Continuity of the Deconfinement Transition in (Super) Yang Mills Theory

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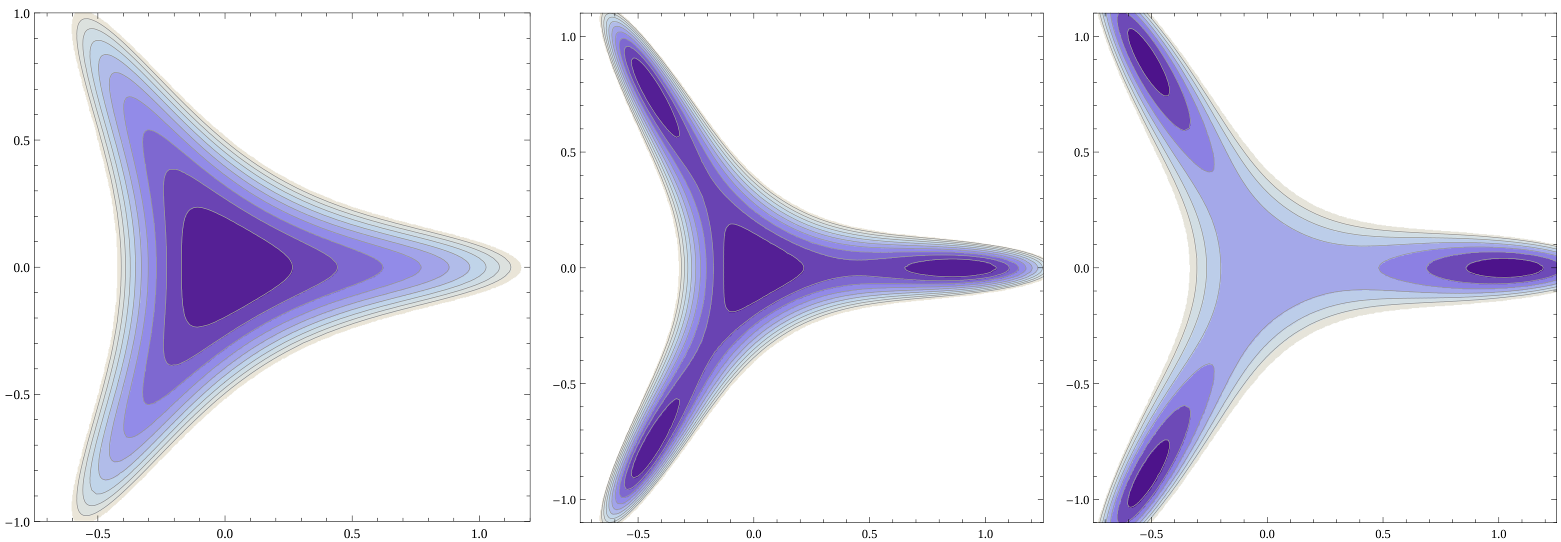
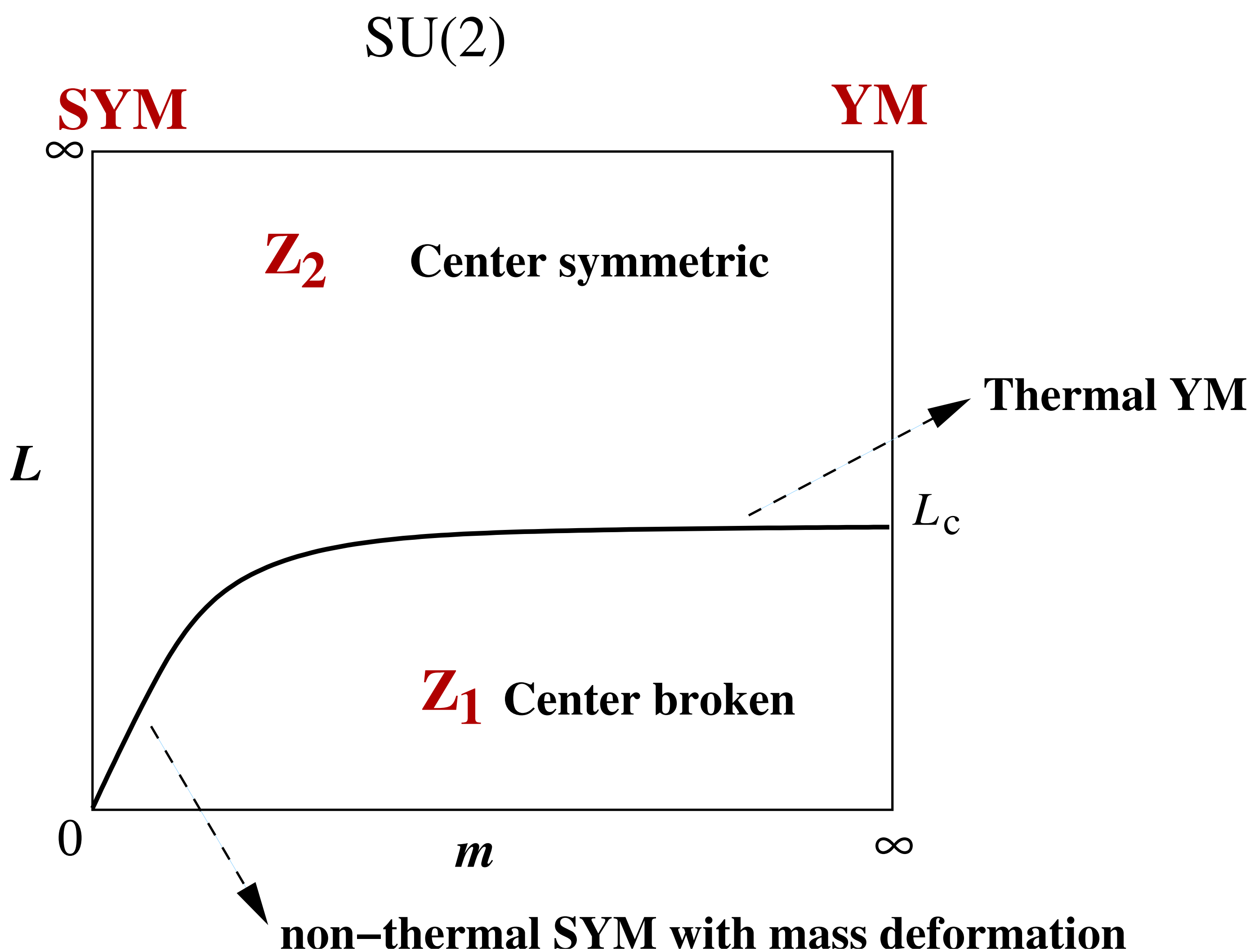


