Instantons and the Spin-Flavor Structure of Hadrons

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Quantumchromodynamics

Elementary fields: **Quarks** **Gluons**

$$(q_\alpha)^a_f \begin{cases} \text{color} & a = r, b, g \\ \text{spin} & \alpha = \uparrow, \downarrow \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

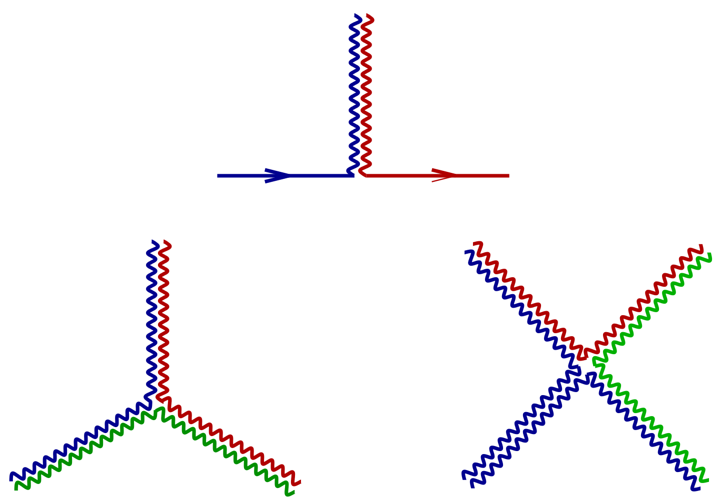
$$A^a_\mu \begin{cases} \text{color} & a = 1, \ldots, 8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

Dynamics: Dirac + generalized Maxwell theory (Yang-Mills theory)

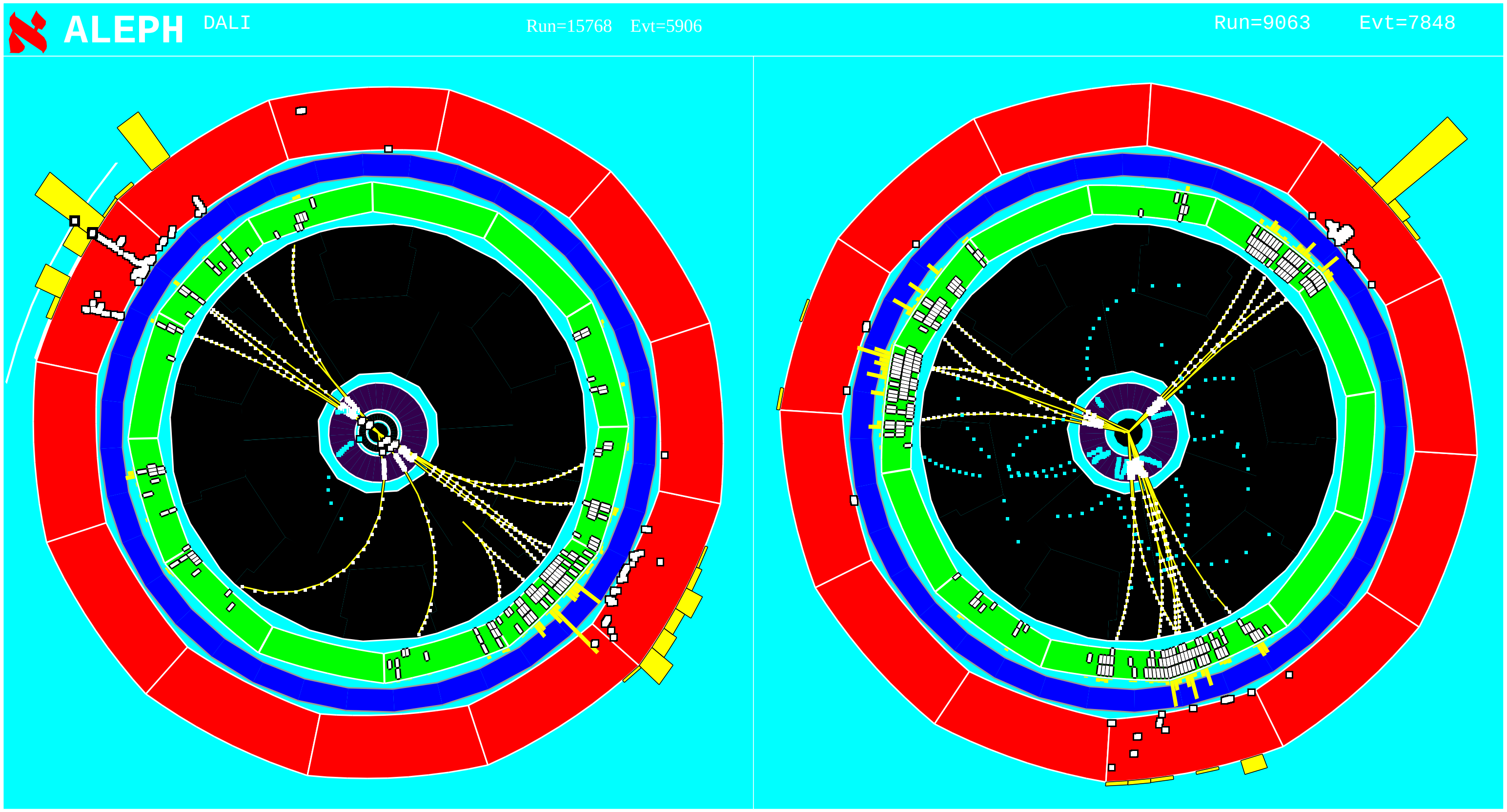
$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a \right) q$$



"Seeing" Quarks and Gluons



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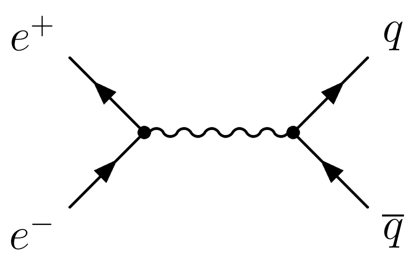
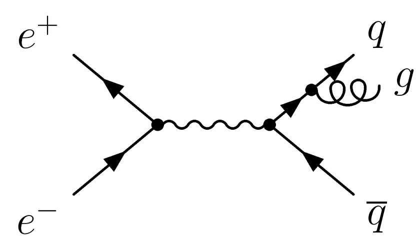
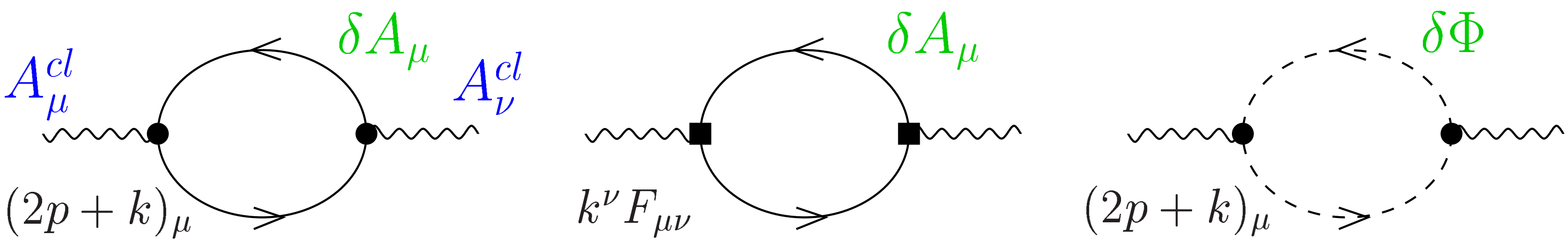




Asymptotic Freedom

Classical field $A_0^{cl} \sim g/r$. Modification due to quantum fluctuations:

$$A_\mu = A_\mu^{cl} + \delta A_\mu \qquad g \to g(\mu) \qquad \beta(g) = \frac{\partial g}{\partial \log(\mu)}$$

