(Super) Fluid Transport in the Unitary Fermi Gas

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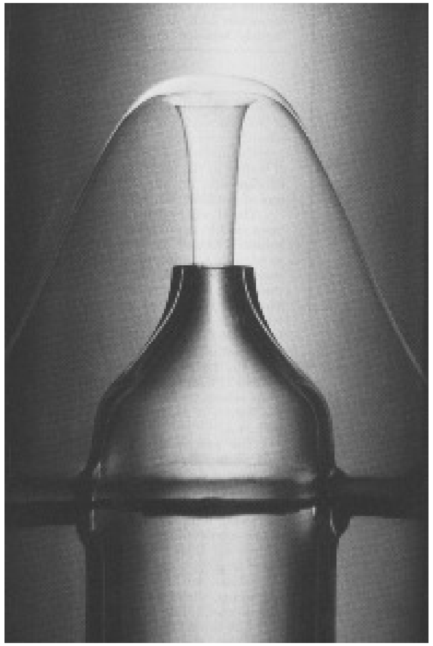


Hydrodynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.

Fluids: Gases, liquids, plasmas, ...

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of <u>any</u> many-body system.

$\tau \sim \tau_{micro}$ $\qquad$ $\tau \sim \lambda$

$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$

Effective theories for fluids (Unitary Fermi Gas, $T > T_F$)

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad \omega < T \frac{s}{\eta}$$

Strongly coupled: $\eta/s \sim 1$

Gradient expansion (simple non-relativistic fluid)

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \vec{j}^{\,\rho} = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\,\epsilon} = 0$$

$$\frac{\partial \pi_i}{\partial t} + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

Ward identity: mass current = momentum density

$$\vec{j}^{\,\rho} \equiv \rho \vec{v} = \vec{\pi}$$

Constitutive relations: Gradient expansion for currents

Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

Gradient expansion, Kubo formula

Consider background metric $g_{ij}(t,x) = \delta_{ij} + h_{ij}(t,x)$. Linear response

$$\delta\Pi^{xy} = -\frac{1}{2} G_R^{xyxy} h_{xy}$$

Harmonic perturbation $h_{xy} = h_0 e^{-i\omega t}$

$$G_R^{xyxy} = P - i\eta\omega + \ldots$$

Kubo relation: $$\eta = -\lim_{\omega\to 0}\left[\frac{1}{\omega}\mathrm{Im}G_R^{xyxy}(\omega,0)\right]$$

Gradient expansion: $\omega \le \frac{P}{\eta} \simeq \frac{s}{\eta} T$.

Superfluid hydrodynamics

Spontaneous symmetry breaking: $\langle\Psi\rangle = v_0 e^{i\theta}$.

Goldstone boson is a new hydro mode: $\vec{v}_s = \frac{\hbar}{m}\vec{\nabla}\theta$

$$\partial_t \vec{v}_s + \frac{1}{2}\vec{\nabla}(v_s^2) = -\vec{\nabla}\mu$$

Momentum density: $\pi_i = \rho_n v_{n,i} + \rho_s v_{s,i}$

$$\rho = \rho_n + \rho_s \qquad \rho_s = \frac{1}{2}\frac{\partial F}{\partial w^2} \qquad \vec{w} = \vec{v}_n - \vec{v}_s$$

Stress tensor and energy current

$$\begin{aligned} \Pi_{ij} &= P\delta_{ij} + \rho_n v_{n,i} v_{n,j} + \rho_s v_{s,i} v_{s,j} \\ \vec{\jmath}^{\,\epsilon} &= sT\vec{v}_n + \left(\mu + \frac{1}{2}v_s^2\right)\vec{\pi} + \rho_n \vec{v}_n \vec{v}_n \cdot \vec{w} \end{aligned}$$

Superfluid hydrodynamics

Dissipative stresses

$$\delta\Pi_{ij} = -\eta\left(\nabla_i v_{n,j} + \nabla_j v_{n,i} - \frac{2}{3}\delta_{ij}\vec{\nabla}\cdot\vec{v}_n\right)$$

$$-\,\delta_{ij}\Big(\zeta_1\vec{\nabla}\left(\rho_s\left(\vec{v}_s - \vec{v}_n\right)\right) + \zeta_2\left(\vec{\nabla}\cdot\vec{v}_n\right)\Big)$$