with Mithat Ünsal and Erich Poppitz

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L-m plane



Ingredients

- $R^3 \times S_1$ circle-compactified gauge theory.
- Small S_1: Effective 3d theory involving holonomy and (dual) photon.
- Double expansion: Perturbative and non-perturbative effects (monopoles, topological molecules).
- Topological molecules: supersymmetry versus BZJ.
- Competition: Center stabilizing molecules, center breaking perturbative (and monopole) effects.

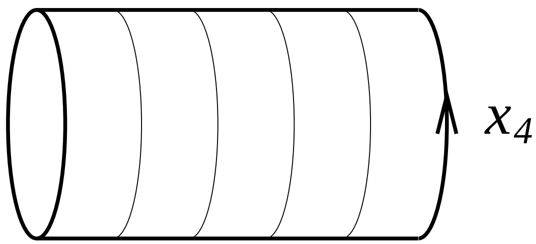
Gauge theory on $R^3 \times S_1$

SU(2) gauge theory, $n_f = 1$ adjoint Weyl fermion

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{i}{g^2}\lambda^a \sigma \cdot D^{ab}\lambda^b + \frac{m}{g^2}\lambda^a\lambda^a$$

$$A^a_\mu(0) = A^a_\mu(L)$$

$$\lambda^a(0) = \lambda^a(L)$$



Vacua labeled by Polyakov line

$$\Omega = \exp\left[i\int A_4 dx_4\right]$$

Center symmetry $\qquad \Omega \to z\Omega \qquad z \in Z_2$

Small S_1: Effective Theory

Consider small S_1 and $\Omega \neq 1$: A_4 is a Higgs field, theory abelianizes.
Bosonic sector of effective 3d theory

$$\mathcal{L} = \frac{g^2}{32\pi^2 L}\left[(\partial_i b)^2 + (\partial_i \sigma)^2\right] + V(\sigma, b)$$

$$\Omega = \begin{pmatrix} e^{i\Delta\theta/2} & 0 \\ 0 & e^{-i\Delta\theta/2} \end{pmatrix} \quad b = \frac{4\pi}{g^2}\Delta\theta \qquad \epsilon_{ijk}\partial_k \sigma = \frac{4\pi L}{g^2} F_{ij}$$

