Instantons in QCD: 25 years later

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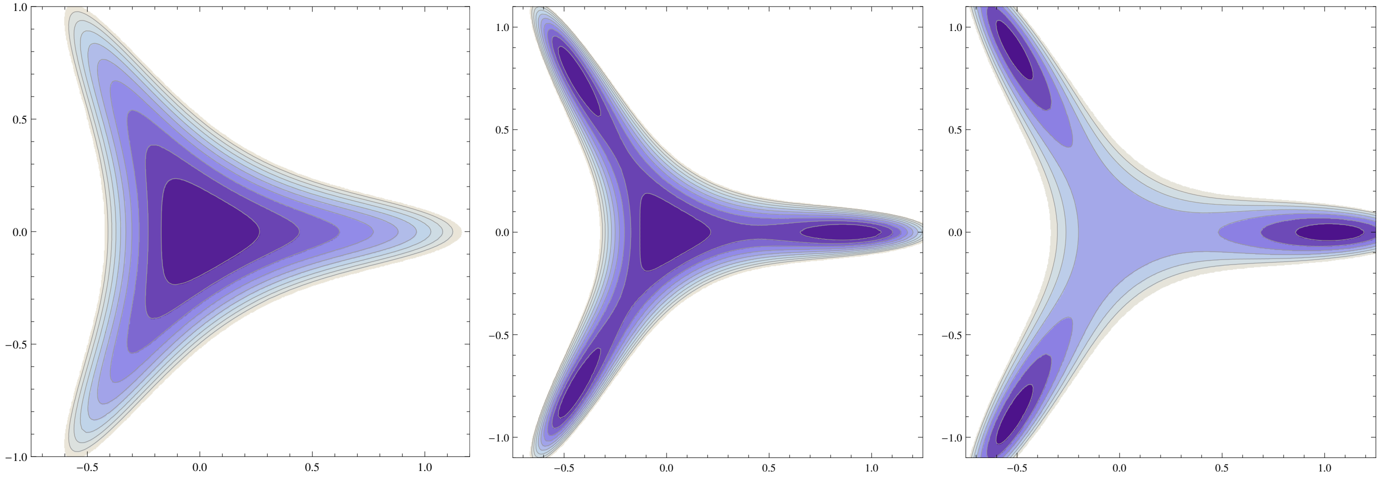
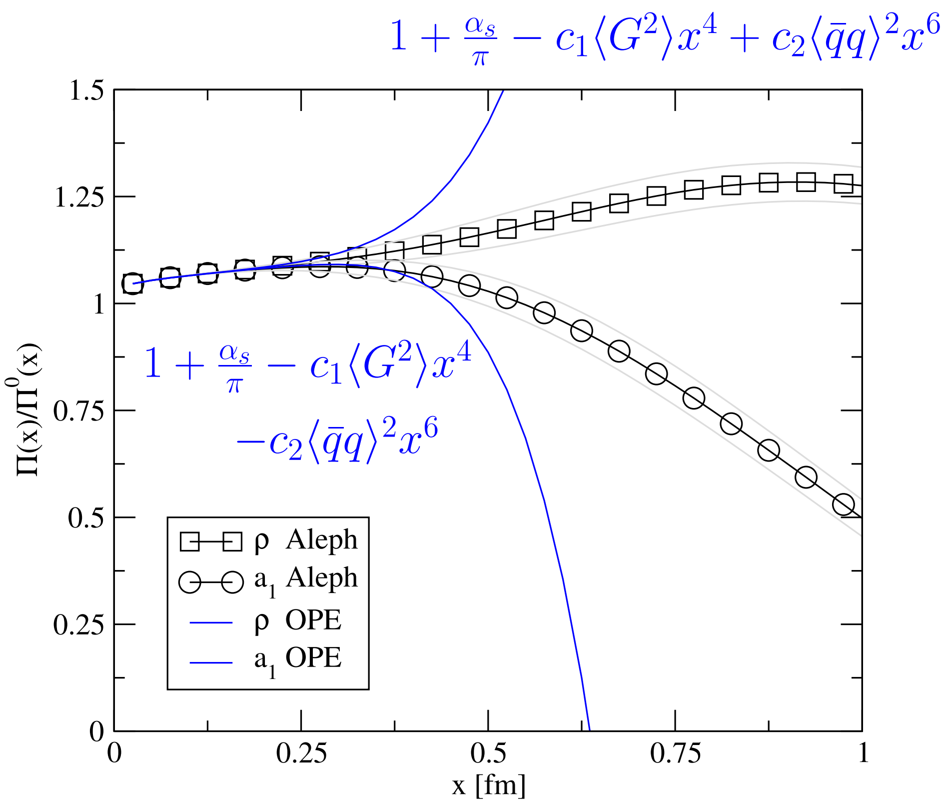
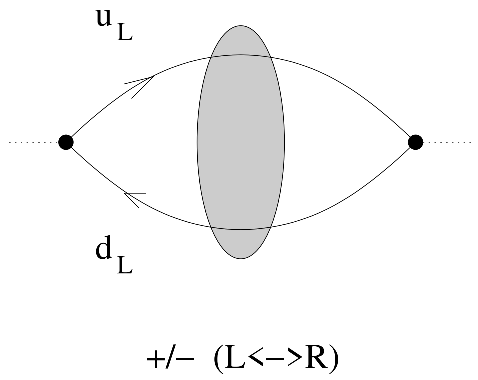


T.S., E. Shuryak, RMP (1997) [hep-ph/9610451], work with A. Cherman, E. Poppitz, and M. Unsal

Are all hadrons alike? Vector Channels (ρ and a_1)