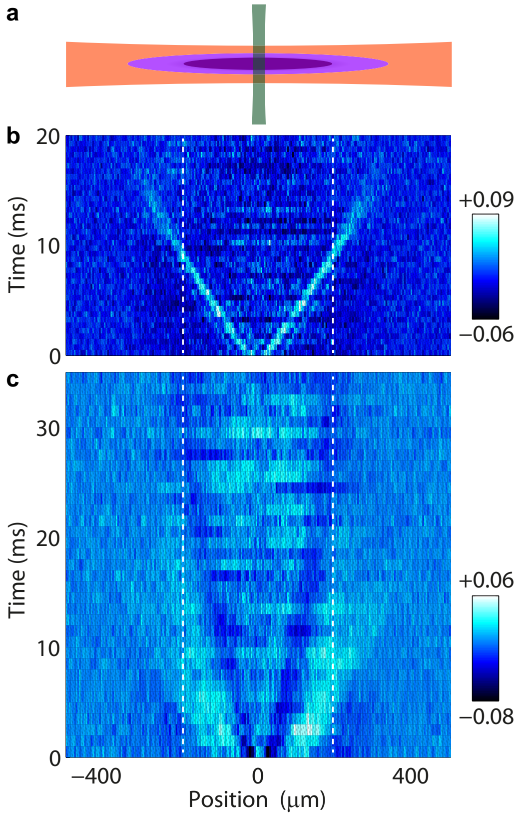
Equation of motions for v_s: $\dot{v}_s + \frac{1}{2}\nabla(v_s^{\,2}) = -\nabla(\mu + H)$ with

$$H = -\zeta_3\vec{\nabla}\left(\rho_s\left(\vec{v}_s - \vec{v}_n\right)\right) - \zeta_4\vec{\nabla}\cdot\vec{v}_n$$

Conformal symmetry: $\zeta_1 = \zeta_2 = \zeta_4 = 0$

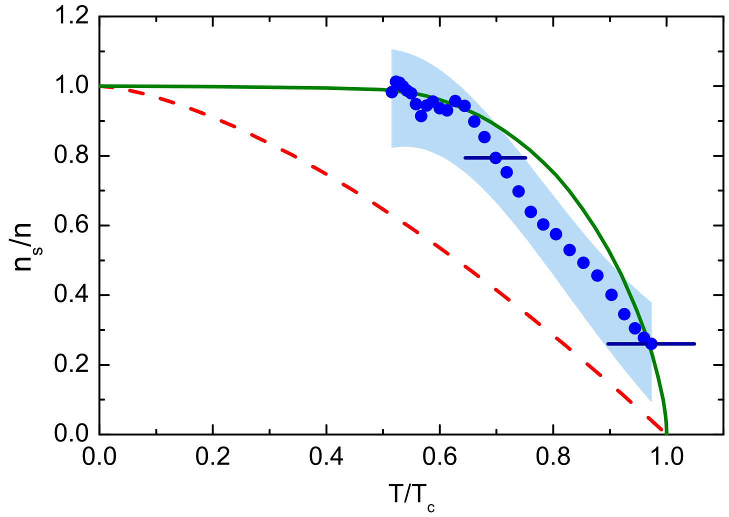
Son (2007)

Superfluid Hydrodynamics: Second Sound



1st (top) 2nd sound (bottom)

in unitary Fermi gas



Superfluid mass fraction

CAG, He, BEC (th)

Grimm et al. (2013)

Fermi gas at unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

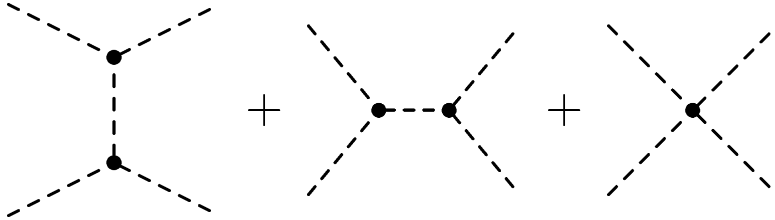
Low T ($T < T_c \sim \mu$): Pairing and superfluidity

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

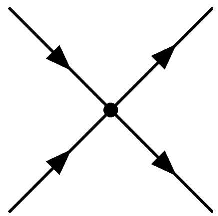
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$