holonomy b dual photon σ

Note: $m = 0$ effective theory can be super-symmetrized

$$\Phi = b + i\sigma + \sqrt{2}\theta^\alpha \lambda^\alpha$$

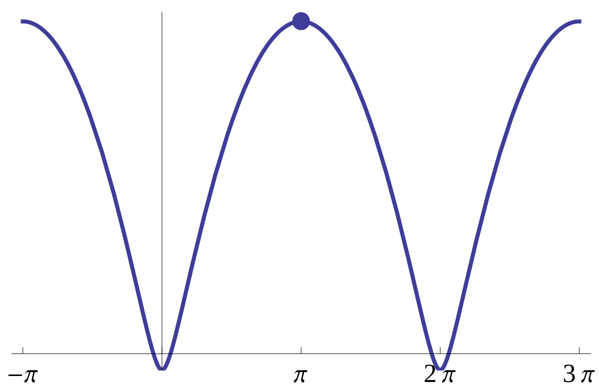
Perturbation Theory

Perturbative potential for holonomy (Gross, Pisarski, Yaffe, 1981)

$$V(\Omega) = -\frac{m^2}{2\pi^2 L^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \, |\mathrm{tr}\, \Omega^n|^2 = -\frac{m^2}{L^2} B_2 \left(\frac{\Delta\theta}{2\pi} \right)$$

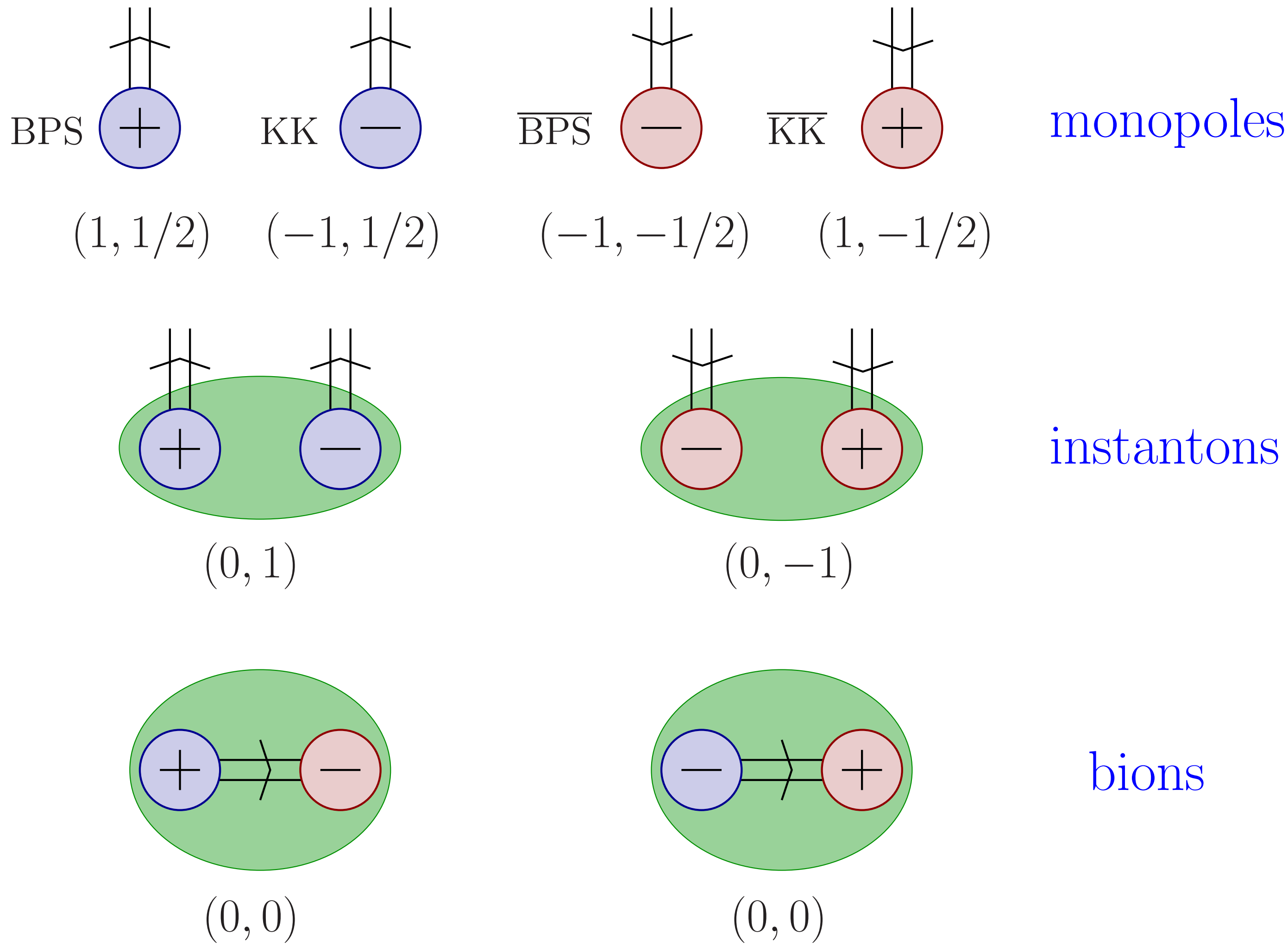
$m = 0$: Bosonic and fermionic terms cancel.