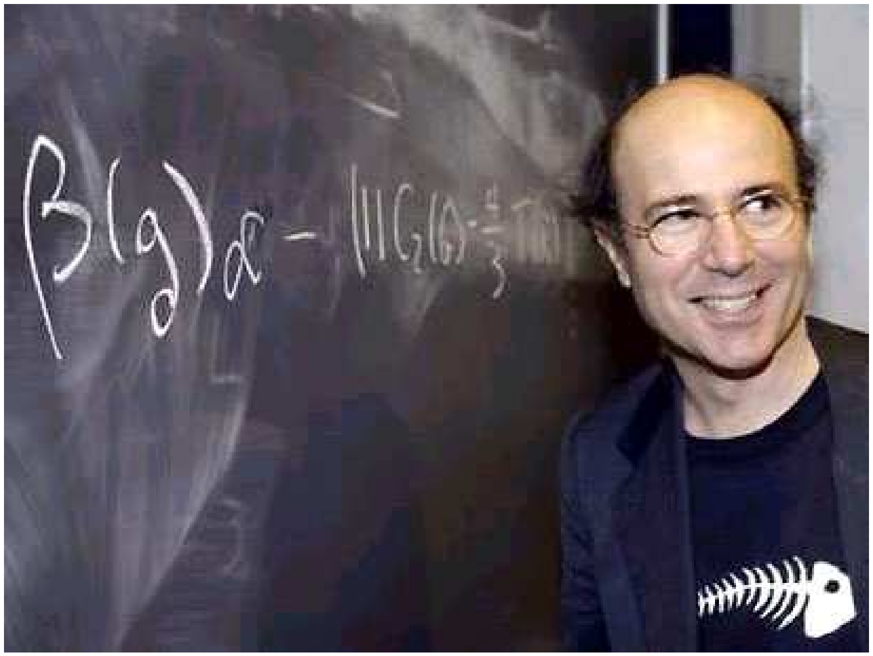


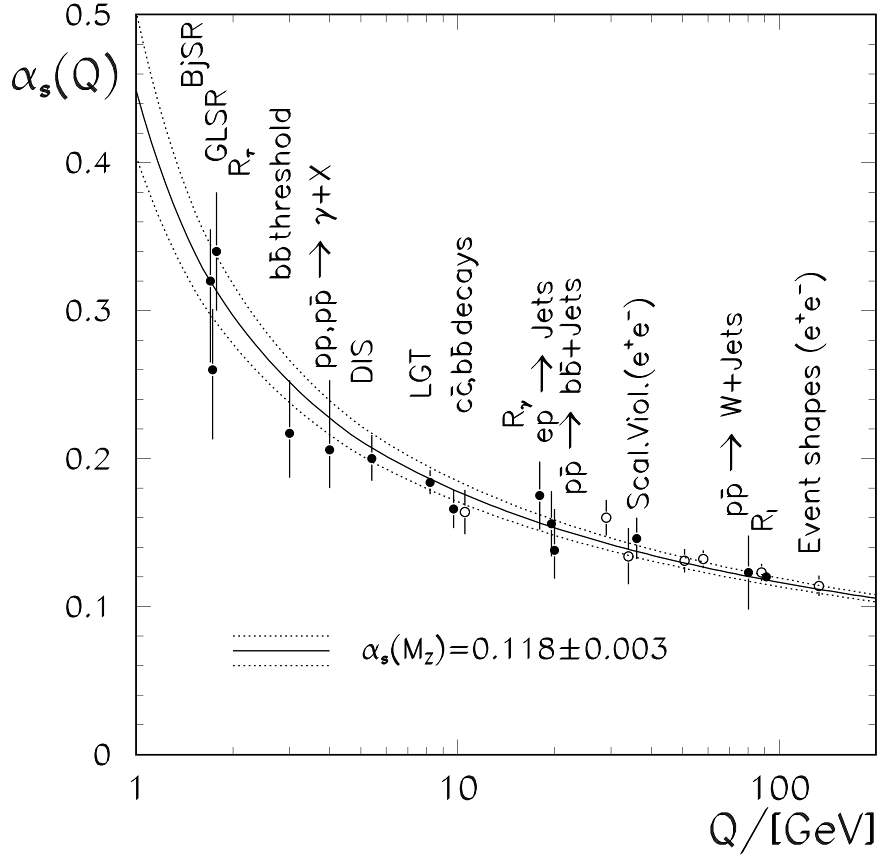
dielectric $\epsilon > 1$ paramagnetic $\mu > 1$ dielectric $\epsilon > 1$

$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

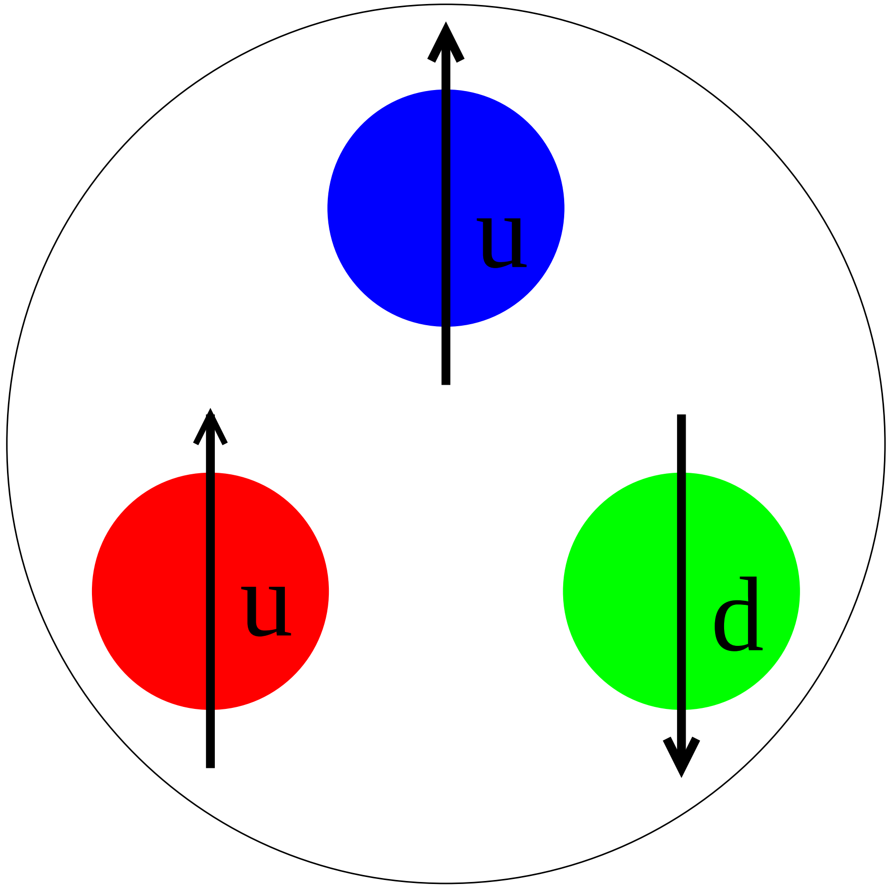
$$\beta(g) = \frac{g^3}{(4\pi)^2}\left\{\left[\frac{1}{3} - 4\right]N_c + \frac{2}{3}N_f\right\}$$

Running Coupling Constant





What is a proton?



Why does this picture "work"? Large N_c limit (?)

Where does it fail? Why? OZI violation, flavor mixing

The Structure of the Proton

The mass of the Proton (from DIS, trace anomaly)

$$
\begin{aligned}
E_q &= \langle p|\int d^3x\,(-i\vec{\alpha}\cdot\vec{D})|p\rangle && \simeq 310\,\mathrm{MeV} \\
E_g &= \langle p|\int d^3x\,\tfrac{1}{2}\left(E^2+B^2\right)|p\rangle + \ldots && \simeq 545\,\mathrm{MeV} \\
E_m &= \langle p|\int d^3x\,(m_u\bar{u}u + m_d\bar{d}d)|p\rangle && \simeq 45\,\mathrm{MeV}
\end{aligned}
$$

Gluon field strength is large

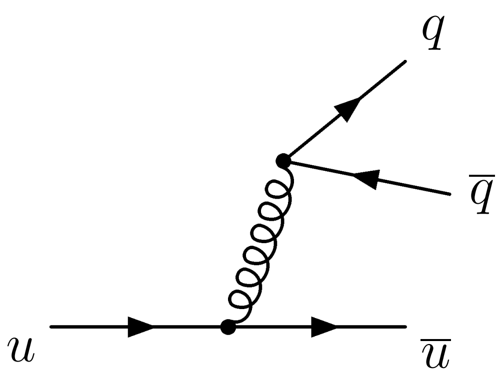
$$\langle p|E^2|p\rangle \simeq 1700\,\mathrm{MeV} \qquad \langle p|B^2|p\rangle \simeq -1050\,\mathrm{MeV}$$

and approximately self-dual

number of quark-anti-quark pairs is large

$$\langle p|\bar{u}u + \bar{d}d|p\rangle = \frac{\Sigma_{\pi N}}{\overline{m}} \simeq 6$$