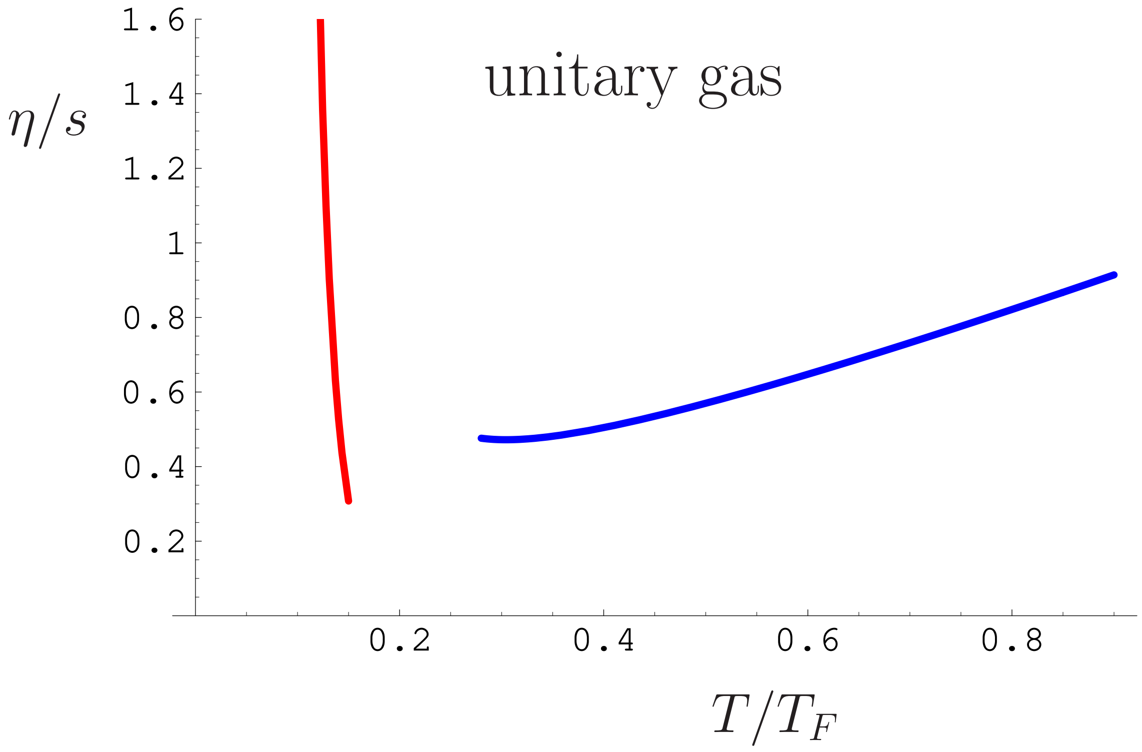


High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



η/s: Kinetic Theory



Low T behavior very steep, no indication for smooth crossover.

Thermal conductivity

Superfluids are very efficient conductors of heat, by a process usually called superfluid convection.

There is a non-zero (but difficult to observe) diffusive contribution

$$\vec{\jmath}^{\,\epsilon} = -\kappa \vec{\nabla} T$$

The calculation of κ is subtle, because quasi-particles with linear dispersion $E_p \sim c_s p$ do not contribute. [Roughly, linear qp's always transport momentum together with energy.]

The dominant process is phonon splitting, made possible by non-linear terms in the dispersion relation.

$$\kappa = \frac{128}{3\pi} \frac{\gamma^2}{g_3^2} \frac{T^2}{c_s^2} D_H = \frac{256\sqrt{2}}{25\pi^3 \xi^2 m} (mT)^{3/2} \left(\frac{T}{T_F} \right)^2 D_H$$

Normal phase $\kappa \sim m^{1/2} T^{3/2}$

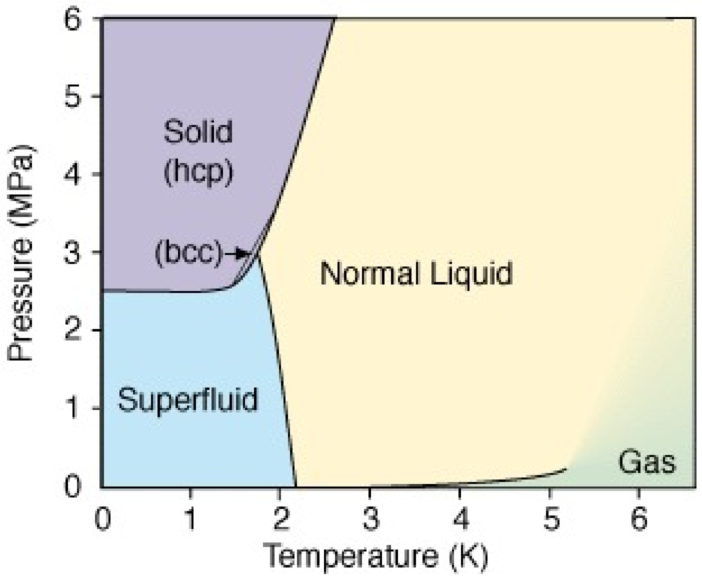
Liquid Helium

Bosons, van der Waals + short range repulsion

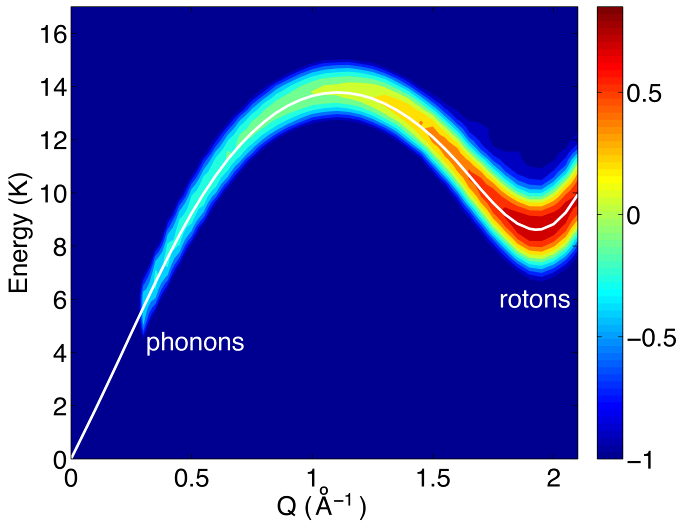
$$S = \int \Phi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \Phi + \int \int \left(\Phi^\dagger \Phi \right) V(x-y) \left(\Phi^\dagger \Phi \right)$$

