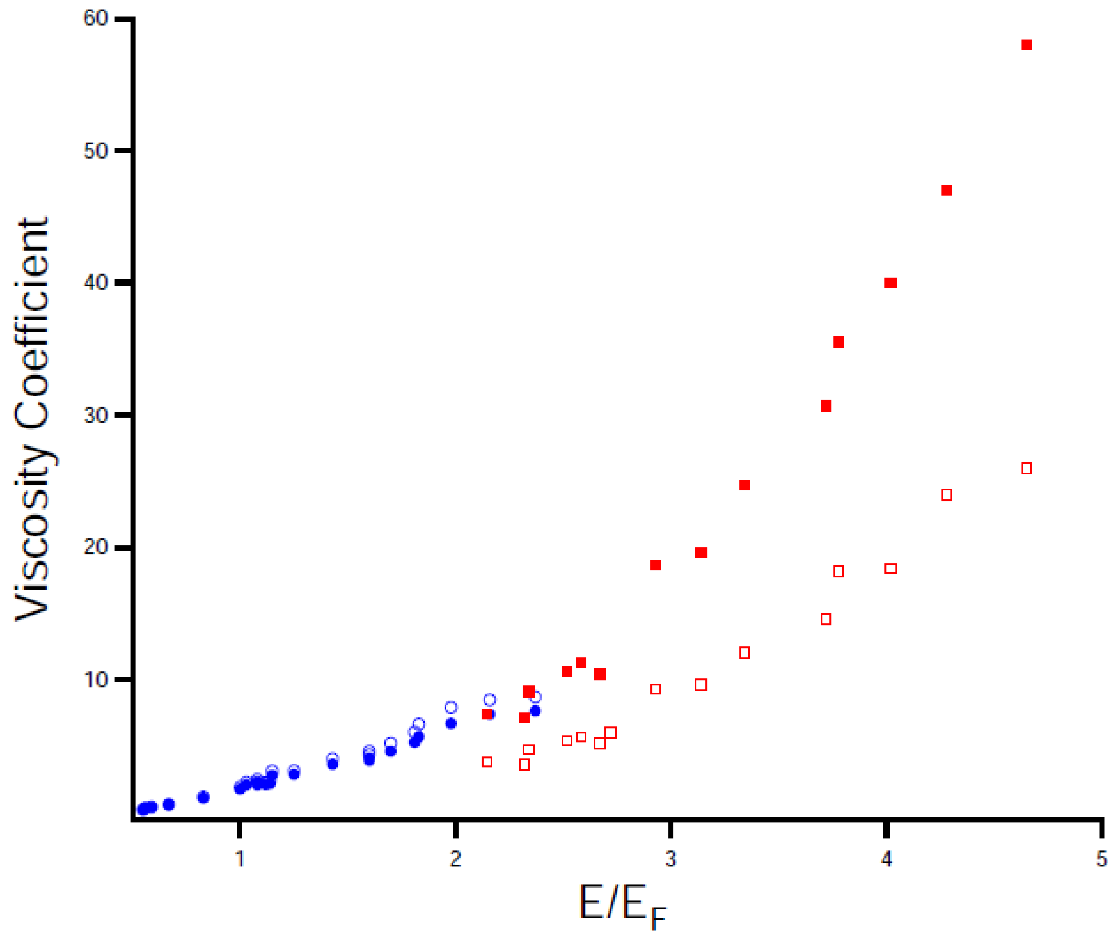
$$\tau_R = \eta / P$$

fit: $\eta_0 = 0.33 \pm 0.04$

theory: $\eta_0 = \dfrac{15}{32\sqrt{\pi}} = 0.26$

Expansion vs Collective Modes

$$\frac{\langle \eta \rangle}{\langle n \rangle}$$



collective modes elliptic flow

filled: full hydro open: no reheating

Cao et al. (2010)

Outlook

The unitary Fermi gas is an important model system for other strongly correlated quantum fluids in nature (the quark gluon plasma, dilute neutron matter).

The equation of state has been determined to a few percent.

Transport properties are more difficult: Kinetic theory at $T \gg T_F$ and $T \ll T_F$. Sum rules constrain spectral fct at all T.

Experimental determination of transport properties: Collective modes give $\eta/s < 0.5$. Analysis of expanding systems still in progress. Requires full second order hydrodynamics.