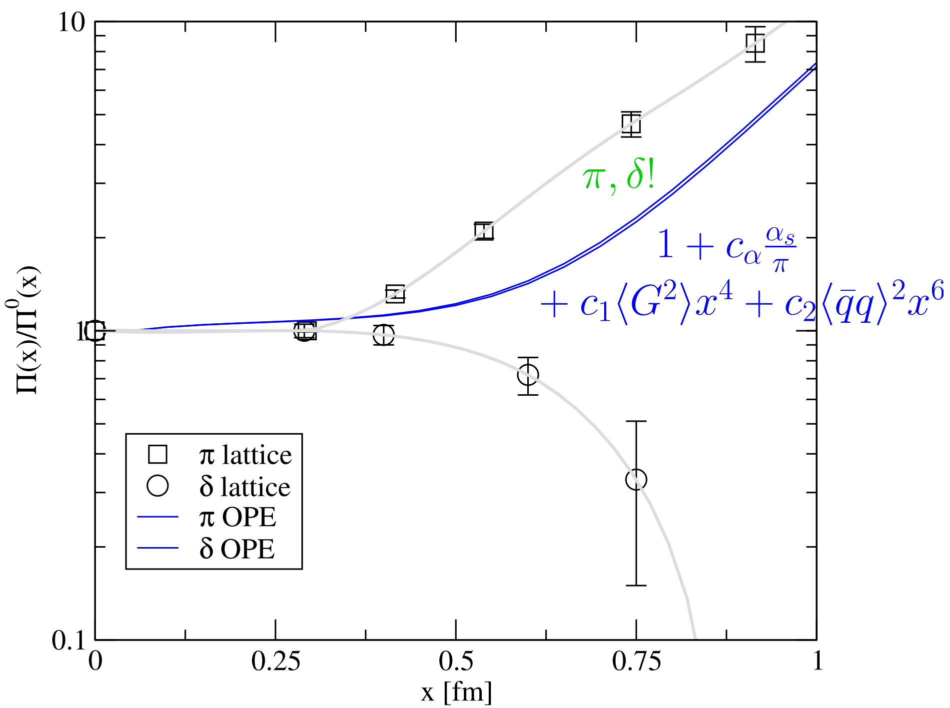


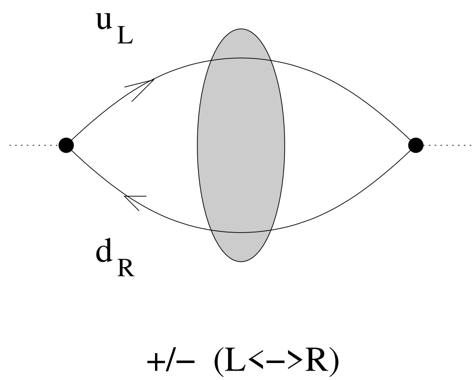


$$\Pi(x)=\langle j_\mu^{V,A}(0)j^{\mu\,V,A}(x)\rangle \qquad j_\mu^{V,A}=\bar{d}_L\gamma_\mu u_L\pm(L\leftrightarrow R)$$

Novikov et al. Nucl.Phys.B 191 (1981) 301. E.S. Rev.Mod.Phys. 65 (1993) 1.

Are all hadrons alike? Scalar Channels (π and a_0)

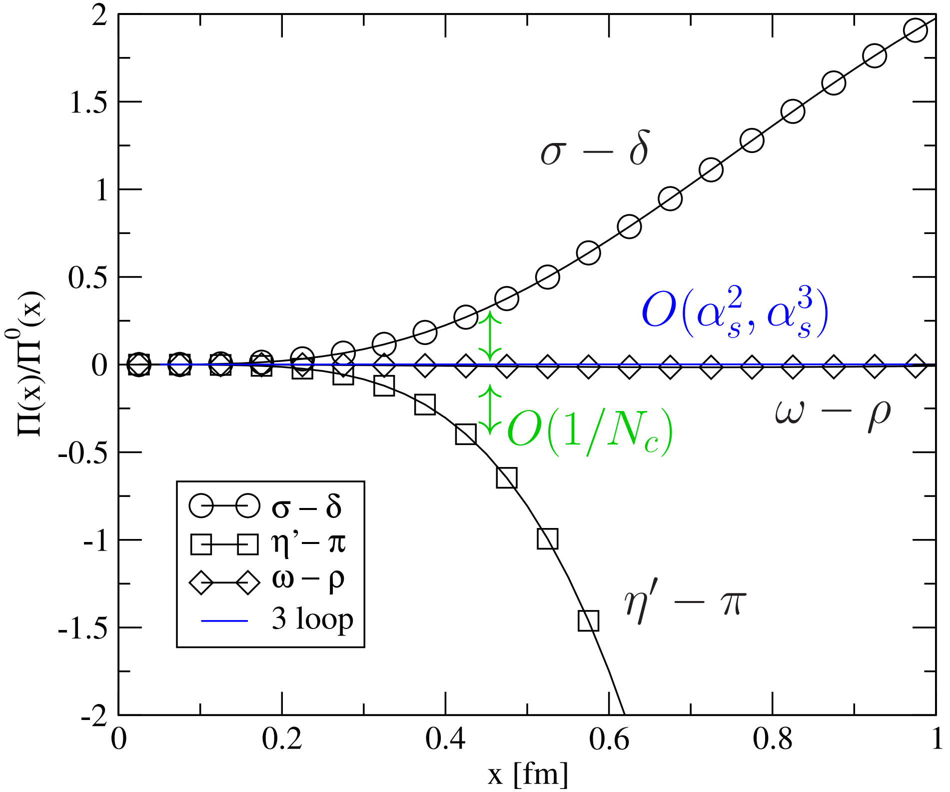


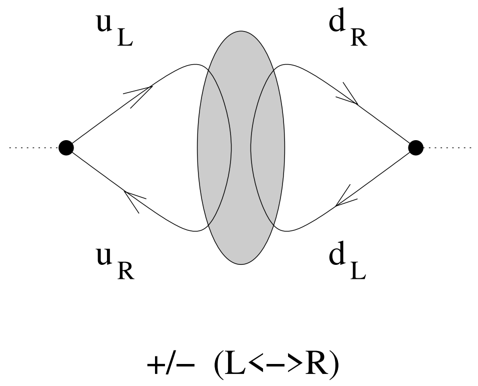


$$\Pi(x) = \langle j^{S,P}(0) j^{\mu\, S,P}(x) \rangle$$

$$j^{S,P} = \bar{d}_L u_R \pm (L \leftrightarrow R)$$

Are all hadrons alike? OZI Violation





$$\Pi(x) = \langle j^\Gamma(0) j^\Gamma(x) \rangle \qquad j^\Gamma = \bar{u}_L \Gamma u_{L,R} + \bar{d}_L \Gamma d_{L,R} \pm (L \leftrightarrow R)$$

Phenomenology: Summary

Only small effects in $(\bar{L}L \pm \bar{R}R)^2$.

Sign changes for $(\bar{L}R + \bar{R}L) \leftrightarrow (\bar{L}R - \bar{R}L)$.

Sign changes for $(\bar{u}d)(\bar{d}u) \leftrightarrow (\bar{u}u)(\bar{d}d)$.

$$\mathcal{L} = G \det_f(\bar{\psi}_L \psi_R) + (L \leftrightarrow R)$$

G.'t Hooft, Phys.Rev.Lett. 37 (1976) 8.