with $V(x) = V_{sr}(x) - c_6/x^6$. Note: $a = 189 a_0 \gg a_0$

Phase Diagram



Excitations



Low T: Phonons and Rotons Effective lagrangian

$$\mathcal{L} = \varphi^*(\partial_0^2 - v^2)\varphi + i\lambda\dot{\varphi}(\vec{\nabla}\varphi)^2 + \ldots$$

$$+ \varphi^*_{R,v}(i\partial_0 - \Delta)\varphi_{R,v} + c_0(\varphi^*_{R,v}\varphi_{R,v})^2 + \ldots$$

Shear viscosity

$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}}\, e^{\Delta/T} + \frac{c_{ph}}{T^5} + \ldots$$

High T: Atoms Viscosity governed by hard core $(V \sim 1/r^{12})$

$$\eta = \eta_0(T/T_0)^{2/3}$$

Experiment: Liquid Helium

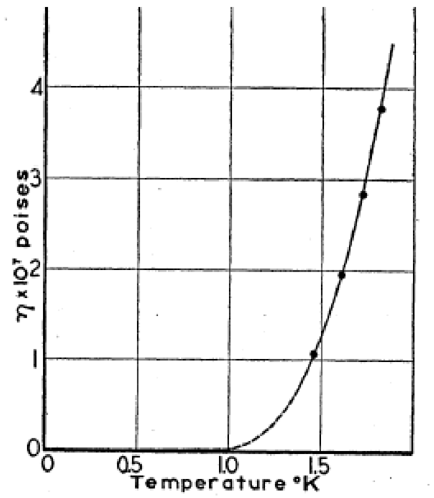
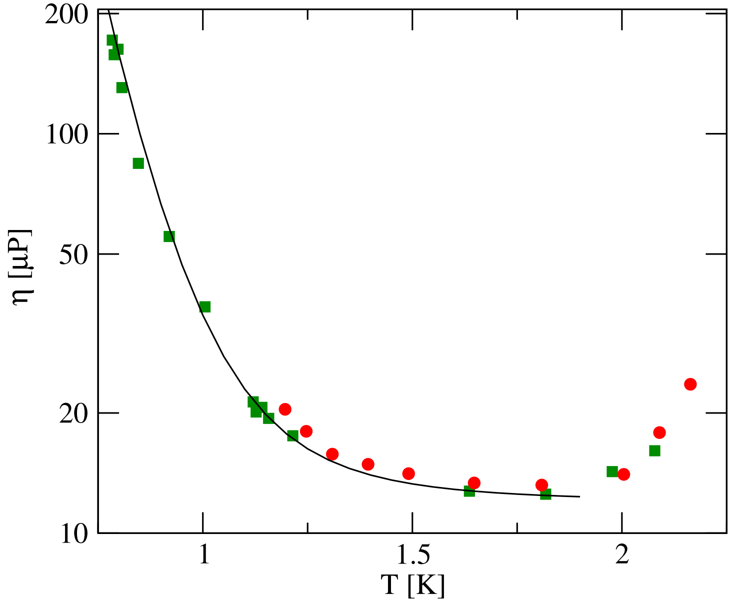


FIG. 1. The viscosity of liquid helium II measured by flow throu[gh]
a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