. . . and not flavor symmetric



$$\frac{\bar{d}(x)}{\bar{u}(x)} \simeq 2 \qquad \text{(NuSea, ...)}$$

quark contribution to proton spin is small

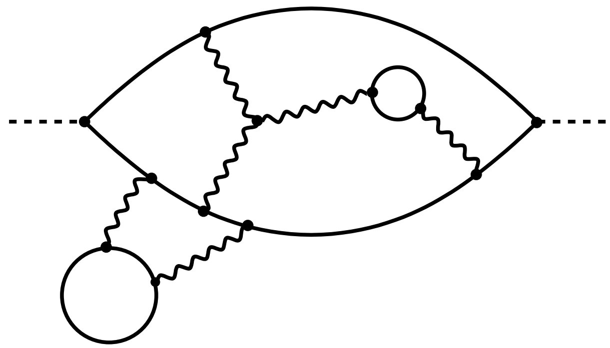
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = (0.25 \pm 0.1) \qquad \text{(SMC, SLAC, Hermes)}$$

$$\langle p|\bar{q}\gamma_\mu\gamma_5 q|p\rangle = \Delta q s_\mu$$

. . . and strange quarks are polarized $\Delta s = -0.12$

Hadronic Correlation Functions

hadronic current $j_M(x) = \bar{q}(x)\Gamma q(x)$



$$\Pi(x) = \langle j(x) j(0) \rangle$$

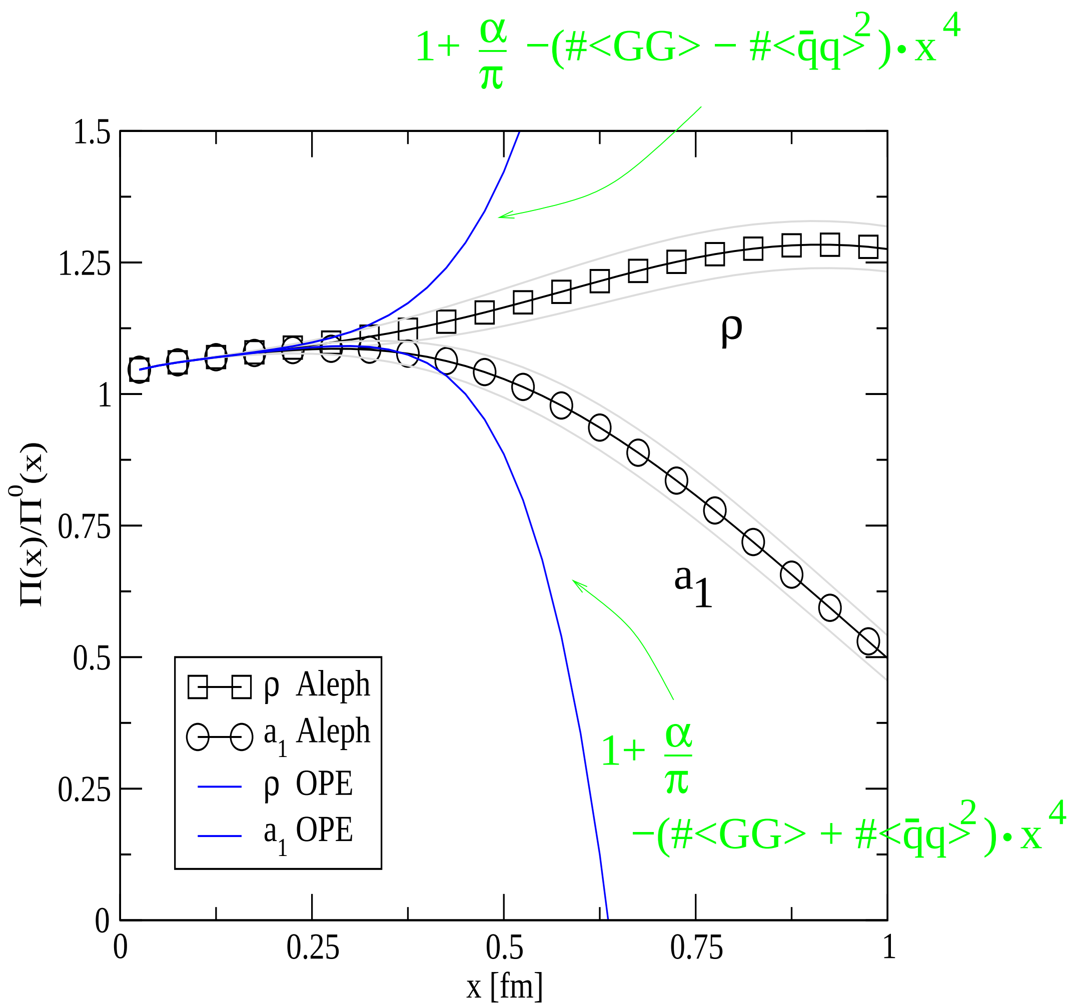
short distance behavior: OPE

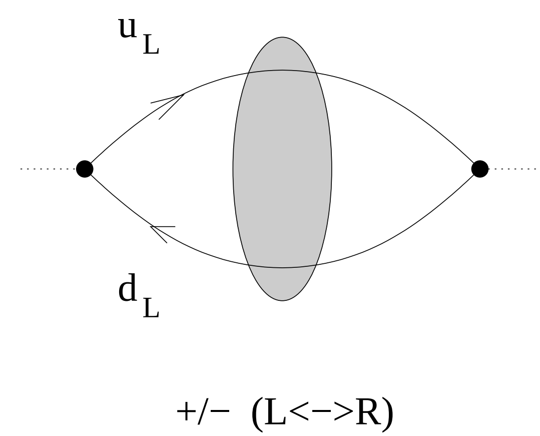
$$\Pi(Q) = c_0 \log(Q^2) + c_4 \frac{\langle \mathcal{O}_4 \rangle}{Q^4} + c_6 \frac{\langle \mathcal{O}_6 \rangle}{Q^6} + \ldots$$

experimental information

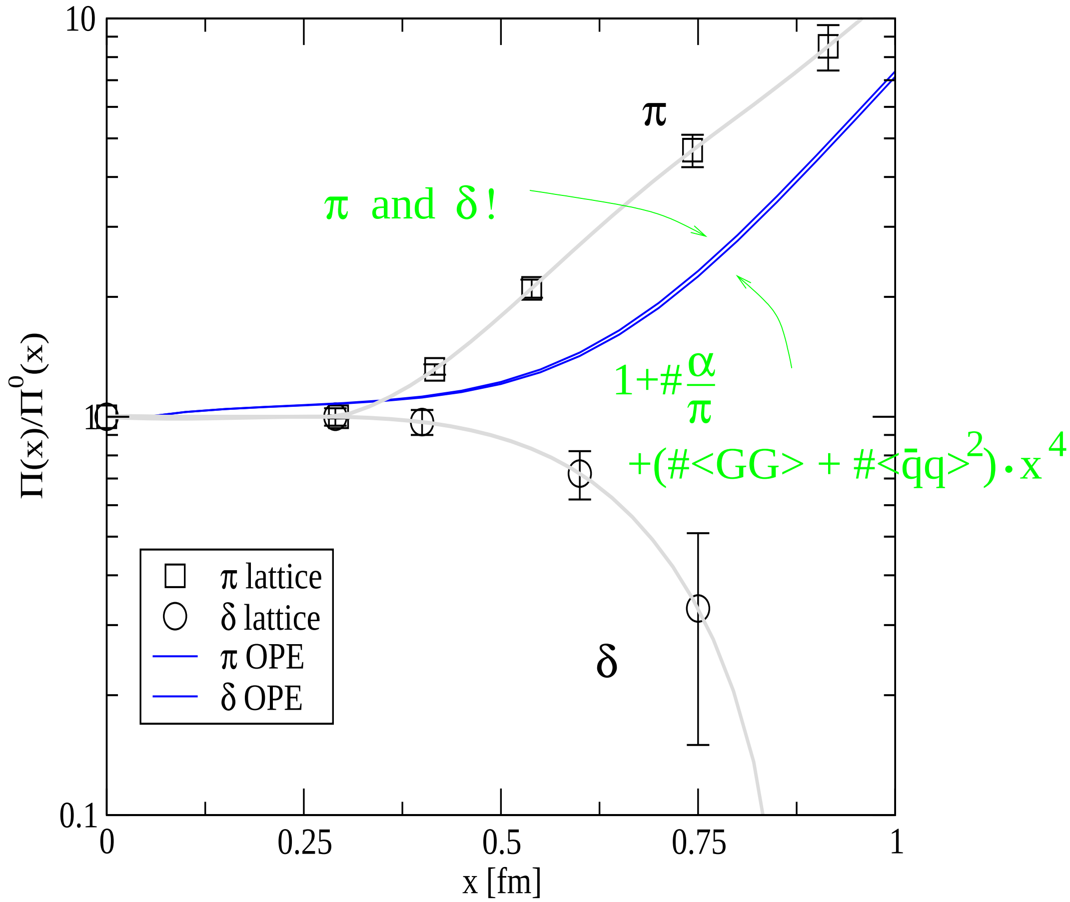
$$\Pi(Q) = \int ds \frac{\rho(s)}{s + Q^2}$$