$m \neq 0$: Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ unstable.



Topological objects

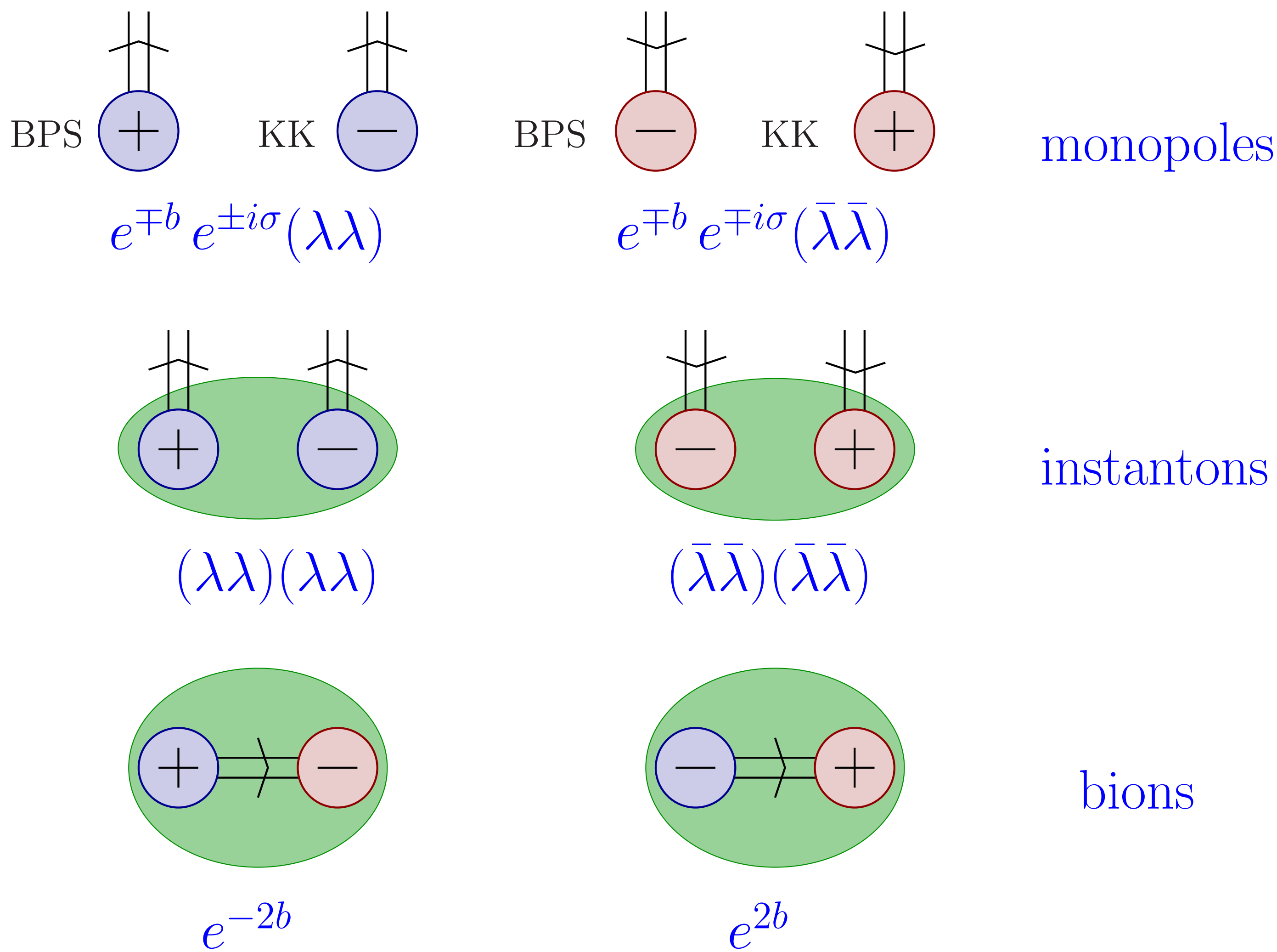
$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