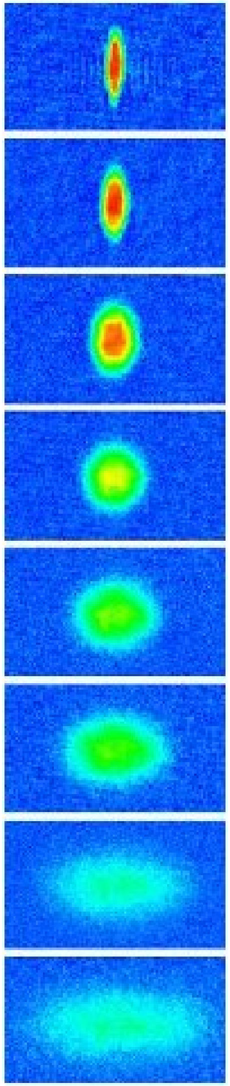
capillary flow viscometer

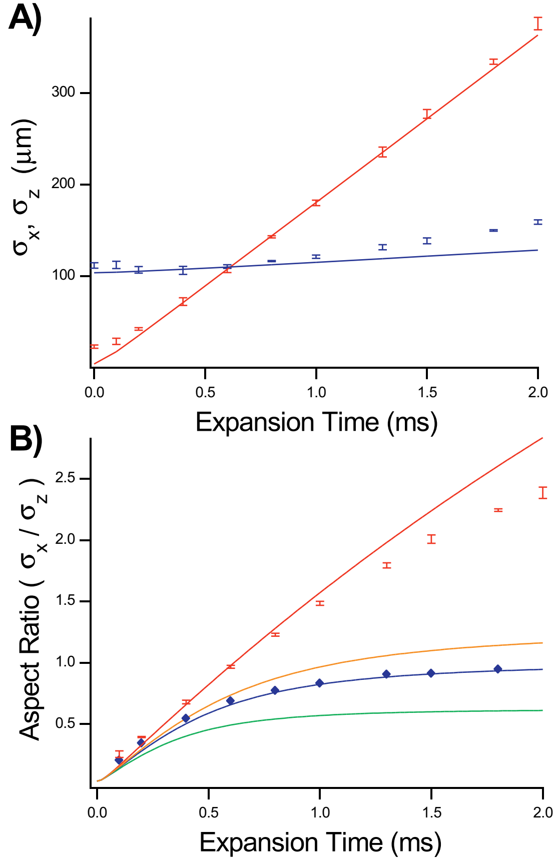
Hollis-Hallett (1955)

roton minimum, phonon rise

rotation viscometer

Experiments: Elliptic flow





Hydrodynamic expansion

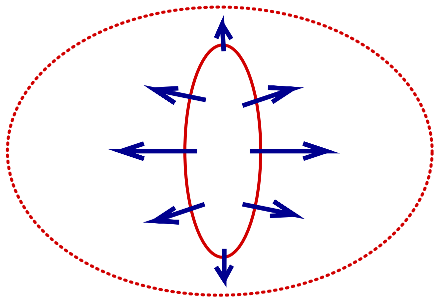
converts

coordinate space

anisotropy

to momentum space

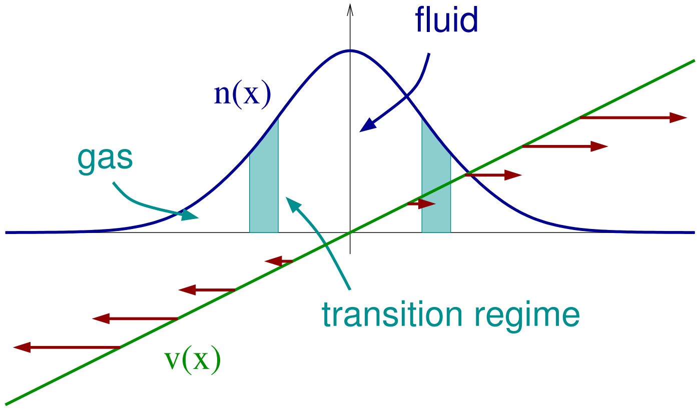
anisotropy



O'Hara et al. (2002)

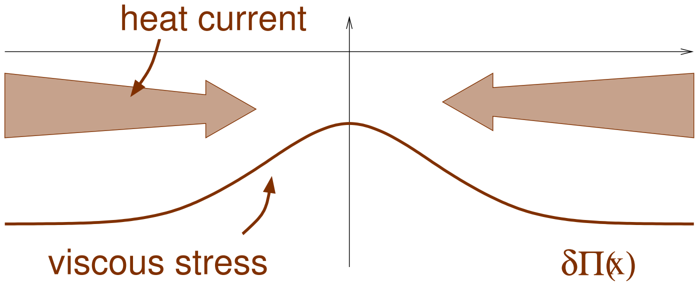
Determination of $\eta(n, T)$

Measurement of $A_R(t, E_0)$ determines $\eta(n, T)$. But:



The whole cloud is not a fluid.

Can we ignore this issue?



No. Hubble flow & low density viscosity $\eta \sim T^{3/2}$ lead to paradoxical fluid dynamics.

$$\dot{Q} = \int \sigma \cdot \delta\Pi = \infty$$

Not a fundamental problem (dilute corona is ballistic) but how to couple fluid and ballistic motion?

Possible approaches to dilute regime

1) Boltzmann equation for the entire cloud. Hard to incorporate $P(n, T)$ and $\eta(n, T)$.

2) Combine hydrodynamics & Boltzmann equation. Not straightforward.

