

(Quasi)Quarks and (Quasi)Baryons at High Baryon Density

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Motivation

- Matter at high baryon density exists in nature

low energy excitations determine physical properties:

specific heat, transport properties, emissivity, ...

- Low energy degrees are composite (at any density)

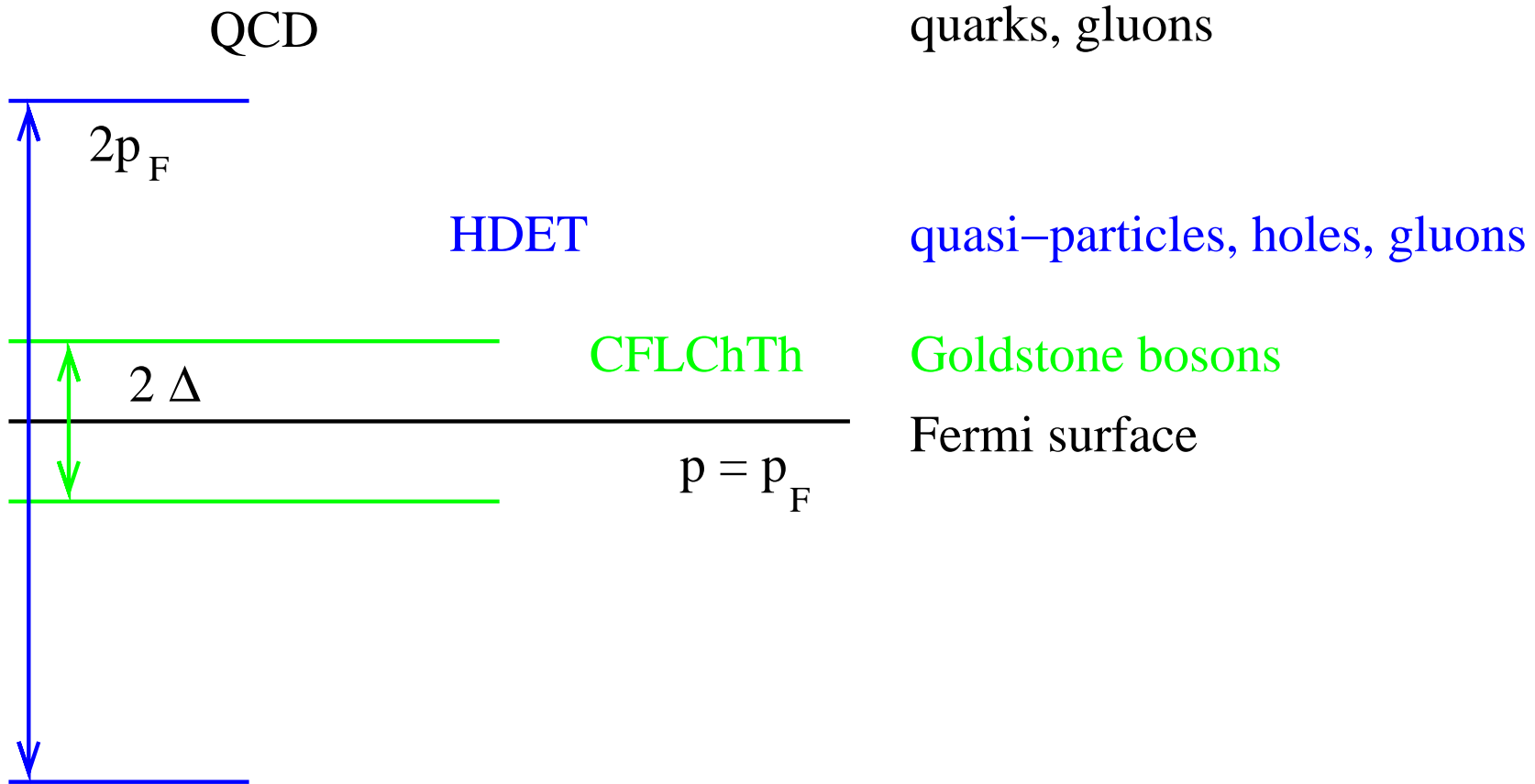
study effective degrees of freedom in a regime

where we can make the connection QCD \rightarrow EFT

Why Effective Field Theories?