Topological objects: Coupling to low energy fields

$$(Q_M, Q_{top}) = \left(\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F}\right)$$



Kraan, van Baal (1998); Lee, Lu (1998); Unsal (2008)

Non-perturbative effects at $m = 0$ from supersymmetry

Monopoles contribute to superpotential: $(\lambda\lambda)e^{-b+i\sigma} \sim \int d^2\theta e^{-\Phi}$

$$\mathcal{W} = \frac{M_{PV}^3 L}{g^2}\left(e^{-b} + e^{-2S_0}e^{b}\right)$$

Scalar potential

$$V(b,\sigma) \sim \left|\frac{\partial \mathcal{W}}{\partial \Phi}\right|^2 \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6}\left[\cosh\left(\frac{8\pi}{g^2}(\Delta\theta - \pi)\right) - \cos(2\sigma)\right]$$

Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\rightarrow$ confinement)

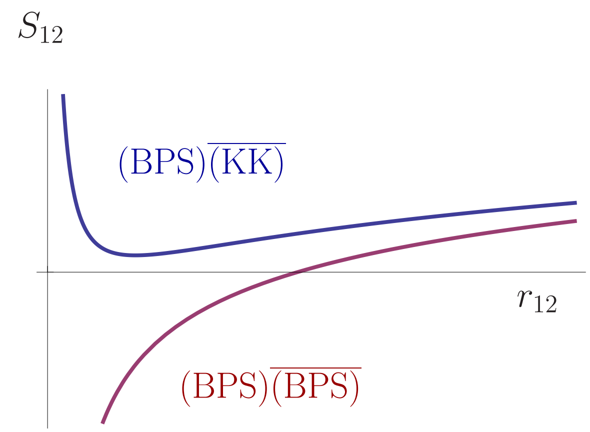
Davies, Hollowood, Khoze, Mattis (1999)

Non-perturbative effects at $m = 0$ from BZJ

Consider magnetically neutral topological molecules. Integrate over near zero-mode:

$$V_{BPS,\overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r \, e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r}(q_m^1 q_m^2 - q_b^1 q_b^2) + 4\log(r)$$



Saddle point integral after analytic continuation $g^2 \to -g^2$ (BZJ)

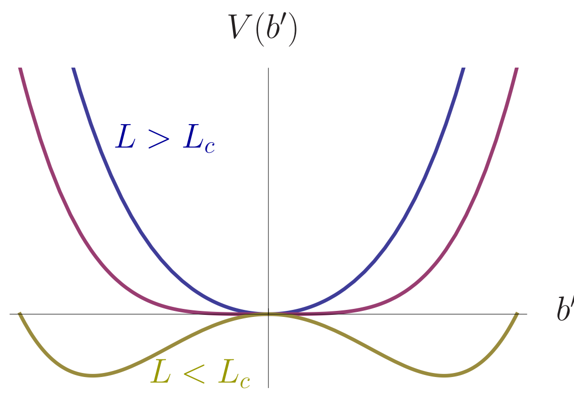
$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \cosh\left(\frac{8\pi}{g^2}(\Delta\theta - \pi)\right)$$

Same for magnetically charged molecules: $V \sim \cos(2\sigma)$.