3) Hydrodynamics + non-hydro degrees of freedom ($\mathcal{E}_a$; $a = x, y, z$)

$$\frac{\partial \mathcal{E}_a}{\partial t} + \vec{\nabla} \cdot \vec{\jmath}_a^{\,\epsilon} = -\frac{\Delta P_a}{2\tau} \qquad \Delta P_a = P_a - P$$

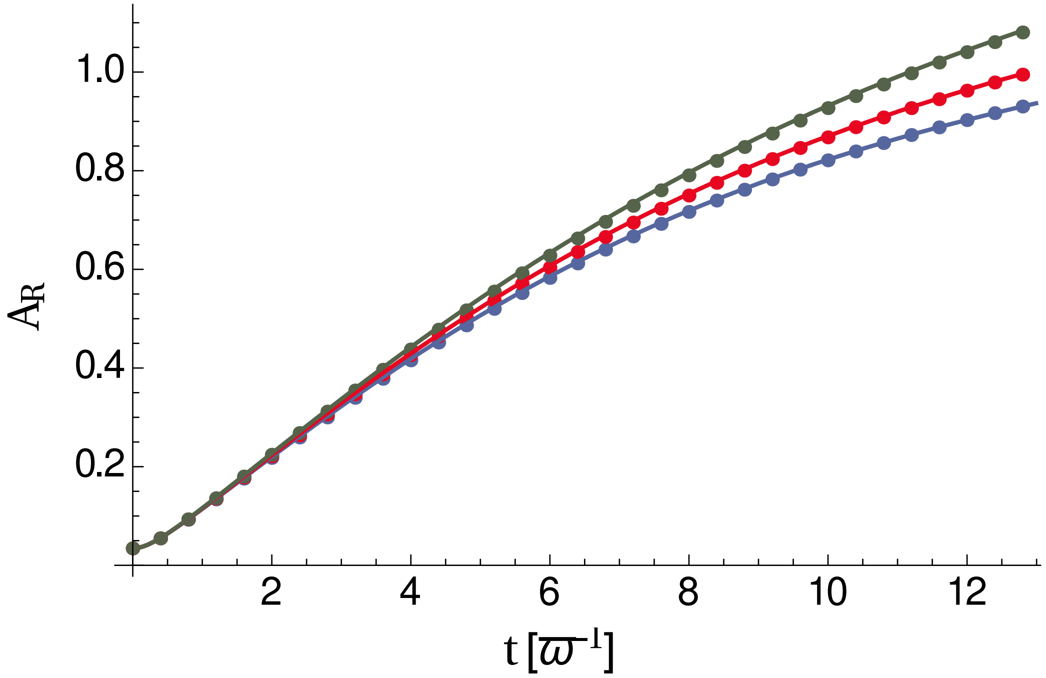
$$\frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla} \cdot \vec{\jmath}^{\,\epsilon} = 0 \qquad \mathcal{E} = \sum_a \mathcal{E}_a$$

τ small: Fast relaxation to Navier-Stokes with $\tau = \eta/P$

τ large: Additional conservation laws. Ballistic expansion.

Anisotropic Hydrodynamics: Comparison with Boltzmann

Aspect ratio $A_R(t) = (\langle r_\perp^2 \rangle / \langle r_z^2 \rangle)^{1/2}$ $\quad (T/T_F = 0.79, 1.11, 1.54)$



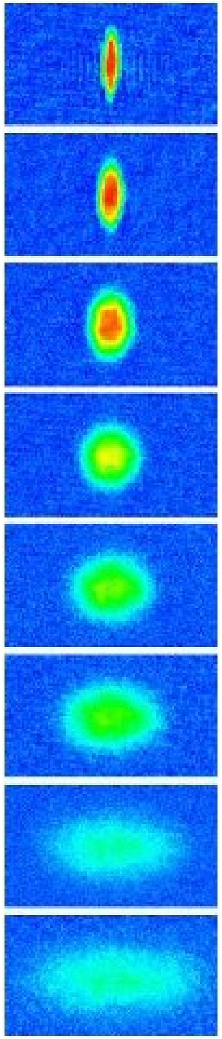
Dots: Two-body Boltzmann equation with full collision kernel

Lines: Anisotropic hydro with η fixed by Chapman-Enskog

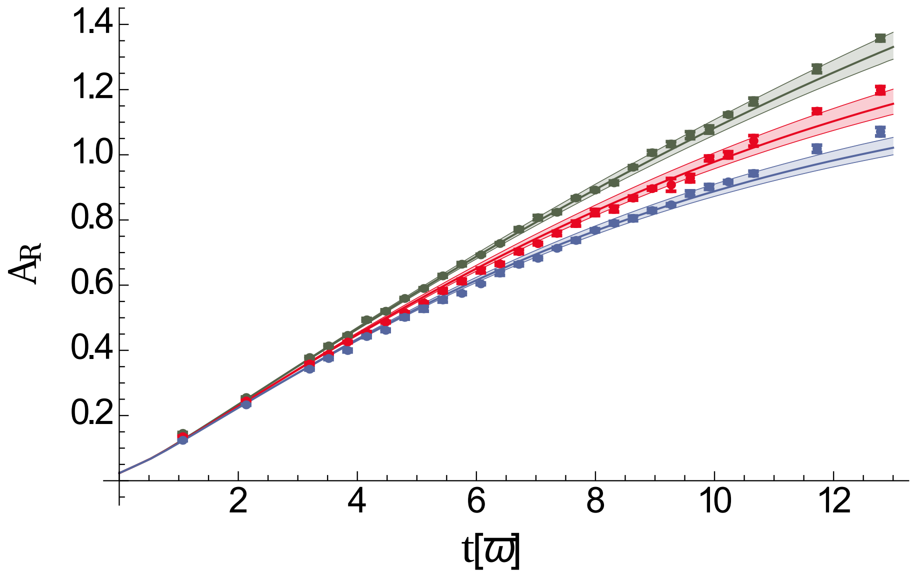
High temperature (dilute) limit: Perfect agreement!