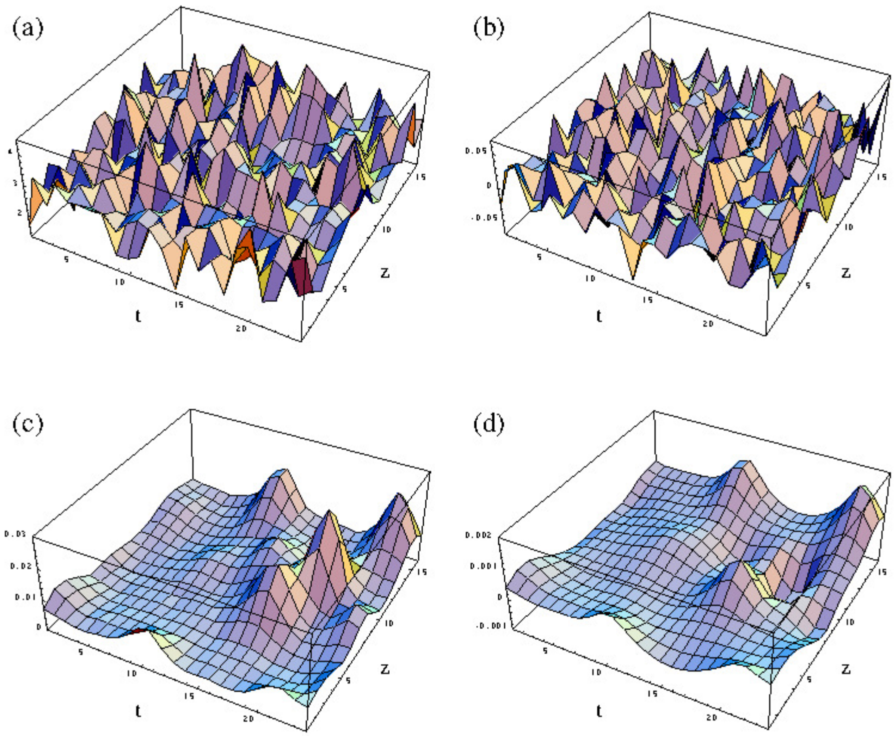
The Instanton Liquid

ES (1982): Instantons provide a quantitative description of QCD correlations functions

$$\rho = 0.3 \text{ fm} \quad \frac{N}{V} = 1 \text{ fm}^{-4}$$

$$S \sim 10 \gg 1$$

$$\delta S \sim 1 \ll S$$



Callan et al. Phys.Rev.D 17 (1978) 2717. Shuryak, Nucl.Phys.B 203 (1982) 93. Dyakonov, Petrov, Nucl.Phys.B 245 (1984) 259-

Successes

Microscopic model for chiral symmetry breaking and the $U(1)_A$ anomaly.

Phenomenology of hadronic correlation functions.

Contact to lattice gauge theory.

Difficulties

Confinement?

Large N_c?

Reliable semi-classics? IA pairs?

Difficulties and Progress

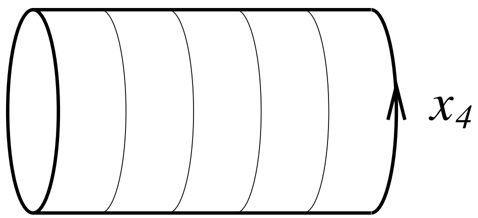
Confinement?	Selfdual monopoles.
Large N_c?	Fractional topological charge.
Reliable semi-classics? IA pairs?	Deformed QCD, resurgence.

Semiclassical Confinement

Consider $SU(2)$ gauge theory with $N_f^{ad} = 1$ on $R^3 \times S_1$

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a$$

$$A^a_\mu(0) = A^a_\mu(L)$$
$$\lambda^a_\alpha(0) = \lambda^a_\alpha(L)$$



Large m: Thermal pure YM Z_β. Small m: Twisted SUSY YM $\tilde{Z}_\beta$.

Small S_1 and m: Confinement can be studied using semi-classical methods, based on monopole-instantons, instantons, and bions.

Theory abelianizes. Low energy fields: Holonomy b and dual photon σ. Perturbative potential vanishes.

Small S_1: Effective Theory

Consider small S_1: Effective theory in 3d

$\Omega \neq 1$: A_4^3 is a Higgs field, theory abelianizes $SU(2) \to U(1)$.

Light bosonic modes: (dual) "photon" σ and holonomy b

$$\mathcal{L} = \frac{g^2}{32\pi^2 L}\left[(\partial_i b)^2 + (\partial_i \sigma)^2\right] + V(\sigma, b)$$

$$\Omega = \begin{pmatrix} e^{i\Delta\theta/2} & 0 \\ 0 & e^{-i\Delta\theta/2} \end{pmatrix} \quad b = \frac{4\pi}{g^2}\Delta\theta \qquad \epsilon_{ijk}\partial_k\sigma = \frac{4\pi L}{g^2}F_{ij}$$

holonomy b dual photon σ

Note: $m = 0$ effective theory can be super-symmetrized

$$B = b + i\sigma + \sqrt{2}\theta^\alpha\lambda^\alpha$$

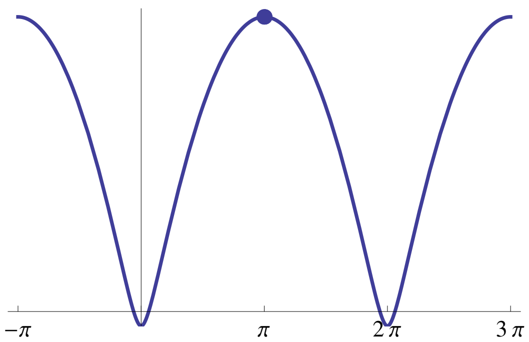
Perturbation Theory

Perturbative potential for holonomy (Gross, Pisarski, Yaffe, 1981)

$$V(\Omega) = -\frac{m^2}{2\pi^2 L^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \, |\mathrm{tr}\,\Omega^n|^2 = -\frac{m^2}{L^2} B_2\left(\frac{\Delta\theta}{2\pi}\right)$$

$m = 0$: Bosonic and fermionic terms cancel.

$m \neq 0$: Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ unstable.



Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x \, F\tilde{F}$$

2. Holonomy (eigenvalues q^α of Polyakov line at spatial infinity)

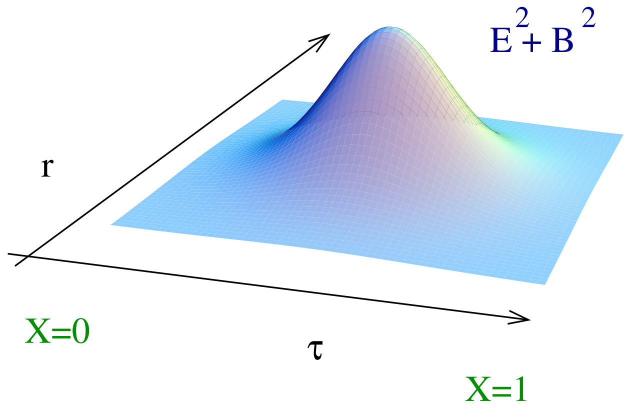
$$\langle \Omega(\vec{x}) \rangle = \left\langle \mathrm{Tr} \exp \left[i \int_0^\beta A_4 dx_4 \right] \right\rangle$$

3. Magnetic charges

$$Q_M^\alpha = \frac{1}{4\pi} \int d^2S \, \mathrm{Tr} \left[P^\alpha B \right]$$

Periodic instantons (calorons)

Instanton solution in R^4 can be extended to solution on $R^3 \times S^1$



$$Q_{top} = \pm 1$$

$$\Omega_\infty = 1 \quad Q_M^\alpha = 0$$

$SU(2)$ solution has $1+3+1+3=8$ bosonic zero modes

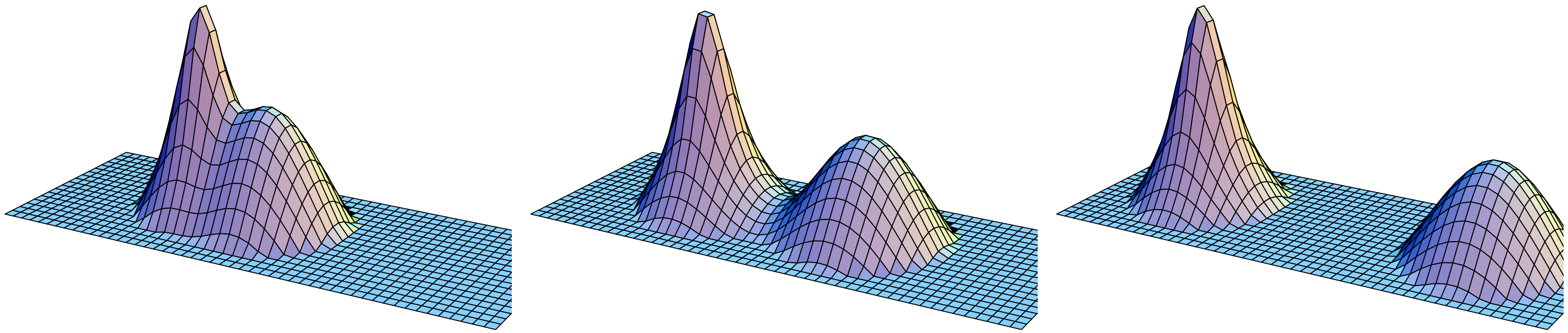
$$\int \frac{d\rho}{\rho^5} \int d^3x\, dx_4 \int dU\, e^{-2S_0} \qquad 2S_0 = \frac{8\pi^2}{g^2}$$

$4n_{adj}$ fermionic zero modes

$$\int d^2\zeta d^2\xi$$

Calorons at finite holonomy: monopole constituents

KvBLL (1998) construct calorons with non-trivial holonomy



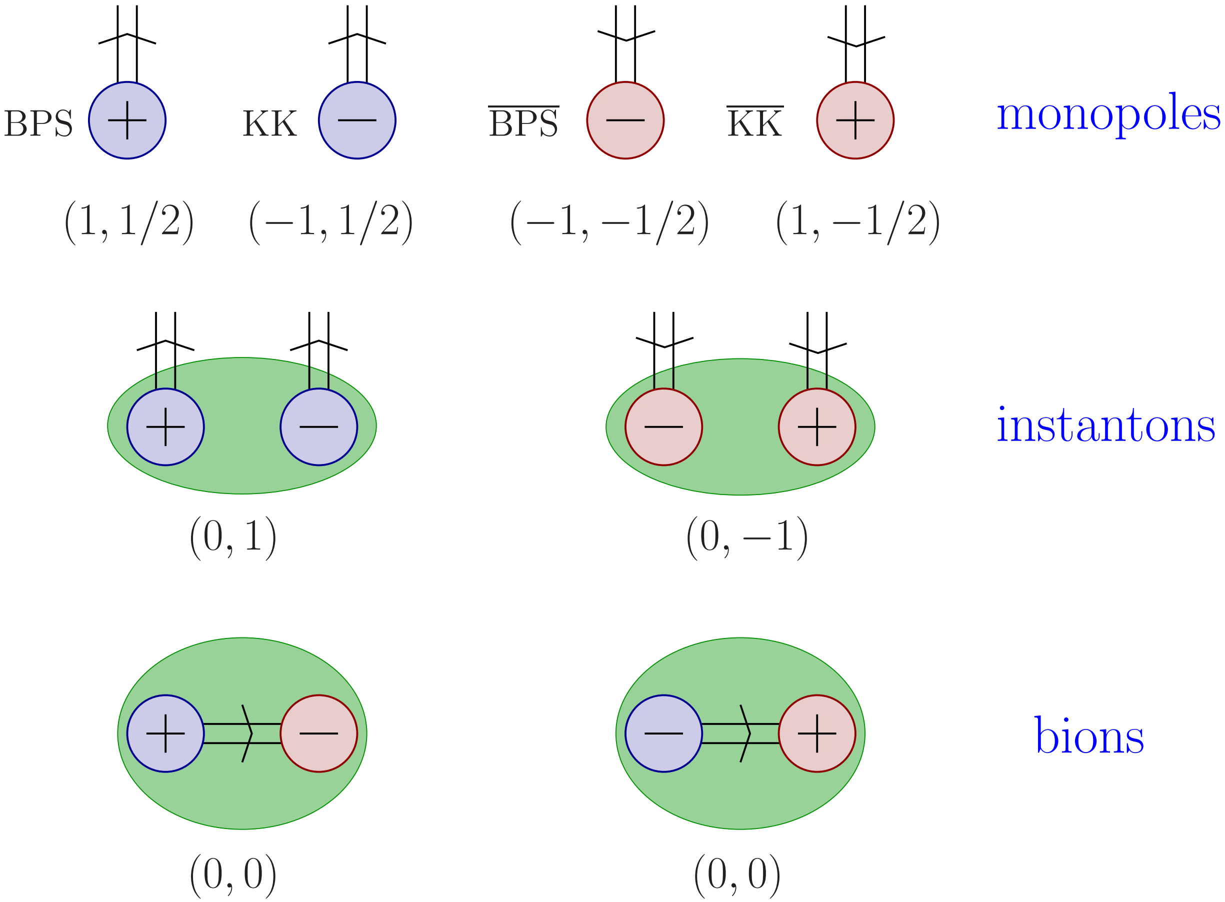
BPS and KK monopole constituents. Fractional topological charge, $1/2$ at center symmetric point.