- weak coupling expansion \neq loop expansion
organize perturbation theory, resum logarithms, etc.
- studying and extending the reach of weak coupling methods
improved dimensional analysis, match to lattice, etc.
- reduce confusion
gauge invariance, off-shell behavior, etc.
- effects of perturbations, external fields
quark masses, electron chemical potentials, etc.

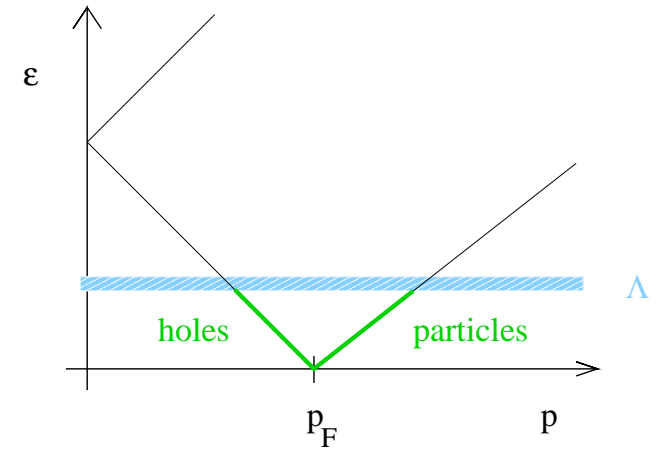
Effective Field Theories



High Density Effective Theory

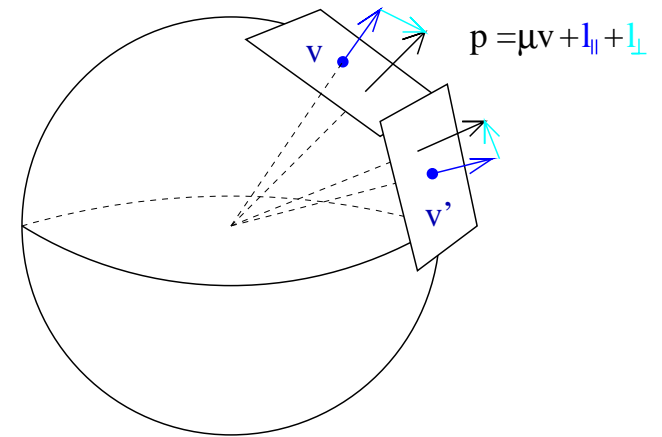
quasi-particles (holes)

$$E_{\pm} = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$



effective field theory on v -patches

$$\psi_{v\pm} = e^{-i\mu v \cdot x} \left(\frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$



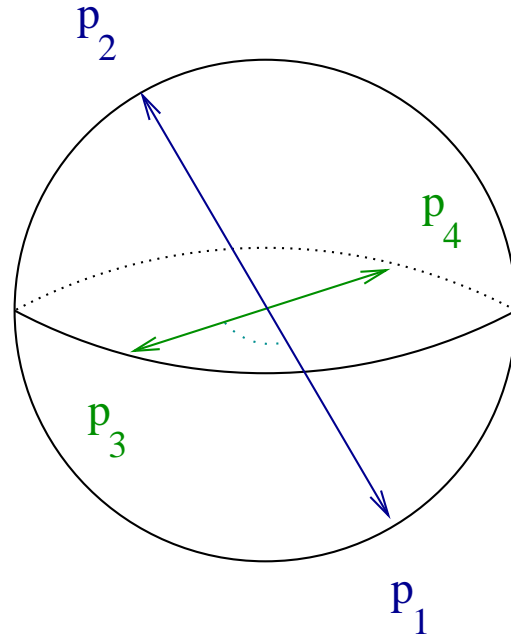
effective lagrangian for ψ_{v+}

$$\mathcal{L} = \sum_v \psi_v^\dagger (i v \cdot D) \psi_v - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + O(1/\mu)$$