Extra

I. Experiment (Liquid Helium)

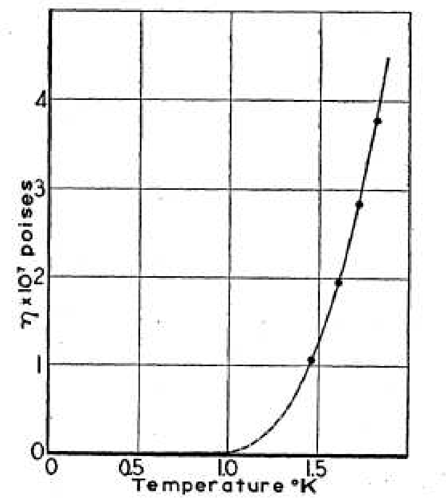
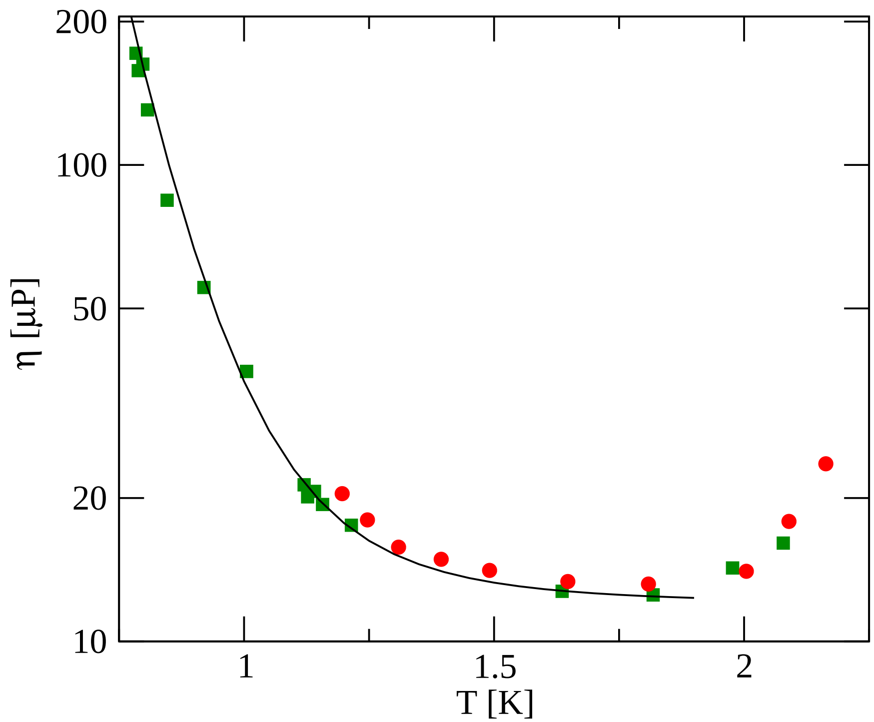