$2 \times (3+1) = 8$ bosonic zero modes, 2×2 fermionic ZM.

$$\int d\phi_1 \int d^3x_1 \int d^2\zeta \, e^{-S_1} \int d\phi_2 \int d^3x_2 \int d^2\xi \, e^{-S_2}$$

Topological objects

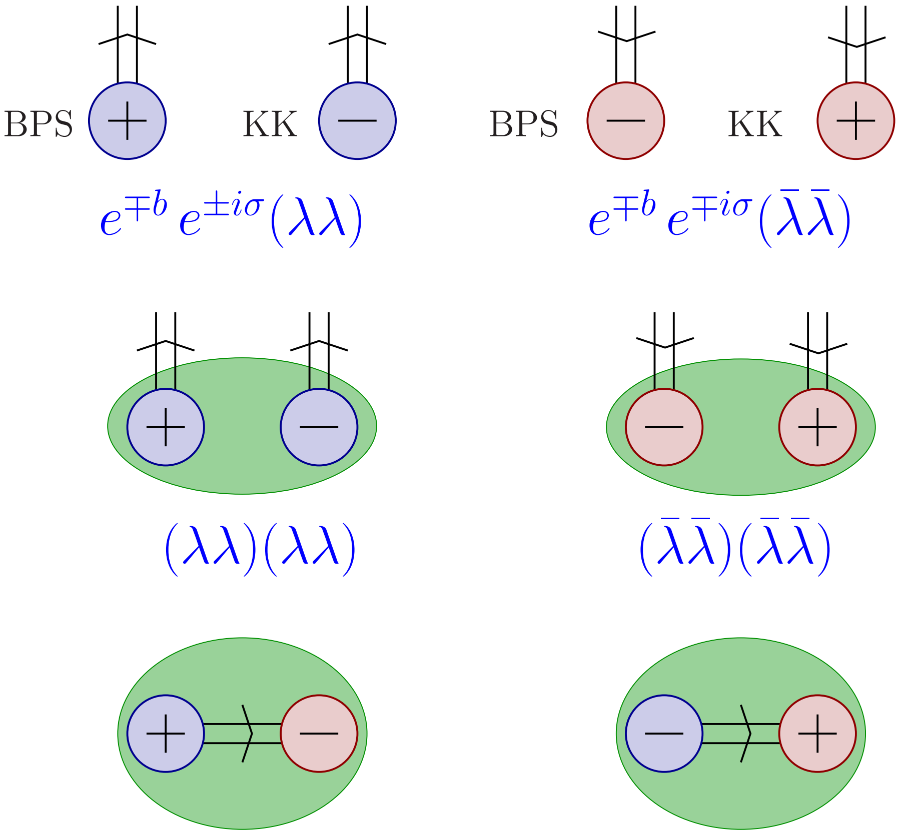
$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

Topological objects: Coupling to low energy fields

$$(Q_M, Q_{top}) = \left(\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F}\right)$$



monopoles: $e^{\mp b}\, e^{\pm i\sigma}(\lambda\lambda)$ $\quad e^{\mp b}\, e^{\mp i\sigma}(\bar{\lambda}\bar{\lambda})$

instantons: $(\lambda\lambda)(\lambda\lambda)$ $\quad (\bar{\lambda}\bar{\lambda})(\bar{\lambda}\bar{\lambda})$

bions: e^{-2b} $\quad e^{2b}$

Kraan, van Baal (1998); Lee, Lu (1998); Unsal (2008)

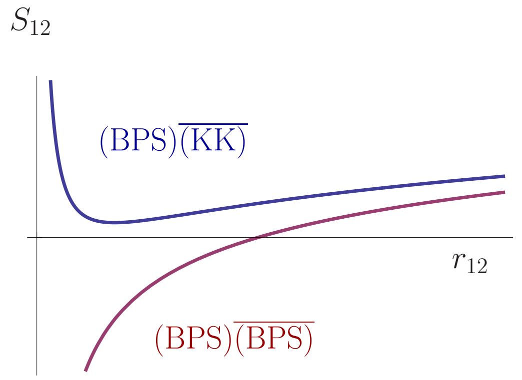
Effective potential

Instantons and monopoles: Exact solutions, but $V(b,\sigma) = 0$.

Bions: Approximate solutions

$$V_{BPS,\overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r \, e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r}(q_m^1 q_m^2 - q_b^1 q_b^2) + 4\log(r)$$



Saddle point integral after resurgent cancellations.

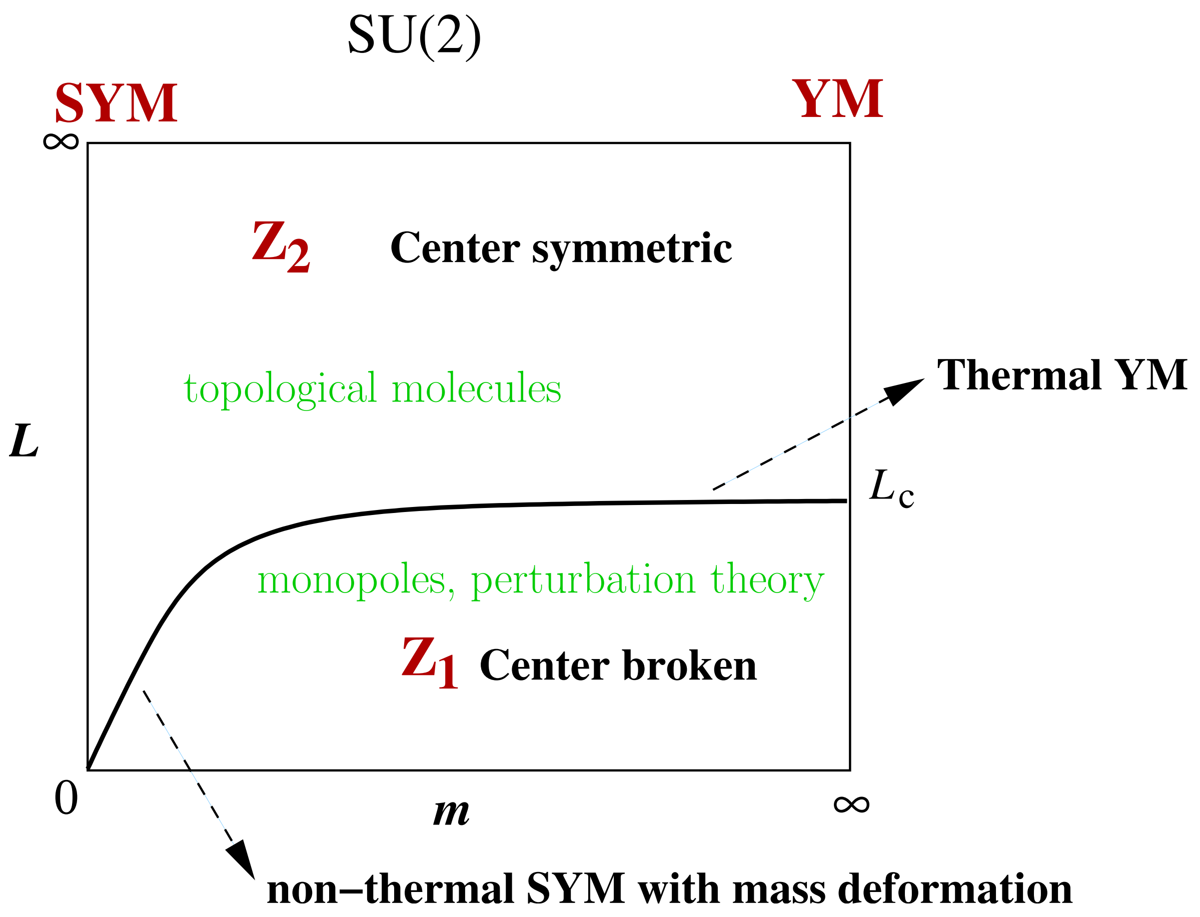
$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \left[\cosh\left(2\left(b - b_0\right)\right) - \cos(2\sigma)\right]$$

Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\rightarrow$ confinement)

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L-m plane



Poppitz, T.S., Unsal, [1205.0290[hep-th]].

What about chiral symmetry breaking?

Original setup: One adjoint fermion, chiral symmetry is discrete.

$$\langle\bar{\lambda}\lambda\rangle \neq 0 \qquad Z_{2N_c} \rightarrow Z_2$$

Light fundamental fermions: Need strong coupling.

$$\mathcal{L} \sim G \det_{N_f}(\bar{\psi}_L \psi_R) + \text{h.c.}$$

Heavy fundamental fermions: Study explicit breaking of Z_N center symmetry.

Role of Boundary Conditions

Consider flavor twisted boundary conditions

$$\psi(\tau+\beta) = \Omega_F \psi(\tau) \qquad \Omega_F = \mathrm{diag}(1, e^{2\pi i/N_f}, \ldots, e^{2\pi i(N_f-1)/N_f})$$

Flavor holonomy Ω_F has several interesting properties:

1. $N_f = N_c$: Respects Z_{N_c} center symmetry.

2. Large L: Breaks flavor symmetry, but in a controlled fashion.

3. Small L: New semi-classical picture of chiral symmetry breaking: Distributed zero modes and color-flavor transmutation.

Large L expectations

Flavor holonomy corresponds imaginary flavor (isospin) chemical potential $\tilde{\mu}_F \sim i/L$.

Can be studied using chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\nabla_\mu U \nabla^\mu U^\dagger\right] - B\mathrm{Tr}\left[MU + h.c\right]$$

with $\nabla_0 U = \partial_0 U + i[\tilde{\mu}_F T_F, U]$.

Consider $N_f = 2$ (isospin chemical potential)

$$m_{\pi^0}^2 = m_\pi^2 \qquad m_{\pi^\pm}^2 = m_\pi^2 + \tilde{\mu}_I^2$$

$N_f - 1$ exact Goldstone modes (m=0), others acquire gaps.

Small L theory: Perturbation theory

Consider center symmetric gauge holonomy (add double trace deformation). For $LN_c \lesssim \Lambda^{-1}$ theory abelianizes

$$SU(N_c) \rightarrow [U(1)]^{N_c-1}$$

Gapless (Cartan) gluons described by dual photon $\vec{\sigma}$

$$S = \frac{g^2}{8\pi^2 L} \int d^3x \, (\partial_\mu \vec{\sigma})^2$$

with $F^i_{\mu\nu} = \frac{g^2}{2\pi L} \epsilon_{\mu\nu\alpha} \partial^\alpha \sigma^i$.

Remain gapless to all orders in perturbation theory due to emergent shift symmetry $\vec{\sigma} \rightarrow \vec{\sigma} + \vec{\epsilon}$.

Small L theory: Semiclassical objects

Center symmetric background, no fermions: Instanton fractionalize into N_c constituents

$$\mathcal{M}_i \sim e^{-S_0} e^{i\vec{\alpha}_i \cdot \vec{\sigma}} \qquad S_0 = \frac{8\pi^2}{g^2 N_c} \qquad \vec{\alpha}_i \;\; SU(N_c) \text{ root vectors}$$

In the ground state these objects proliferate: The monopole-anti-monopole gas.

$$V(\vec{\sigma}) \sim m_W^3 e^{-S_0} \sum_i \cos(\vec{\alpha}_i \cdot \vec{\sigma})$$

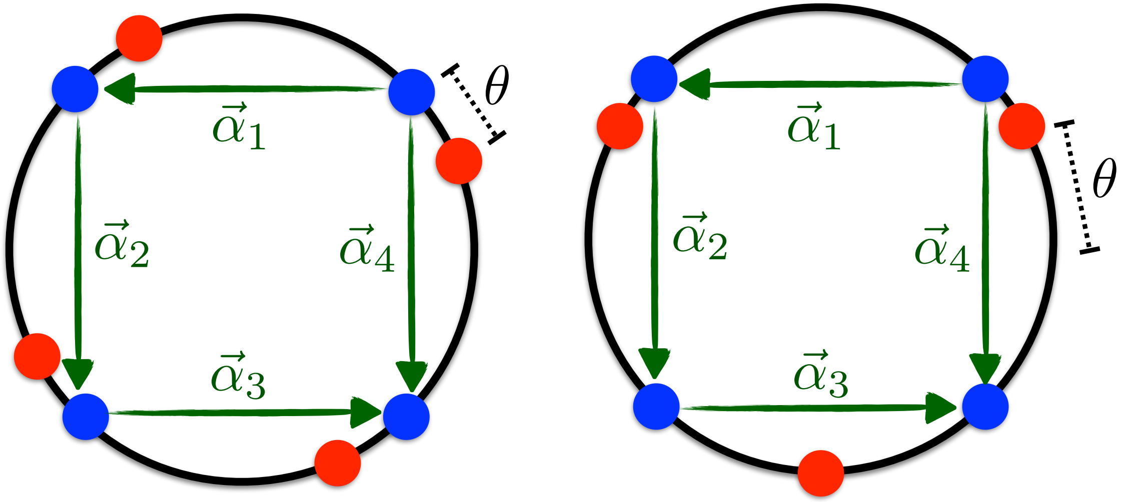
Mass gap for the dual photon, continuous shift symmetry broken.

Massless fermions: Take into account fermion zero modes.