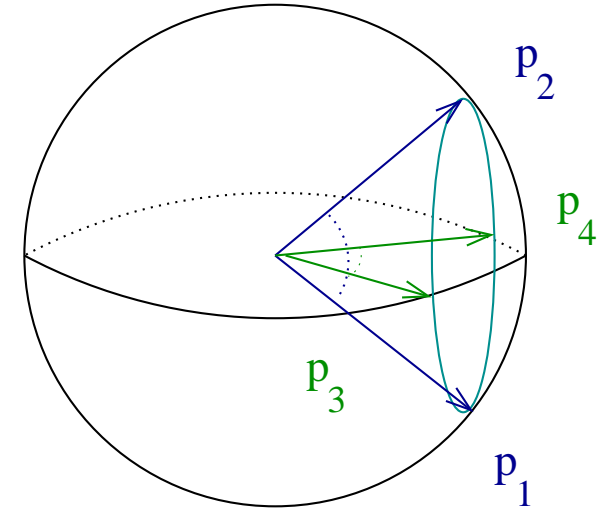
Four Quark Operators

quark-quark scattering

$$(v_1, v_2) \rightarrow (v_3, v_4)$$



BCS



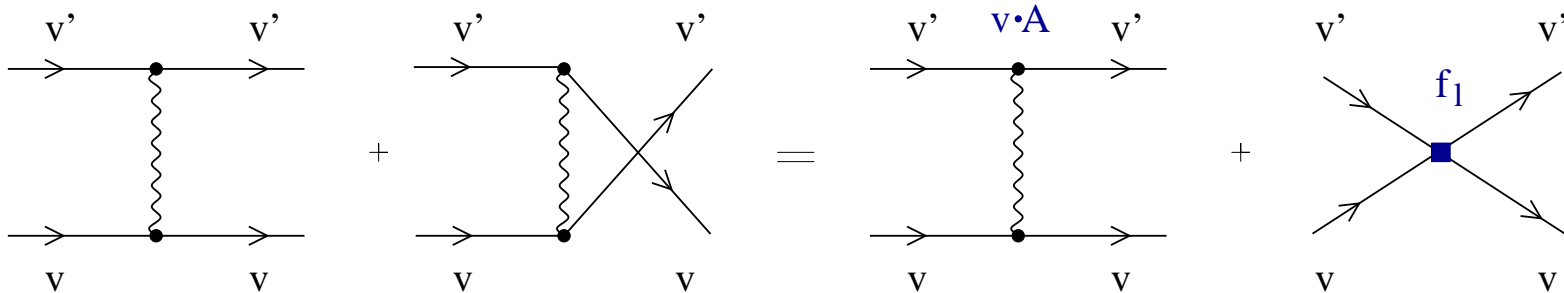
Landau

$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'} (\vec{v} \cdot \vec{v}') (\psi_v \Gamma \psi_{-v}) (\psi_{v'}^\dagger \Gamma' \psi_{-v'}^\dagger),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'} (\vec{v} \cdot \vec{v}') (\psi_v \Gamma \psi_{v'}) (\psi_{\tilde{v}}^\dagger \Gamma' \psi_{\tilde{v}'}^\dagger)$$

Four Fermion Operators: Matching

- match scattering amplitudes on Fermi surface: forward scattering



- color-flavor-spin symmetric terms

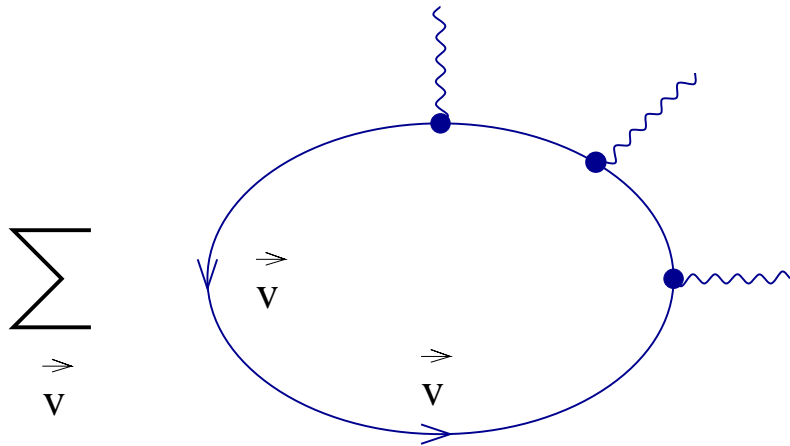
$$f_0^s = \frac{C_F}{4N_c N_f} \frac{g^2}{p_F^2}, \quad f_i^s = 0 \quad (i > 1)$$

Power Counting

- naive power counting

$$\mathcal{L} = \hat{\mathcal{L}} \left(\psi, \psi^\dagger, \frac{D_{\parallel}}{\mu}, \frac{D_{\perp}}{\mu}, \frac{\bar{D}_{\parallel}}{\mu}, \frac{m}{\mu} \right)$$