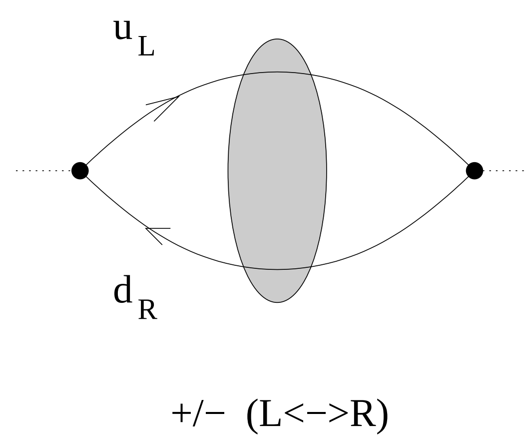
Vector Channels: ρ and a_1



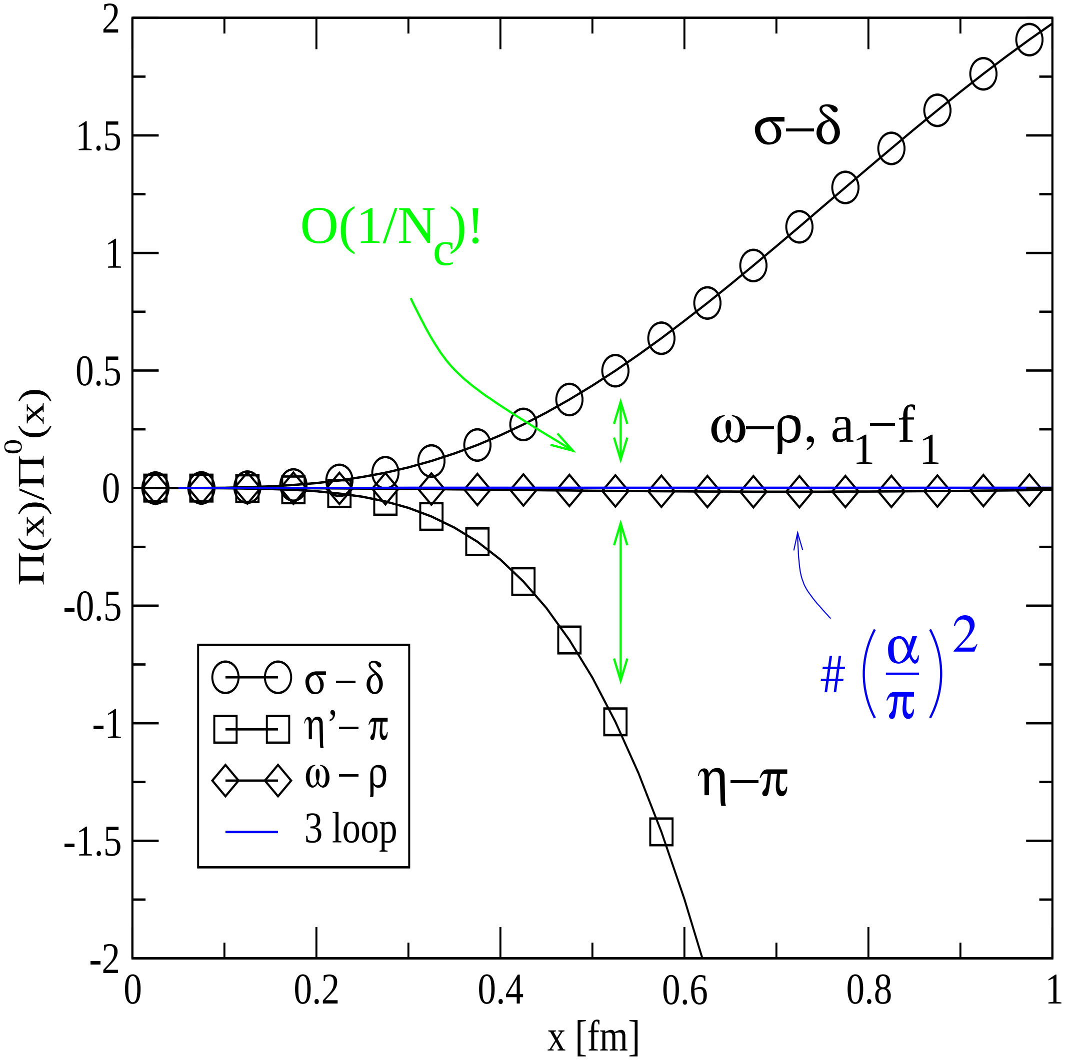


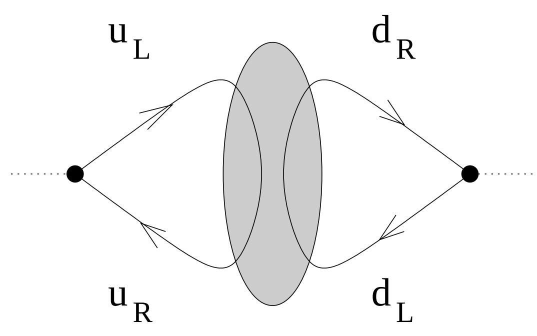
Scalar Channels: π and δ





OZI violation: $\eta' - \pi$, $\sigma - \delta$, $\omega - \rho$, $a_1 - f_1$





Summary

Only small effects in $(\bar{L}L \pm \bar{R}R)^2$.

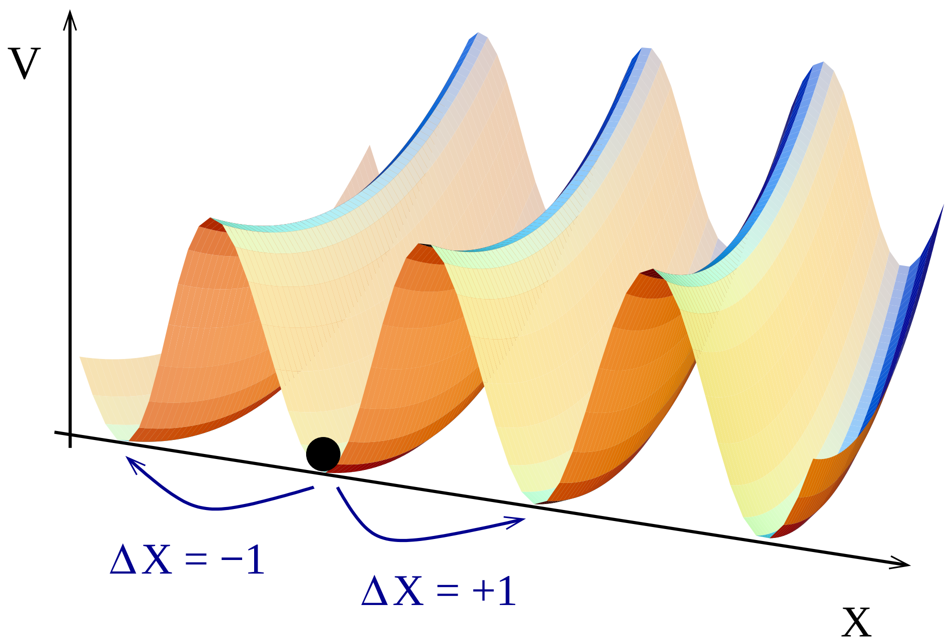
Sign changes for $\bar{L}R \leftrightarrow \bar{R}L$.

Sign changes for $(\bar{u}d)(\bar{u}d) \leftrightarrow (\bar{u}u)(\bar{d}d)$.

$$\mathcal{L} = G \det_f(\bar{\psi}_L \psi_R) + (L \leftrightarrow R)$$

Topology in QCD

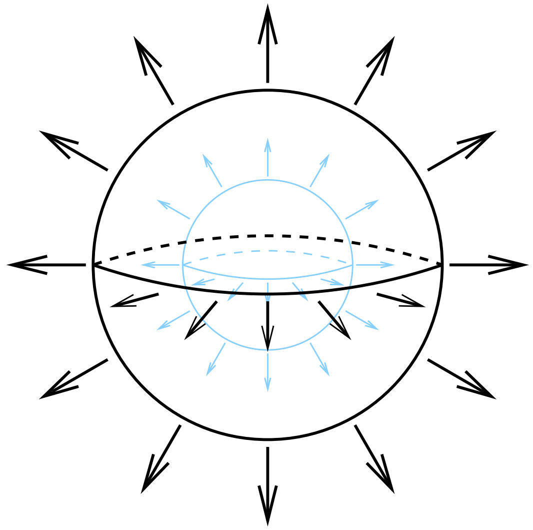
classical potential is periodic in variable X