Small L theory: Fermion zero modes

Many eigenvalue circles: Polyakov line Flavor holonomy

Instanton-monopoles θ flavor singlet twist



$N_c = N_f = 4$ $N_c = 4\ N_f = 3$

Zero modes localize on monopoles jumping over flavor eigenvalues

Bruckmann, Nogradi, van Baal (2003); Moore et al. (2014)

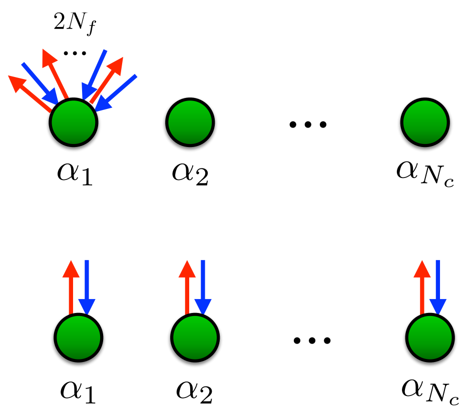
Two basic scenarios ($N_c = N_f$)

No flavor twist: Standard 't Hooft vertex carried by one monopole

$$\mathcal{M}_1 \sim e^{-S_0}\, e^{i\vec{\alpha}_1\cdot\vec{\sigma}}\, \det_F(\bar{\psi}_L^f \psi_R^g) \qquad \mathcal{M}_{i>1} \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}$$

Center symmetric flavor holonomy: Single flavor 't Hooft vertex carried by each monopole

$$\mathcal{M}_i \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\, (\bar{\psi}_L^i \psi_R^i)$$



Spontaneous symmetry breaking

Unbroken symmetries of flavor twisted theory

$$[U(1)_J]^{N_c-1} \times [U(1)_V]^{N_f-1} \times [U(1)_A]^{N_f-1} \times U(1)_Q$$

Shift symmetry　　　　Exact flavor symmetry

Symmetries of monopole vertex

$$\mathcal{M}_i \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\, (\bar{\psi}^i_L \psi^i_R)$$

Preserves vectorial symmetry $[U(1)_V]^{N_f-1} \times U(1)_Q$. Breaks axial symmetry

$$[U(1)_A]^{N_f-1}: \qquad (\bar{\psi}^f_L \psi^f_R) \to e^{i\epsilon_i} (\bar{\psi}^i_L \psi^i_R)$$

Spontaneous symmetry breaking, continued

Monopole vertex is invariant provided $[U(1)_A]^{N_f-1}$ is combined with $[U(1)_J]^{N_c-1}$ shift symmetry

$$[\tilde{U}(1)_A]^{N_f-1}: \quad \begin{cases} (\bar{\psi}_L^f \psi_R^f) & \rightarrow \quad e^{i\epsilon_i}(\bar{\psi}_L^i \psi_R^i) \\ e^{i\vec{\alpha}_i\cdot\vec{\sigma}} & \rightarrow \quad e^{-i\epsilon_i} e^{i\vec{\alpha}_i\cdot\vec{\sigma}} \end{cases}$$

Ground state $\langle e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\rangle \rightarrow 1$. Breaks

$$[U(1)_V]^{N_f-1} \times [\tilde{U}(1)_A]^{N_f-1} \rightarrow [U(1)_V]^{N_f-1}$$

For $m=0$ the ground state is degenerate. Massless Goldstone boson

$$S_\sigma = L \int d^3x \left\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] - B\mathrm{Tr}\left[M\Sigma + h.c.\right] \right\}$$

Microscopically $\Sigma = e^{i\Pi/f_\pi}$ with $\Pi = \pi^a T^a$ and $\pi^a = \frac{g}{2\pi L}\sigma^a$

Color-flavor transmutation

Discrete symmetries and anomaly matching

Discrete symmetries

$Z_{2N_f} \in U(1)_A$ $\qquad\qquad$ $Z_d \in Z_{N_c} \times Z_{N_f}^{perm}$

't Hooft vertex $\qquad\qquad$ color-flavor center

Mixed $[Z_d]^2 \times Z_{2N_f}$ anomaly can be studied along the lines of Gaiotto et al. Introdude 1 and 2-form gauge fields, obtain anomaly

$$\mathcal{A} = -\frac{N}{2\pi}\int B_c^{(1)} \wedge B_f^{(2)} \in \frac{2\pi}{N} Z$$

Matching requires Z_d or Z_{2N_f} to be broken (or more exotic phases)

Here: Z_d preserved, Z_{2N_f} broken, and $U(1)_L^{N-1} \times U(1)_R^{N-1}$ breaking comes along for the ride.

Tanizaki et al. 1711.10487, Cherman et al. 1711.10567.

Chiral Lagrangian

Chiral lagrangian has calculable coefficients

$$S_\sigma = L \int d^3x \left\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] - B\,\mathrm{Tr}\left[M\Sigma + h.c.\right] \right\}$$

$$f_\pi^2 = \left(\frac{g}{\sqrt{6}\pi L}\right)^2 = \frac{N_c \lambda m_W^2}{24\pi^2}$$

$$B = -\frac{1}{2}\langle \bar{\psi}\psi \rangle \sim m_W^{-3} e^{-\frac{8\pi^2}{\lambda}}$$

Also note: VEV of monopole operator can be viewed as effective constituent quark mass

$$m_Q \sim m_W e^{-\frac{8\pi^2}{\lambda}}$$

Cherman, T.S., Unsal [1604.06108 [hep-th]].

Conclusions and Outlook

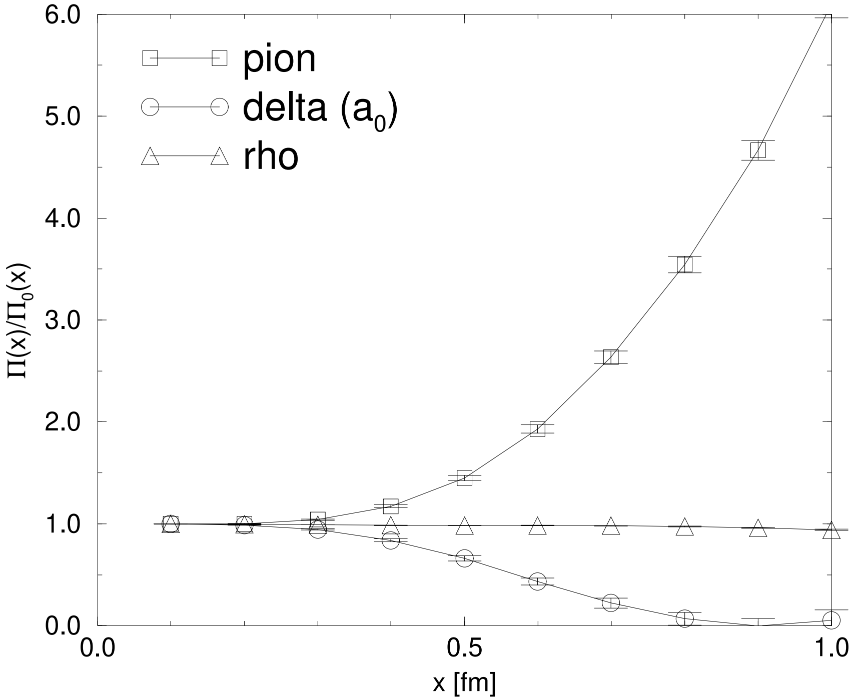
Calculable mechanism for chiral symmetry breaking and confinement in compactified versions of QCD.