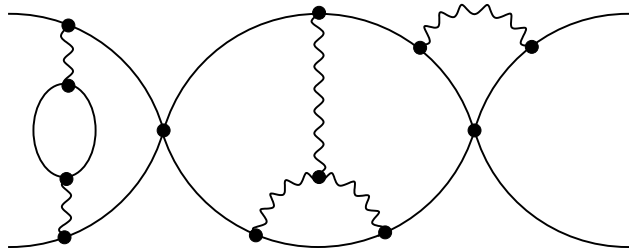
- problem: hard loops



$$\frac{1}{2\pi} \sum_{\vec{v}} \int \frac{d^2 l_{\perp}}{(2\pi)^2} = \frac{\mu^2}{2\pi^2} \int \frac{d\Omega}{4\pi}.$$

Modified Power Counting: $\mathcal{A} \sim l^\delta$

$$\delta = \sum_i [(k-4)V_k^S + (k-2-f_k)V_k^H] + E_Q + 4 - 2N_C$$



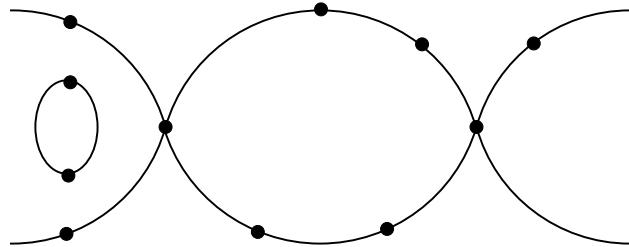
V_k^S soft vertices of $O(l^k)$

V_k^H hard vertices of $O(l^k)$

f_k number of fermion fields in V_k^H

E_Q external quark lines

N_C connected graphs in hard graph



- quark loops in gluon n-pt fcts blow up at $l \sim g\mu$
- four quark operators are leading order
- six quark operators (etc) are suppressed

Effective Theory for $l < g\mu$

$$\mathcal{L} = \psi_v^\dagger (iv \cdot D) \psi_v + f_0^s (\psi_v^\dagger \psi_v) (\psi_{v'}^\dagger \psi_{v'}) - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G_{\mu\alpha}^a \frac{v^\alpha v^\beta}{(v \cdot D)^2} G_{\mu\beta}^b$$

- transverse gauge boson propagator

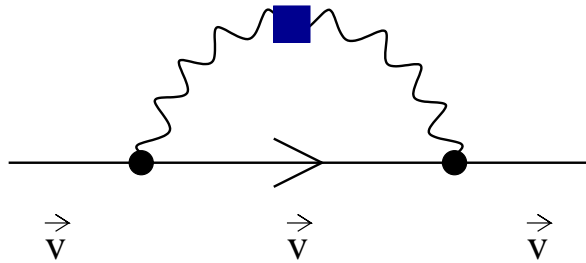
$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0| / |\vec{k}|},$$

- scaling of gluon momenta

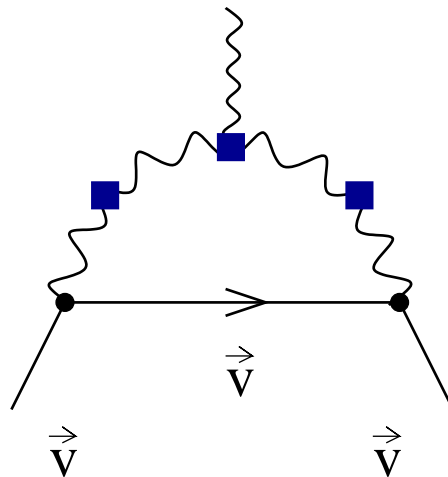
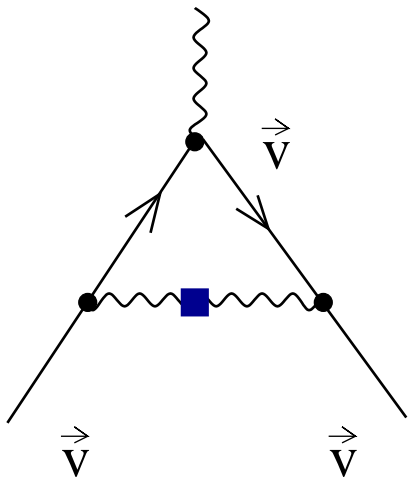
$$|\vec{k}| \sim k_0^{1/3} \eta^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Quasi-Quarks at Large Density:
Non-Fermi Liquid Effects