Boltzmann, Pantel et al (2015); AVH1 hydro code, M. Bluhm & T.S. (2015)

Elliptic flow: High T limit



Quantum viscosity $\eta = \eta_0 \dfrac{(mT)^{3/2}}{\hbar^2}$



$T/T_F = 0.79, 1.11, 1.54$

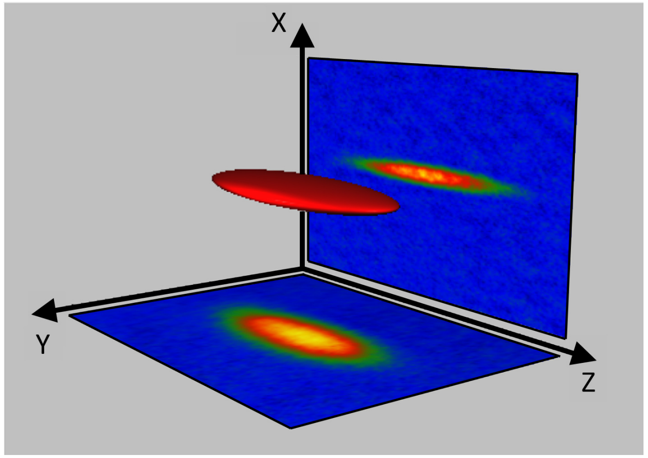
Cao et al., Science (2010)

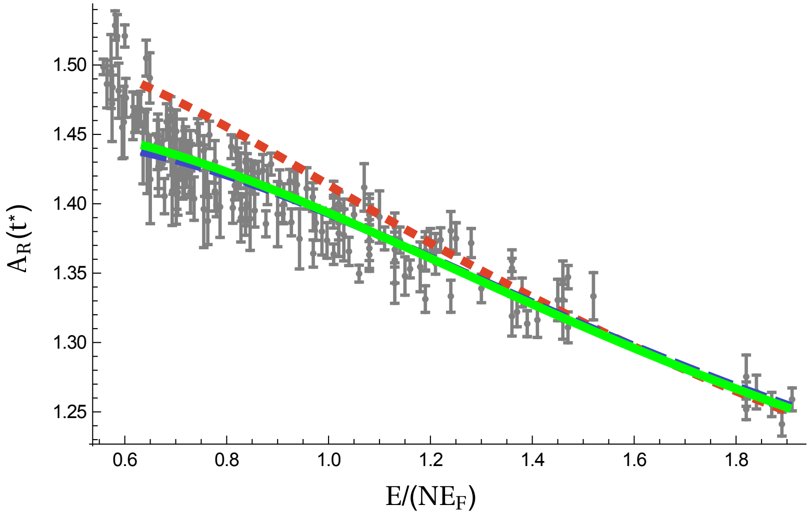
Bluhm et al., PRL (2016)

fit: $\eta_0 = 0.282 \pm 0.02$

theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.269$

Anisotropic fluid dynamics analysis





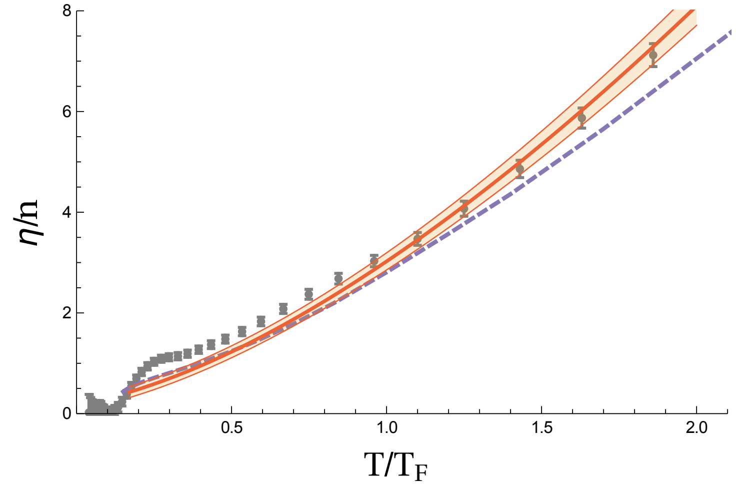
$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