$$X = \int d^3x\, K_0(x,t)$$

$$\partial^\mu K_\mu = \frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

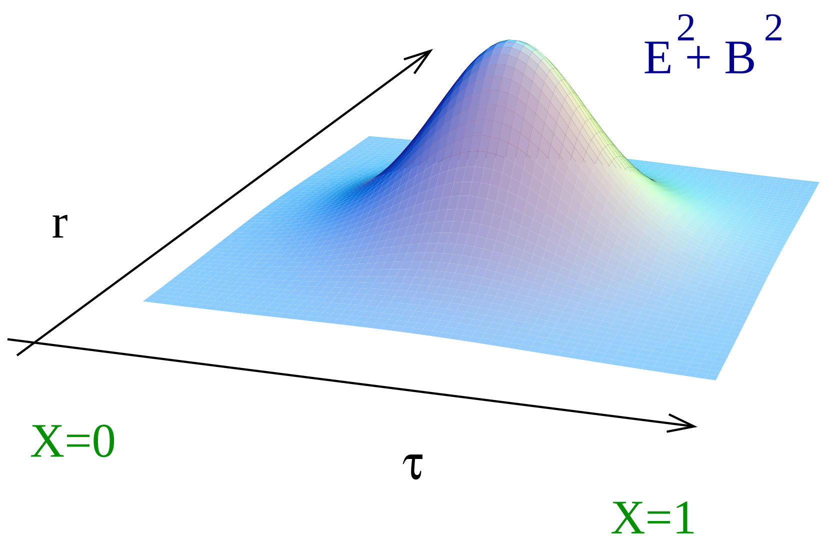
classical minima correspond to pure gauge configurations



$$A_i(x) = iU^\dagger(x)\partial_i U(x)$$

$$E^2 = B^2 = 0$$

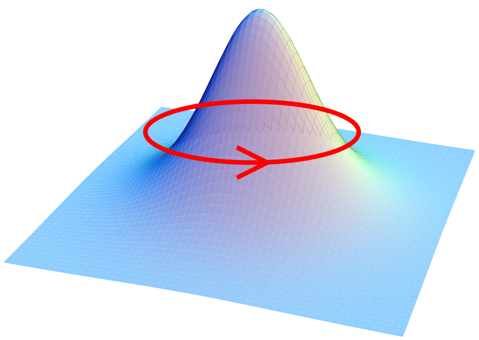
semi-classical tunneling paths: Instantons



$$A^a_\mu(x) = 2\frac{\eta_{a\mu\nu}x_\nu}{x^2+\rho^2},$$

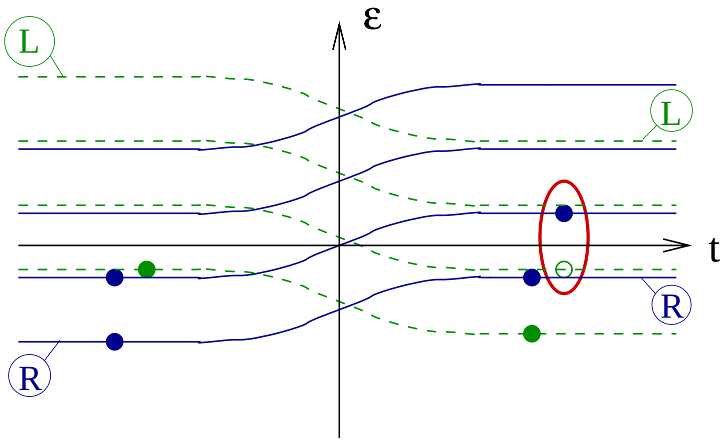
$$G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = \frac{192\rho^4}{(x^2+\rho^2)^4}.$$

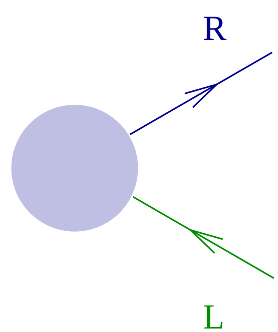
(Anti)Instantons: Dirac operator has a L/R zero mode.



$$\gamma \cdot (\partial + A_{I,A})\, \psi^0_{L,R} = 0$$

spectrum of Hamiltonian

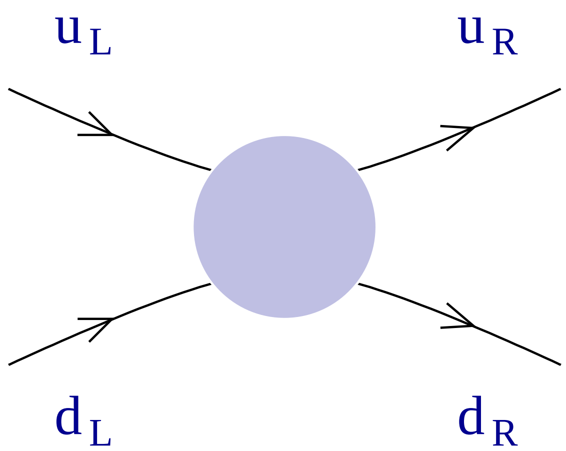




axial charge violation:

$$\Delta Q_A = 2$$

instanton induced quark interaction ($N_f = 2$)



$$\mathcal{L} = G \det_f(\bar\psi_{L,f}\psi_{R,g})$$

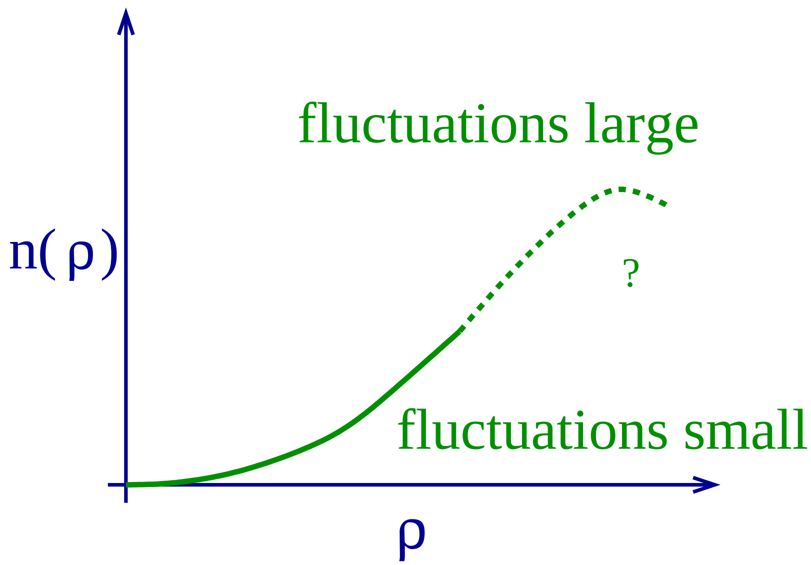
$$G = \int d\rho\, n(\rho)$$

violates $U(1)_A$ but

preserves $SU(2)_{L,R}$

... and contributes to the η' mass

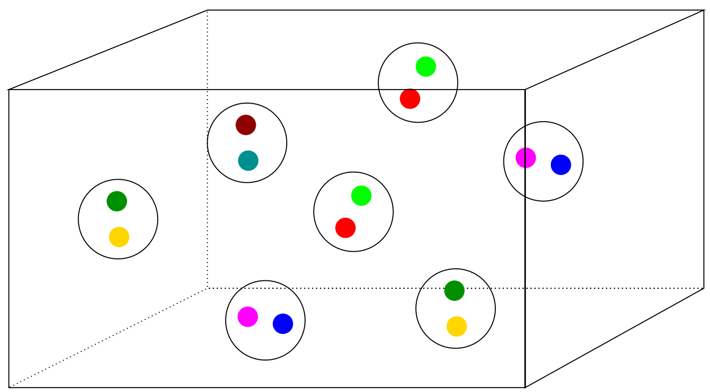
tunneling rate (barrier penetration factor)



$$n(\rho) \sim \exp\left[-\frac{8\pi^2}{g^2(\rho)}\right] \sim \rho^{b-5}$$

Instanton Ensemble

instanton liquid described by partition function



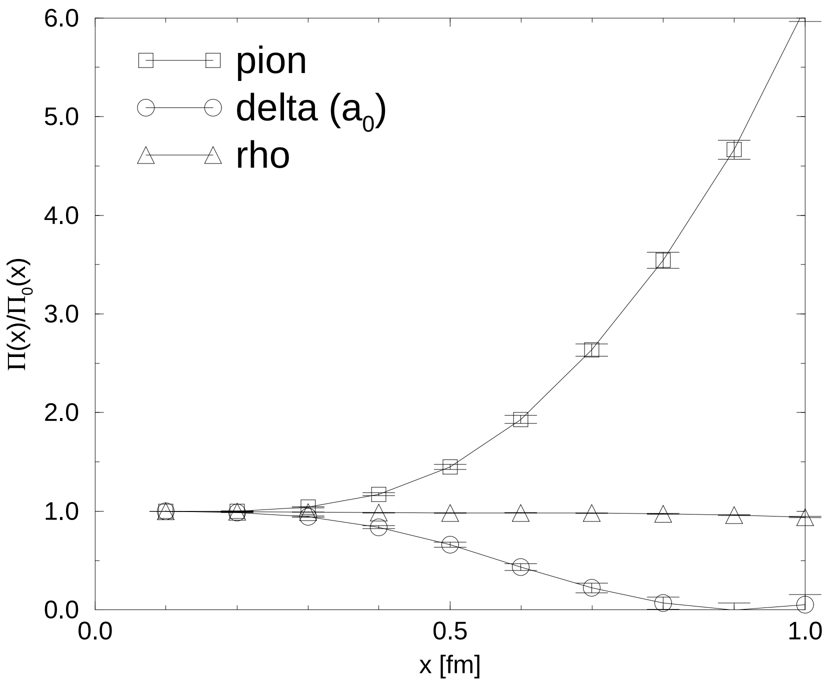
$$Z = \frac{1}{N_I! N_A!} \prod_I^{N_I+N_A} \int [d\Omega_I \, n(\rho_I)]$$