Loop Corrections



$$\Sigma(\omega) \simeq \frac{g^2 C_F}{12\pi^2} \omega \log\left(\frac{\Lambda}{\omega}\right)$$



$$\Gamma_\alpha = \frac{g^3 C_F v_\alpha}{12\pi^2} \log\left(\frac{\Lambda}{\omega}\right) \quad \text{time like}$$

like

$$\Gamma_\alpha = O(g^3)$$

space

like

“Migdal’s Theorem” for QCD

- self energy has to be resummed for $\omega \sim \Lambda \exp(-9\pi^2/g^2)$
- coupling has no logs ($k \gg k_4!$)

Renormalization Group

renormalized parameters

$$\psi_{0,v} = Z^{1/2} \psi_v, \quad v_{0,F} = Z_F v_F, \quad g_0 = \frac{Z_g}{Z Z_F} g, \quad \alpha = \frac{g^2 v_F}{4\pi},$$

one loop calculation $(\beta = \frac{\partial \alpha}{\partial \log \Lambda}, \gamma = \frac{\partial \log Z}{\partial \log \Lambda}, \gamma_F = \frac{\partial \log Z_F}{\partial \log \Lambda})$

$$\gamma(\alpha) = -\gamma_F(\alpha) = \frac{4\alpha}{9\pi}, \quad \beta(\alpha) = -\gamma_F(\alpha)\alpha \quad \text{IR free !!}$$

RG equation

$$\left\{ \Lambda \frac{\partial}{\partial \Lambda} + \beta(\alpha) \frac{\partial}{\partial \alpha} - \gamma_F(\alpha) l_i \frac{\partial}{\partial l_i} + \frac{n}{2} \gamma(\alpha) \right\} G^{(n)}(\omega_i, l_i, \alpha) = 0,$$

$\gamma \ll 1$: RG equation can be solved exactly

$$S^{-1}(\omega, l) = \omega \left(1 + \gamma \log \left(\frac{\Lambda}{\omega} \right) \right) - v_F l$$

no terms $\alpha^2 \log^2(\omega)$, etc.

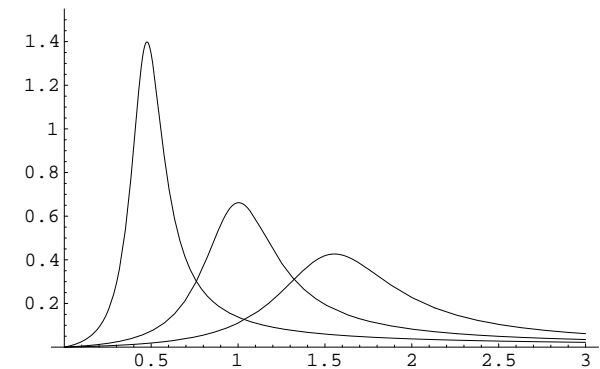
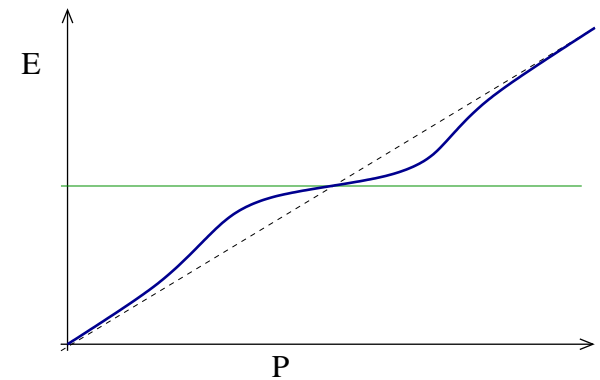
- quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$

- anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$

- enhanced corrections to the gap
 $\log(\mu/\Delta) = \log(\mu/\Delta_0)(1 - O(\gamma g))$



Quasi-Baryons at Large Density:

CFL Phase

CFL Phase

- Consider $N_f = 3$ ($m_i = 0$)

$$\langle q_i^a q_j^b \rangle = \phi \epsilon^{abI} \epsilon_{ijI}$$

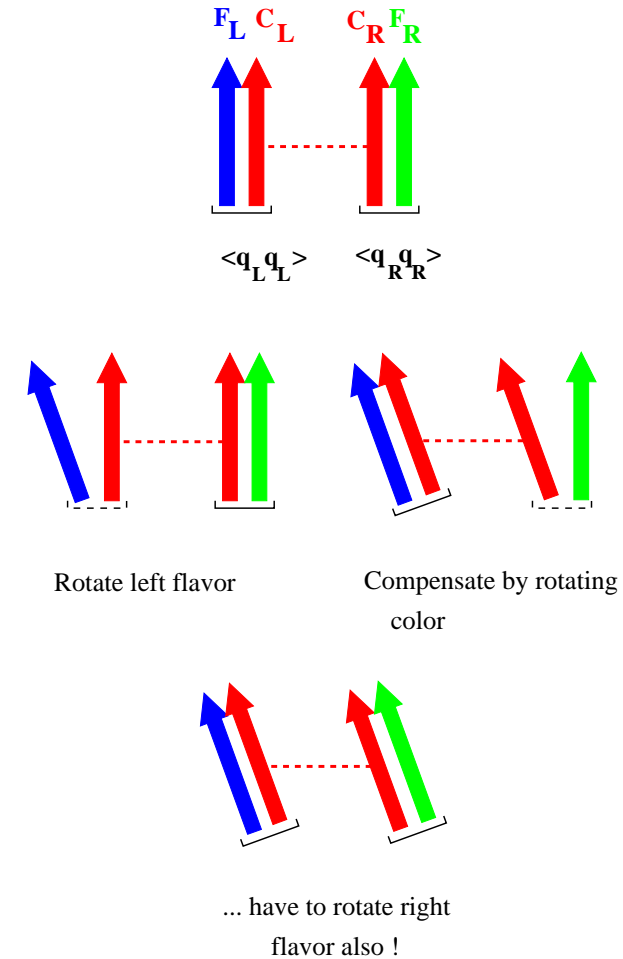
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

- symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \\ \times U(1) \rightarrow SU(3)_{C+F}$$

- all quarks and gluons acquire a gap



$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

EFT in the CFL Phase

consider HDET with a CFL gap term

$$\mathcal{L} = \text{Tr} \left(\psi_L^\dagger (i v \cdot D) \psi_L \right) + \frac{\Delta}{2} \left\{ \text{Tr} (X^\dagger \psi_L X^\dagger \psi_L) - \kappa [\text{Tr} (X^\dagger \psi_L)]^2 \right\} \\ + (L \leftrightarrow R, X \leftrightarrow Y)$$

$$\psi_L \rightarrow L \psi_L C^T, \quad X \rightarrow L X C^T, \quad \langle X \rangle = \langle Y \rangle = \mathbb{1}$$

quark loops generate a kinetic term for X, Y

$$\mathcal{L} = -\frac{f_\pi^2}{2} \left\{ \text{Tr} \left((X^\dagger D_0 X)^2 + (Y^\dagger D_0 Y)^2 \right) \right\} + \dots$$

integrate out gluons, identify low energy fields

$$\Sigma = X Y^\dagger \quad [8] + [1] \text{ GBs}, \quad N_L = \xi (\psi_L X^\dagger) \xi^\dagger \quad [8] + [1] \text{ Baryons}$$

effective theory: CFL(B) χ PTh

$$\begin{aligned} \mathcal{L} = & \frac{f_\pi^2}{4} \left\{ \text{Tr} (\nabla_0 \Sigma \nabla_0 \Sigma^\dagger) - v_\pi^2 \text{Tr} (\nabla_i \Sigma \nabla_i \Sigma^\dagger) \right\} \\ & + \text{Tr} (N^\dagger i v^\mu D_\mu N) - D \text{Tr} (N^\dagger v^\mu \gamma_5 \{ \mathcal{A}_\mu, N \}) \\ & - F \text{Tr} (N^\dagger v^\mu \gamma_5 [\mathcal{A}_\mu, N]) + \frac{\Delta}{2} \left\{ \text{Tr} (N N) - [\text{Tr} (N)]^2 \right\} \end{aligned}$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

$$\mathcal{V}_\mu = -\frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi)$$