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

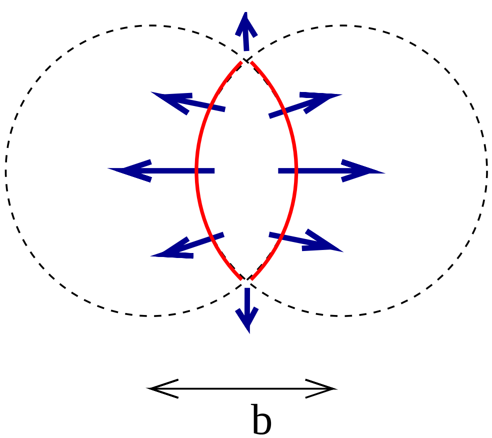
roton minimum, phonon rise

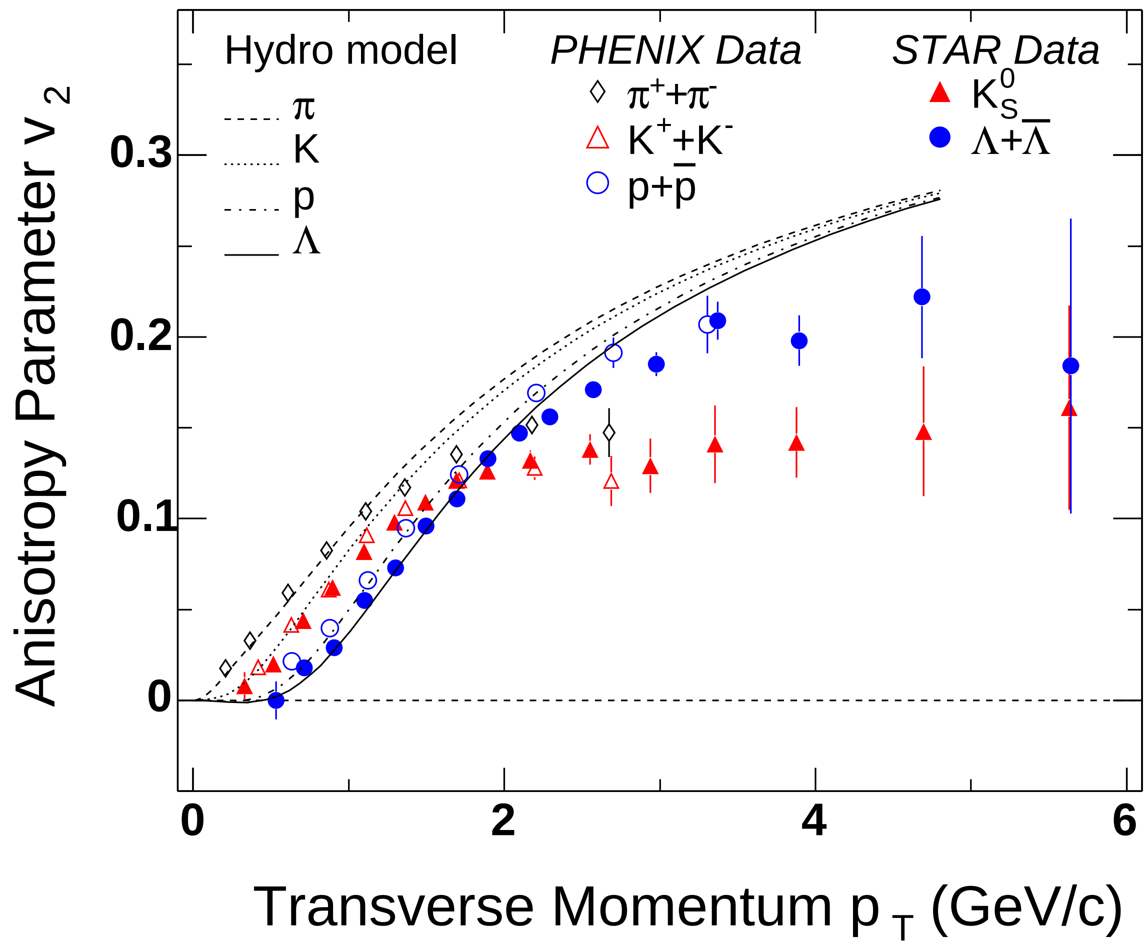
rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