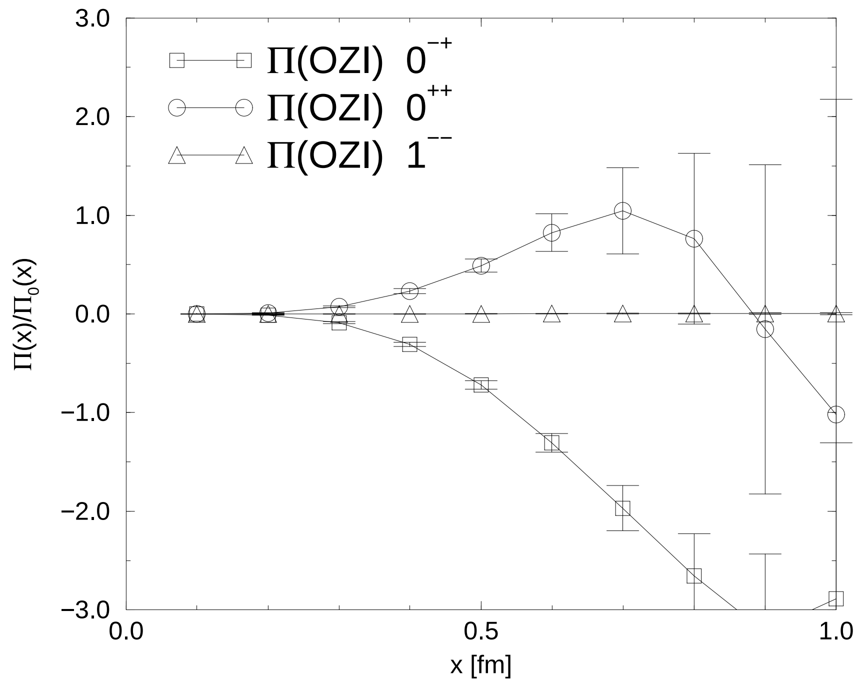
$$\times \det(\not{D}) \exp(-S_{int})$$

quark propagator

$$S(x,y) = \sum_{IJ} \psi_I(x) \left(\frac{1}{T + im} \right)_{IJ} \psi_J^\dagger(y) + S_{NZM}(x,y)$$