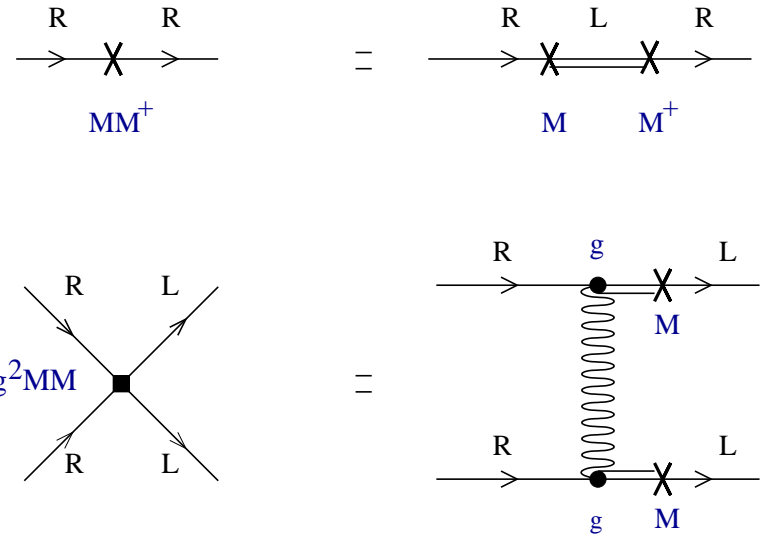
$$\mathcal{A}_\mu = -\frac{i}{2} \xi (\partial_\mu \Sigma^\dagger) \xi$$

$$f_\pi^2 = \frac{21 - 8 \log 2}{18} \frac{\mu^2}{2\pi^2} \quad v_\pi^2 = \frac{1}{3} \quad D = F = \frac{1}{2}$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L$$

$$+ \frac{C}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L) (\psi_R^\dagger M \lambda^a \psi_L)$$



Mass Terms: Match HDET to CFL χ Th

kinetic term: $\psi_L^\dagger X_L \psi_L + \psi_R^\dagger X_R \psi_R$

$$D_0 N = \partial_0 N + i[\Gamma_0, N], \quad \Gamma_0 = \mathcal{V}_0 + \frac{1}{2} (\xi X_R \xi^\dagger + \xi^\dagger X_L \xi)$$

$$\nabla_0 \Sigma = \partial_0 \Sigma + i X_L \Sigma - i \Sigma X_R$$

vector (axial) potentials

contact term: $(\psi_R^\dagger M \psi_L)(\psi_R^\dagger M \psi_L)$

$$\mathcal{L} = \frac{3\Delta^2}{4\pi^2} \{ [\text{Tr}(M\Sigma)]^2 - \text{Tr}(M\Sigma M\Sigma) \}$$

meson mass terms

Phase Structure and Spectrum

phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2} \text{Tr} (X_L \Sigma X_R \Sigma^\dagger) - A \text{Tr}(M \Sigma^\dagger) - B_1 [\text{Tr}(M \Sigma^\dagger)]^2 + \dots$$

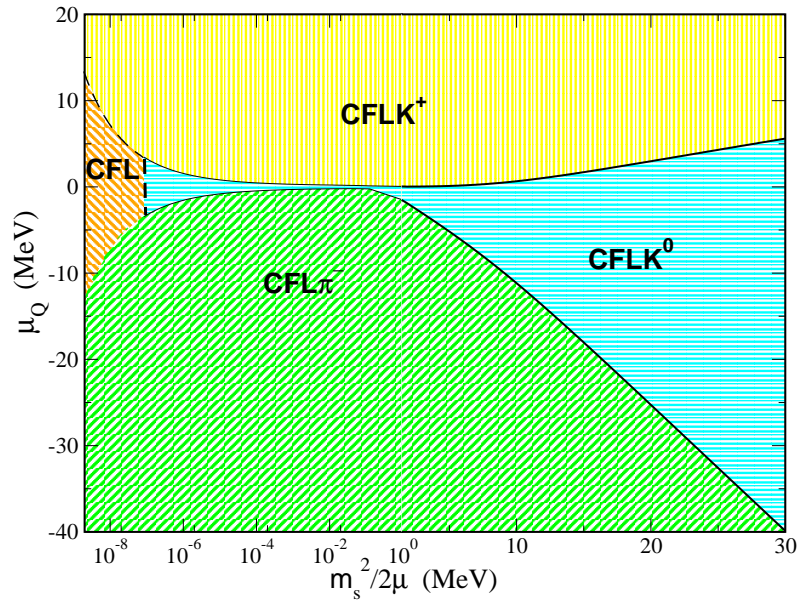
$$V(\Sigma_0) \equiv \text{min}$$

fermion spectrum determined by

$$\mathcal{L} = \text{Tr} (N^\dagger i v^\mu D_\mu N) + \text{Tr} (N^\dagger \gamma_5 \rho_A N) + \frac{\Delta}{2} \left\{ \text{Tr} (N N) - [\text{Tr} (N)]^2 \right\},$$