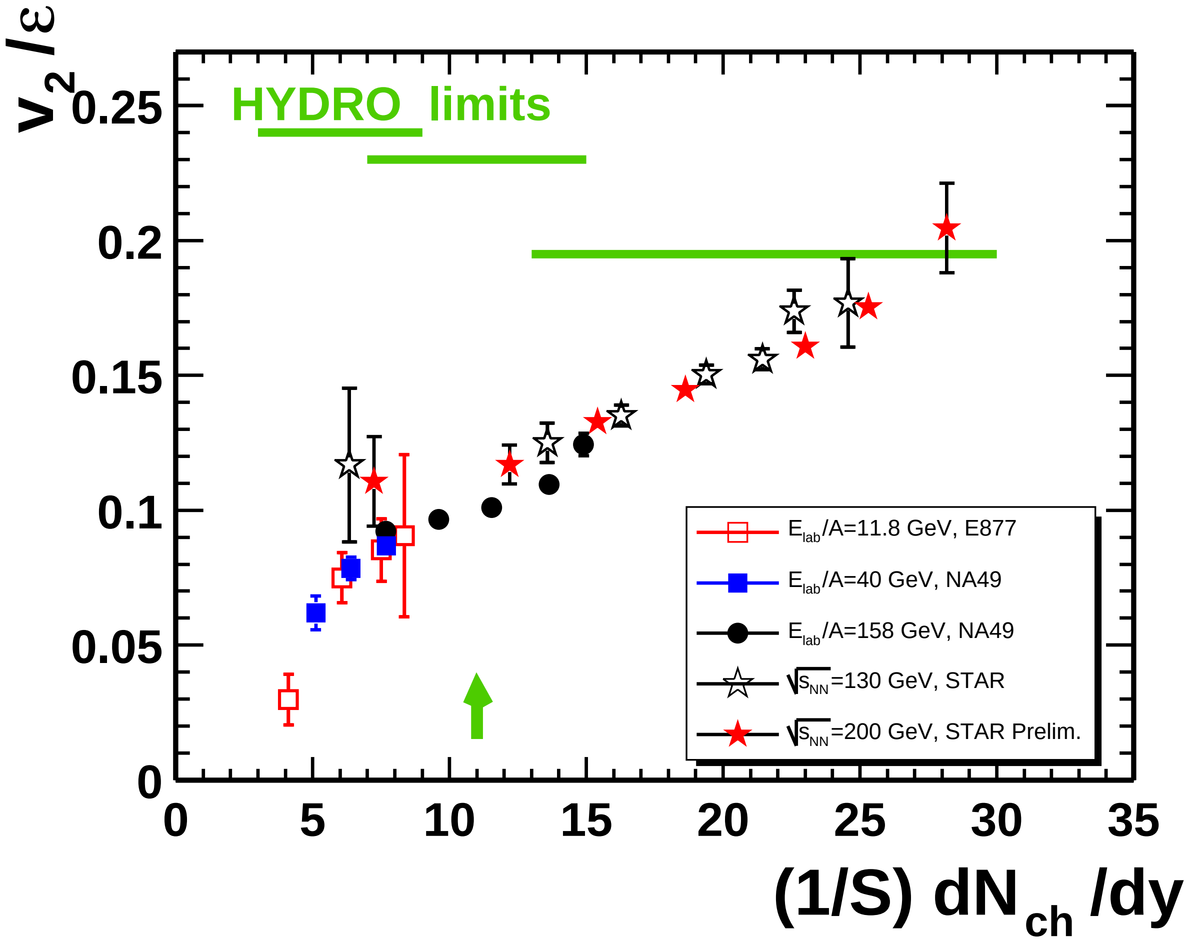




source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Elliptic flow: initial entropy scaling



source: U. Heinz (2005)

Viscosity and Elliptic Flow

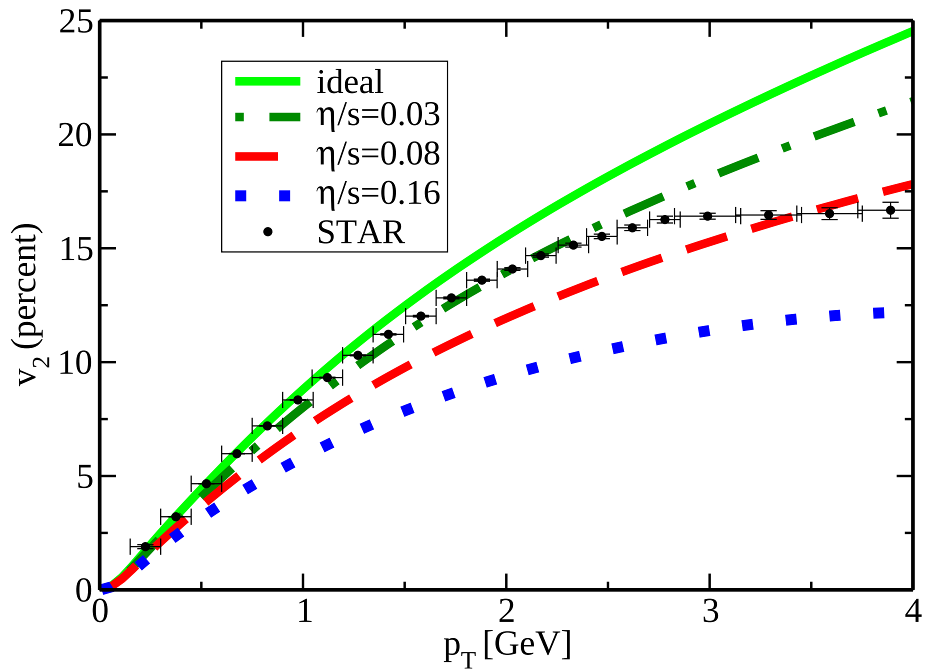
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound $\boxed{\dfrac{\eta}{s} < 0.4}$