Meson Correlation Functions





$$m_\pi = 140^* \text{ MeV} \quad (f_\pi = 71\,\text{MeV})$$

$$m_\rho = 795\,\text{MeV}$$

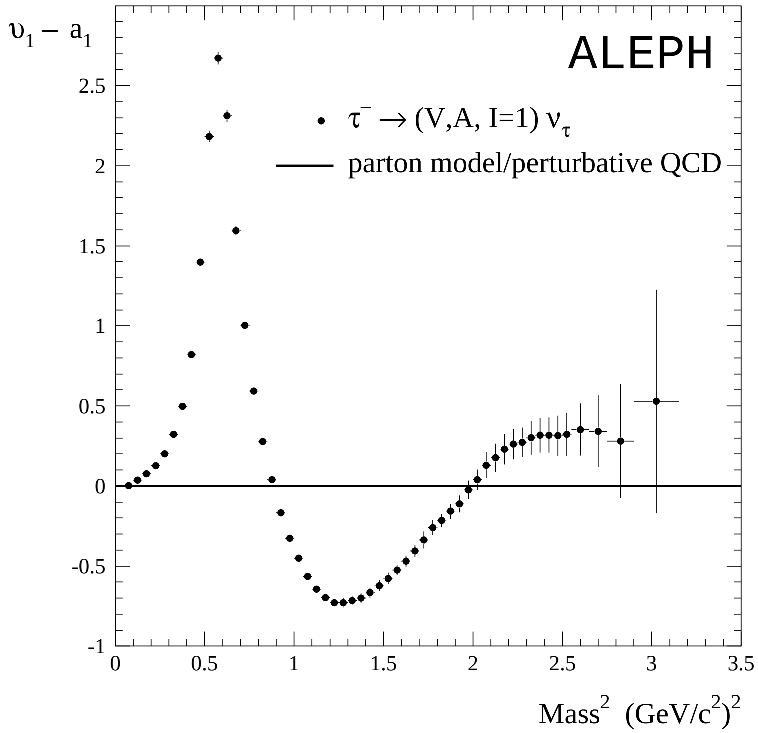
$$m_{a_0} \simeq 1\,\text{GeV}$$

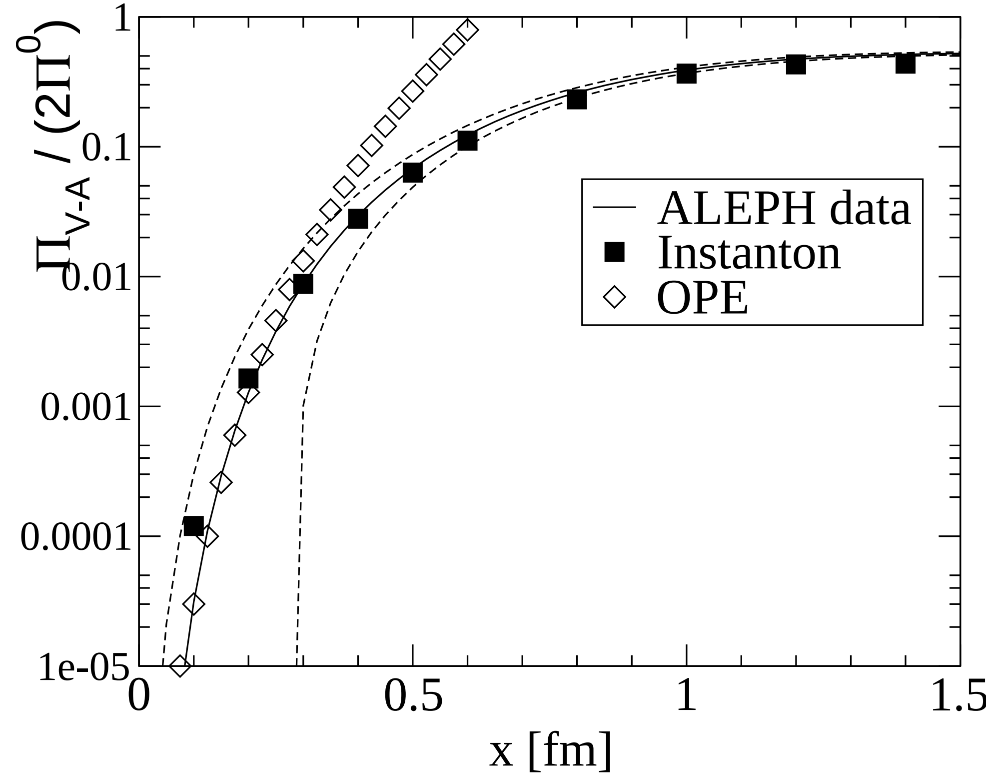
$$m_\rho \simeq m_\omega$$

$$m_\sigma \simeq 580\,\text{MeV}$$

$$m_{\eta'} \simeq 1\,\text{GeV}$$

V−A Correlation Functions





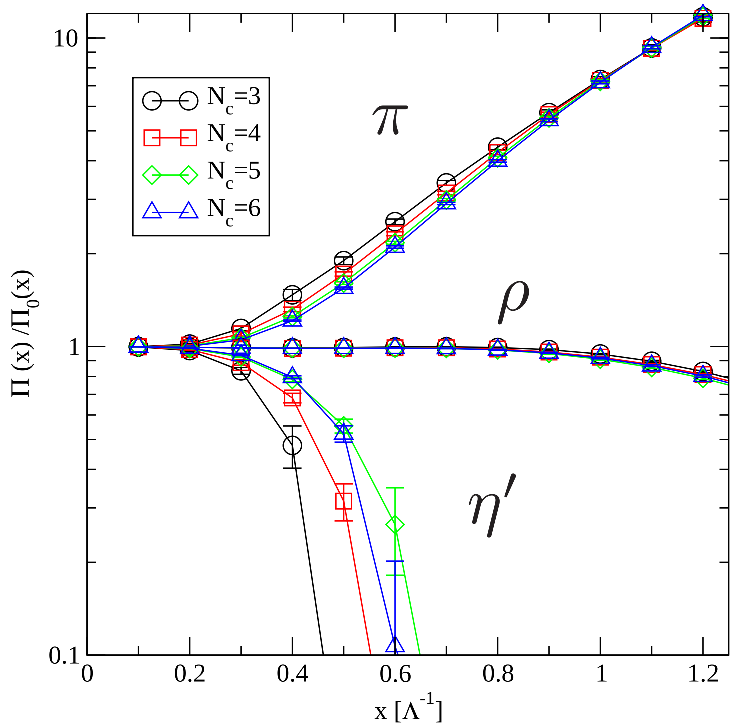
Aleph spectral function

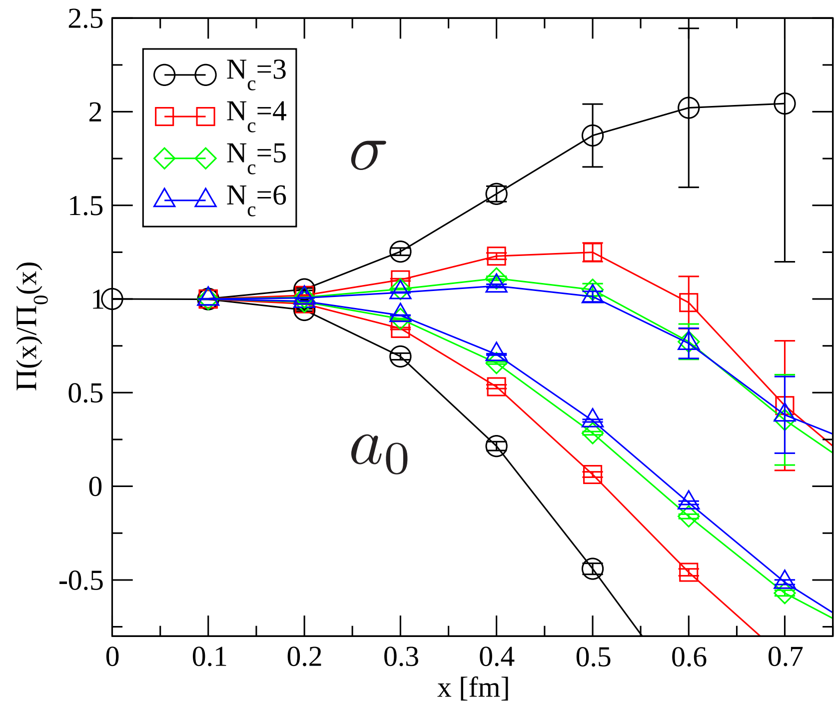
$\tau \to (V, A, I=1)\nu_\tau$

coordinate space correlator

OPE, instanton liquid, data

Large N$_c$: From extraordinary to ordinary hadrons



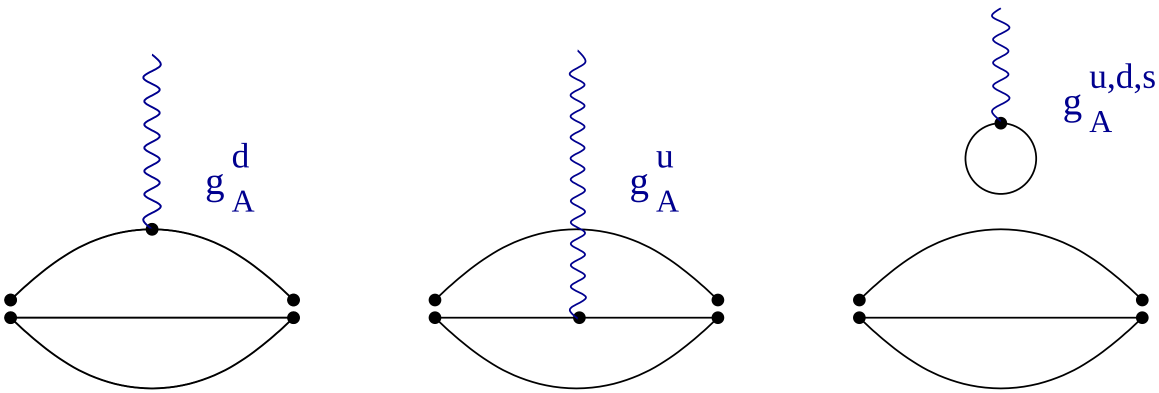


η' becomes light, light σ disappears

(quenching artifacts in a_0 disappear)

Quark Contribution to Nucleon Spin

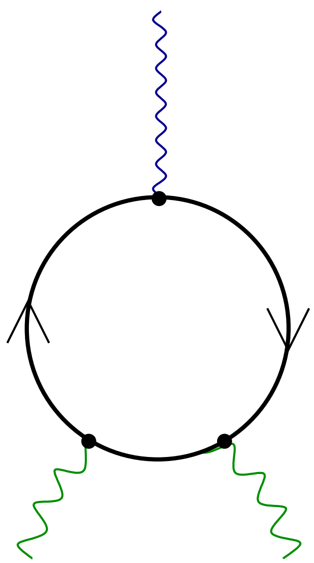
polarized DIS implies large OZI violation



$$g_A^0 = \Delta u + \Delta d + \Delta s \simeq 0.25$$

$$g_A^8 = \Delta u + \Delta d - 2\Delta s \simeq 0.65$$

related to axial anomaly and instantons?

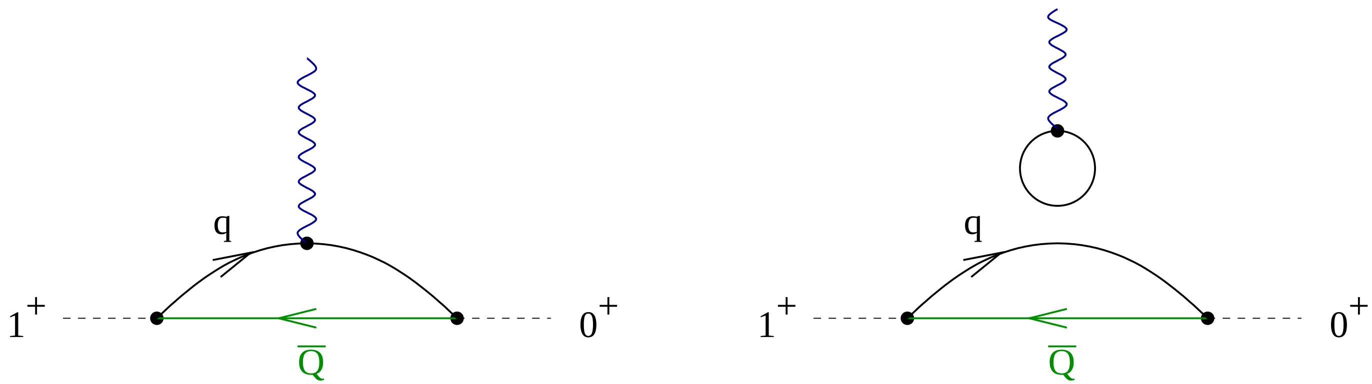


$$\partial^\mu A_\mu^0 = \frac{N_f g^2}{16\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$$

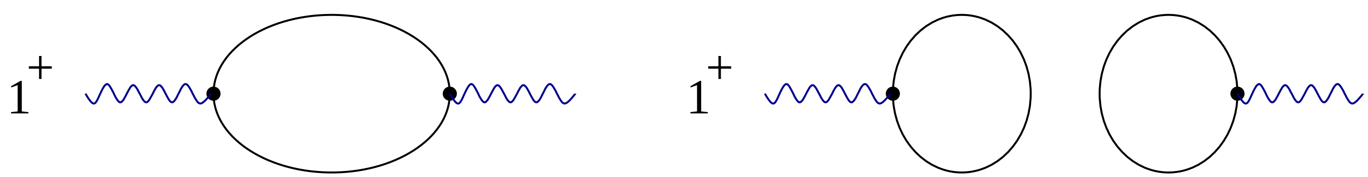
$$g_A^0 = \frac{N_f}{32\pi^2 m_N} \langle p | g^2 G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a | p \rangle$$

OZI violation

Suppression of g_A^0 property of the nucleon or of the QCD vacuum?