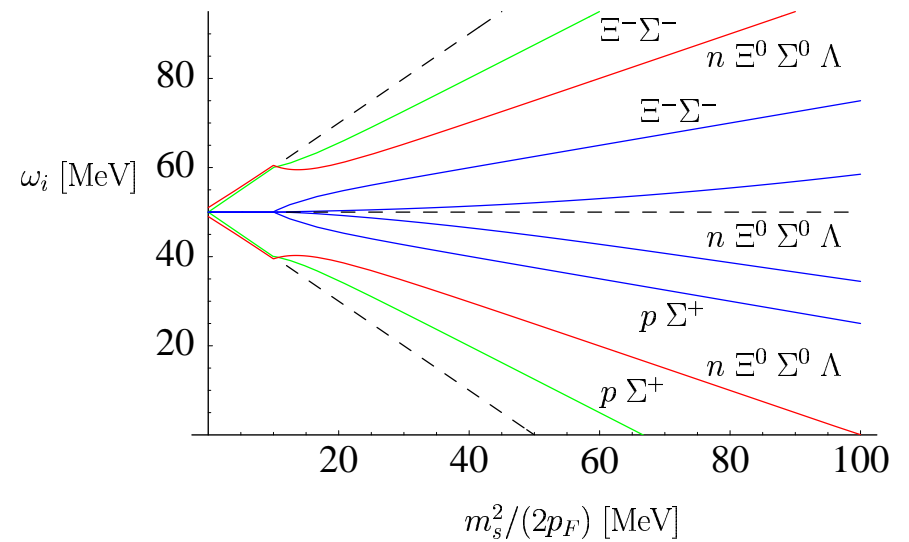
$$\rho_{V,A} = \frac{1}{2} \left\{ \xi \frac{M^\dagger M}{2p_F} \xi^\dagger \pm \xi^\dagger \frac{M M^\dagger}{2p_F} \xi \right\} \quad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum



meson condensation: CFLK

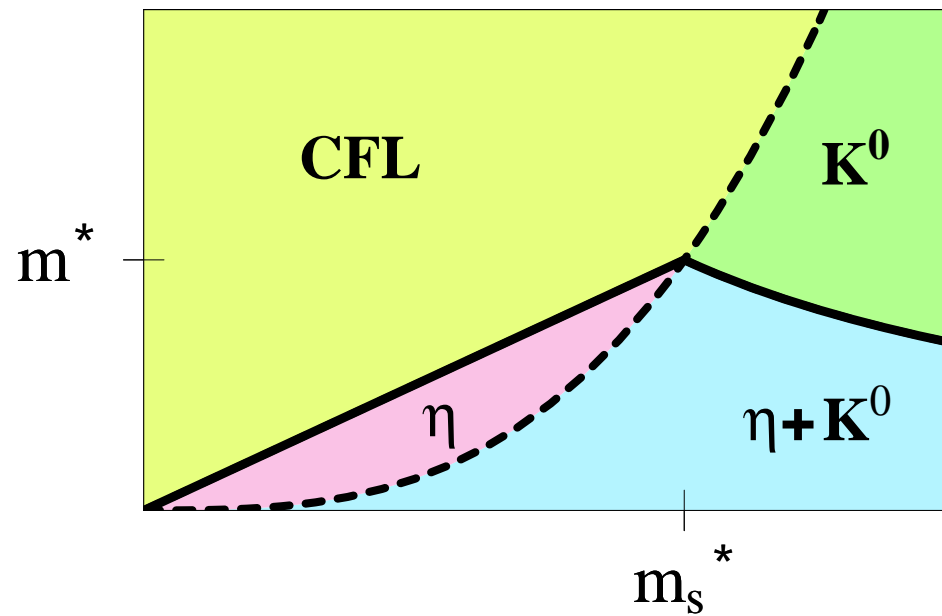
reliable: yes!



gapless modes? (gCFLK)

reliable: not clear yet

Phase Structure (continued)



phases with non-zero theta angle (CFL_η)

Summary

- EFT/RG methods provide powerful tools
 - phase structure and spectrum at large density
- normal phase: non-Fermi liquid behavior due to unscreened transverse gauge bosons
 - perturbation theory reliable (no rainbows, etc.)
- superfluid phase: effective chiral theory with calculable coefficients
 - kaon condensation, possibility of gapless modes