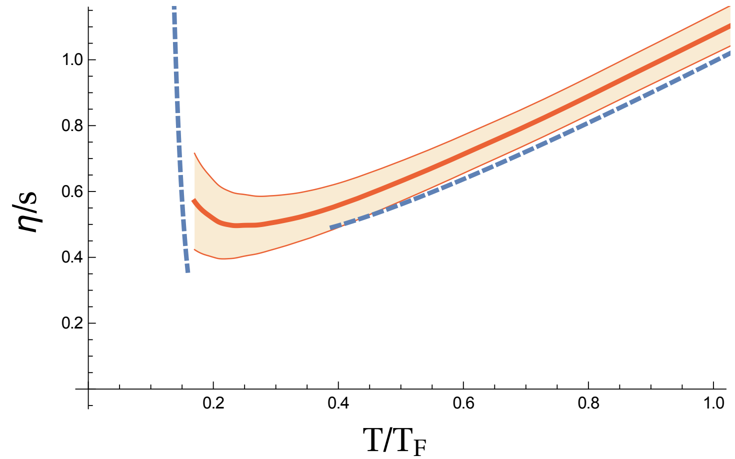
Red, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0 (mT)^{3/2} \left\{ 1 + \eta_2 n\lambda^3 + \eta_3 (n\lambda^3)^2 + \ldots \right\}$$

Converges rapidly above T_c. (Discontinuity below T_c?)

Reconstruct η/n and η/s





Left: η/n (Red band)

Right: η/s (Red band) $T_c \sim 0.17 T_F$.

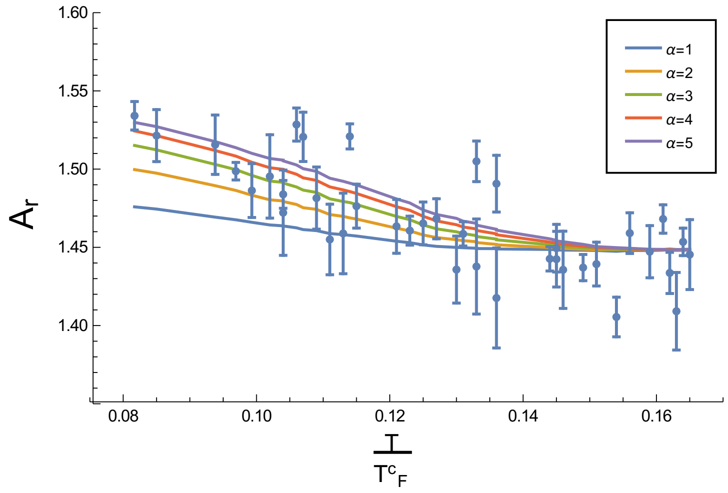
Joseph et al. (Black points). Enss et al. (Dashed line).

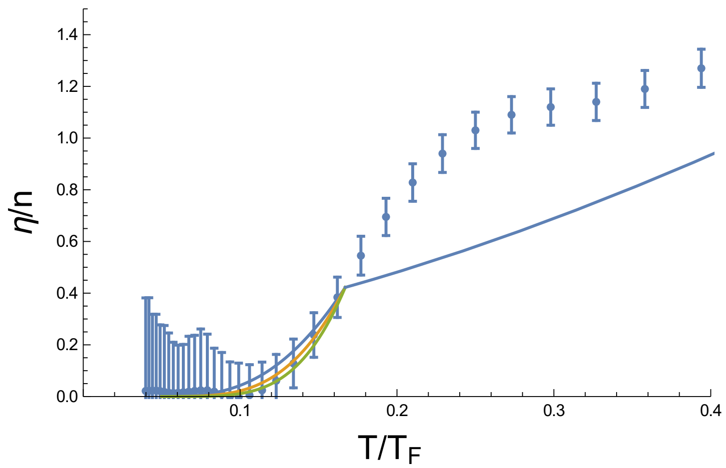
Kinetic theory at low and high T (blue dashed)

$$\eta(T \gg T_c) = (0.265 \pm 0.02)(mT)^{3/2} \qquad \eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

$$\eta/n|_{T_c} = 0.41 \pm 0.15 \qquad \eta/s|_{T_c} = 0.56 \pm 0.20$$

Extend analysis to $T < T_c$





Left: $A_R = \sigma_x/\sigma_y$ as function of T/T_F. Data: Joseph et al (2016). Right: Points, Joseph et al. Lines, Hou & Schaefer.

Analyzed using one-fluid dynamics. No sign of rise in η/n as $T \to 0$.

Outlook

Perform analysis using two-fluid hydrodynamics.

Data from homogeneous systems (box potentials).

Other transport properties: Spin diffusion, thermal conductivity, bulk viscosity near unitarity.