$$(g_A^Q)^0 - (g_A^Q)^3$$

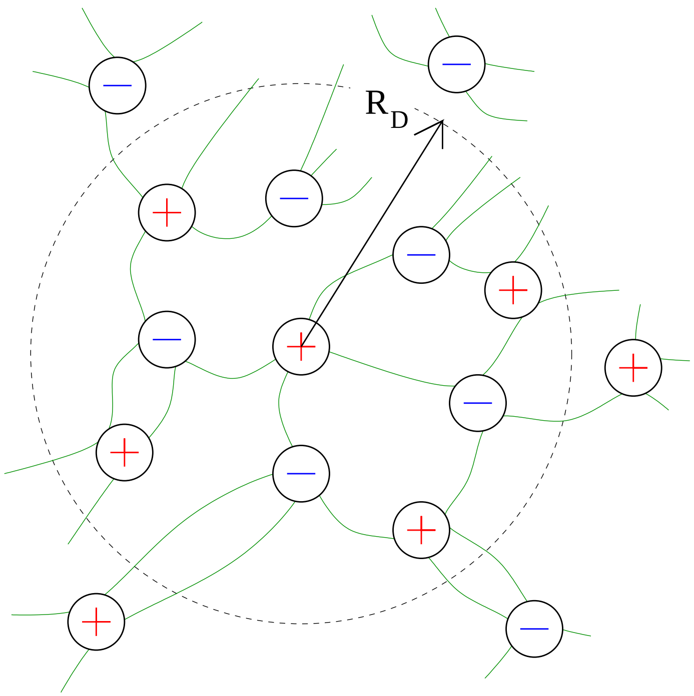


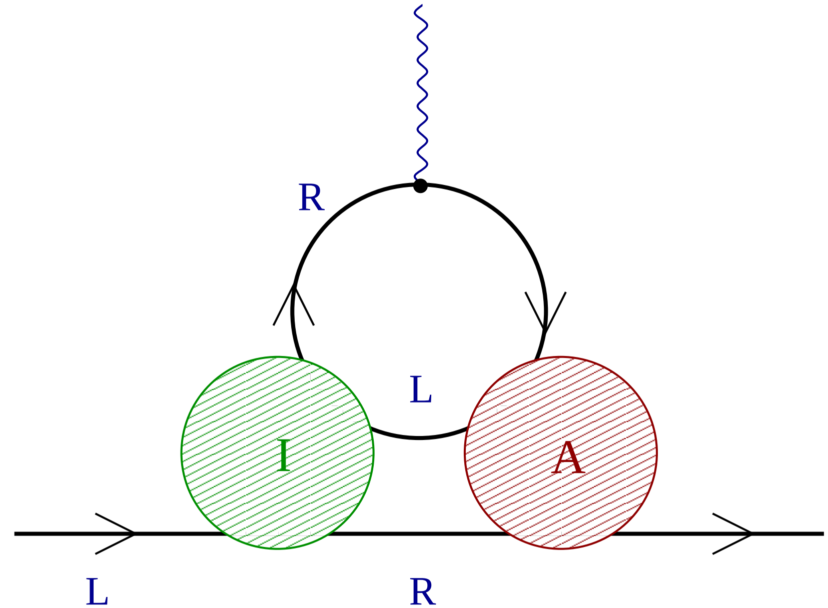
$$(f_a^2 m_a^2)^0 - (f_a^2 m_a^2)^3$$

Study singlet correlators in $\bar{q}q$ and $\bar{Q}q$ (or QQq) channel

Vacuum Properties

Axial charge screening related to topological charge screening?



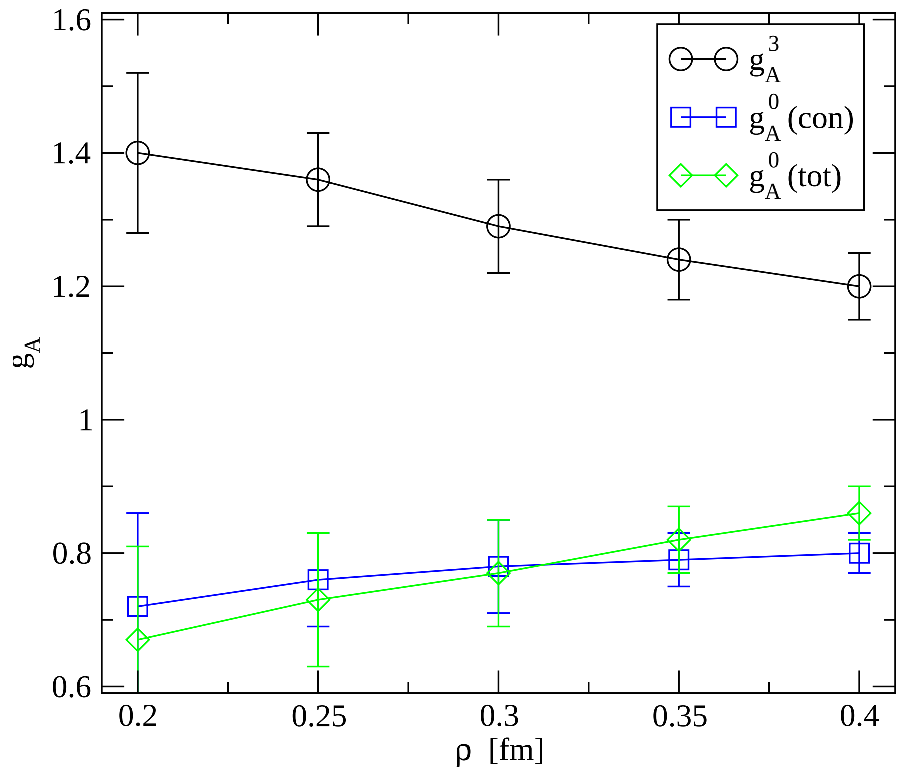


$$\chi_{top} = \frac{1}{V}\langle Q^2_{top}\rangle = 0 \qquad\qquad L \to R(\bar{L}R)$$

e.g. Veneziano and Shore $g_A^0 = g_A^8 \sqrt{\frac{6\chi'_{top}(0)}{f_\pi^2}}$ (target independent)

also: Shuryak and Forte, Dorokhov and Kochelev

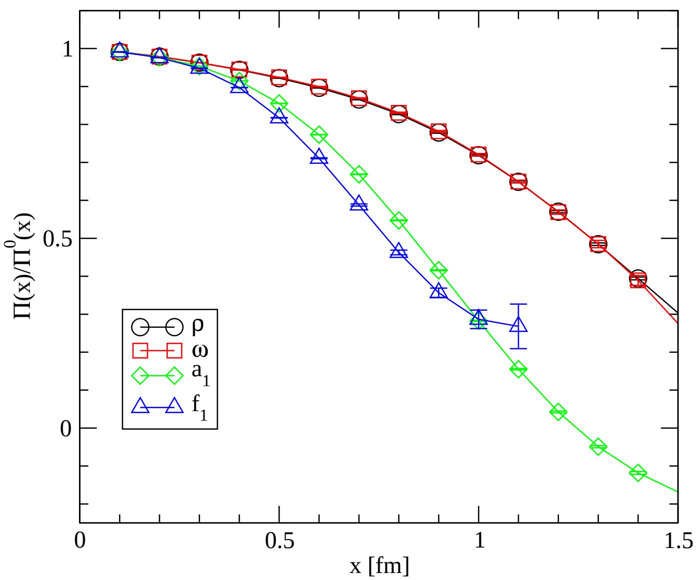
Numerical Study

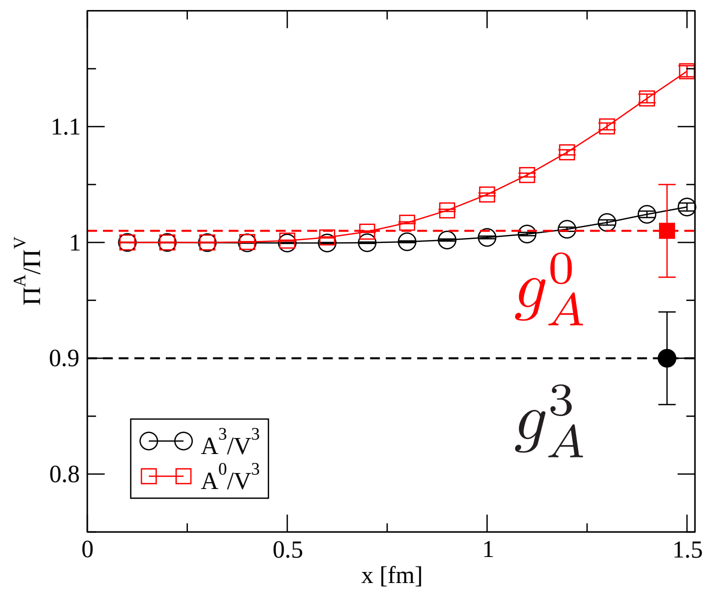


$g_A^3 \simeq 1.25$ agrees with experiment

$g_A^0 \simeq 0.75$ too large (little OZI violation)

$(\bar{q}q)$ and $(\bar{Q}q)$ states





$(f^2m^2)^0 < (f^2m^2)^3$

$(g_A^Q)^0 > (g_A^Q)^3$

Note: 1. f_1/a_1 and g_A^0/g_A^3 anti-correlated ($\to$ NJL studies)

2. sign fixed by QCD inequalities

Summary and Outlook

instantons account for OZI violation in meson sector

Not all hadrons are alike

instanton liquid reproduces axial vector coupling g_A

But: $g_A^8 \simeq g_A^0 \simeq 0.75$

no evidence that suppression of g_A^0 is a vacuum effect

$$[(g_A^Q)^0 \sim 1] > [(g_A^Q)^3 \sim 0.9]$$

OZI violation? Go back and look at g_A^8

Large $SU(3)_F$ violation?