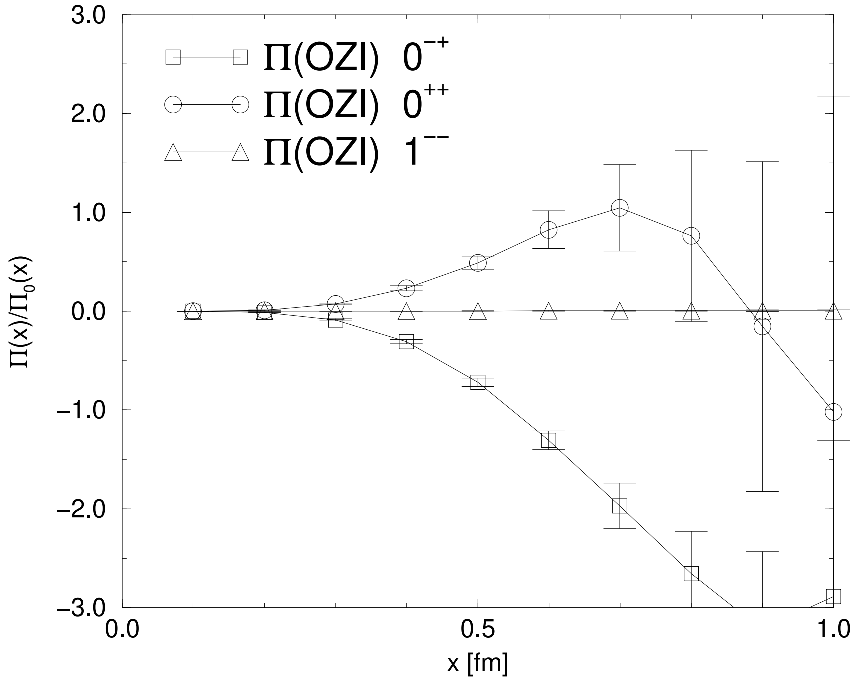
Results consistent with continuity between large L, m (full QCD) and small L, m theory.

Mechanism based on selfdual monopoles and color flavor transmutation.

Difficulty: From weak-coupling (quasi-abelian) confinement to strong-coupling (non-abelian) confinement. Other weak coupling limits? (selfdual vortices?)

Meson Correlation Functions





$m_\pi = 140^* \, \mathrm{MeV} \quad (f_\pi = 71 \, \mathrm{MeV})$

$m_\rho = 795 \, \mathrm{MeV}$

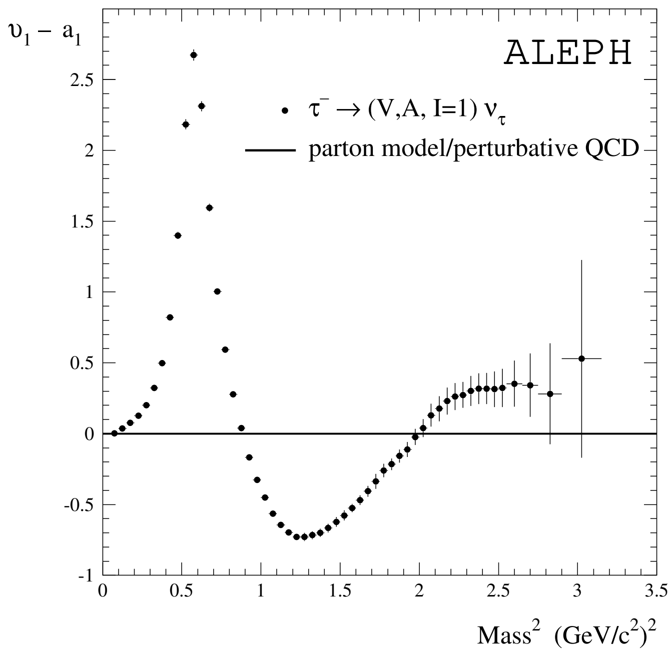
$m_{a_0} \simeq 1 \, \mathrm{GeV}$

$m_\rho \simeq m_\omega$

$m_\sigma \simeq 580 \, \mathrm{MeV}$

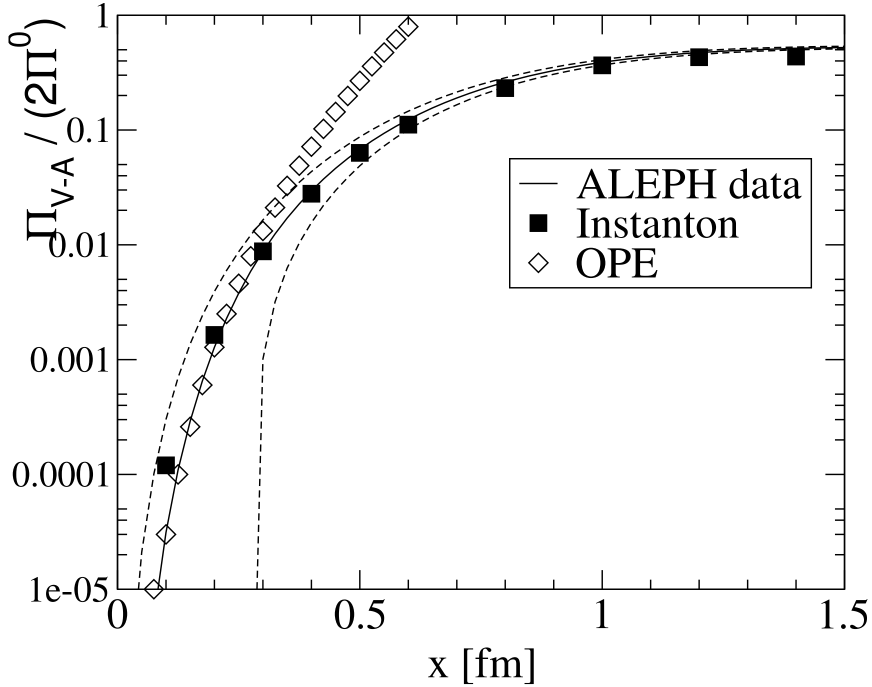
$m_{\eta'} \simeq 1 \, \mathrm{GeV}$

V−A Correlation Functions



Aleph spectral function

$\tau \to (V, A, I\!=\!1)\nu_\tau$



coordinate space correlator

OPE, instanton liquid, data