Effective potential for $m \neq 0$

Effective potential: molecules, monopoles, perturbation theory

$$
\begin{aligned}
\tilde{V} &= \cosh 2b' - \cos 2\sigma \\
&+ \frac{\tilde{m}}{2\tilde{L}^2} \cos\sigma \left(\cosh b' - \frac{b' \sinh b'}{3 \log \tilde{L}^{-1}} \right) \\
&- \frac{1}{1728} \left(\frac{\tilde{m}}{\tilde{L}^2} \right)^2 \frac{1}{\log^3 \tilde{L}^{-1}} \, (b')^2 \, .
\end{aligned}
$$



$\tilde{L} = L\Lambda$, $\tilde{m} = m/\Lambda$, $b' = \frac{4\pi}{g^2}(\Delta\theta - \pi)$

Critical S_1 size $\qquad \tilde{L}_c^2 = \frac{\tilde{m}}{8}\left[1 + \mathcal{O}\left(\frac{1}{\log \tilde{L}}, \frac{\tilde{m}}{\tilde{L}^2}\right)\right]$,

Corresponds to $T_c = \sqrt{\frac{8}{\tilde{m}}}\Lambda_{QCD}$

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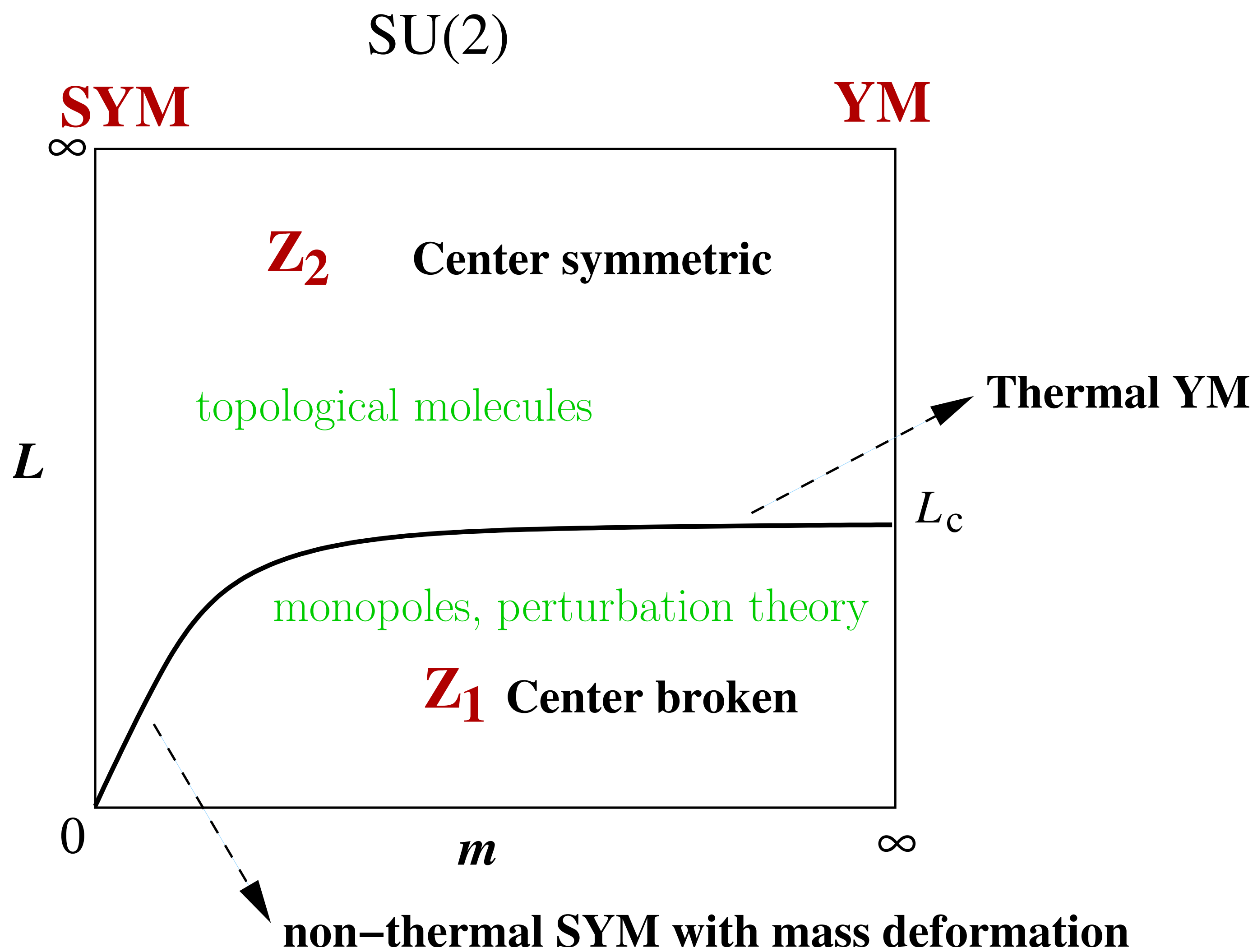
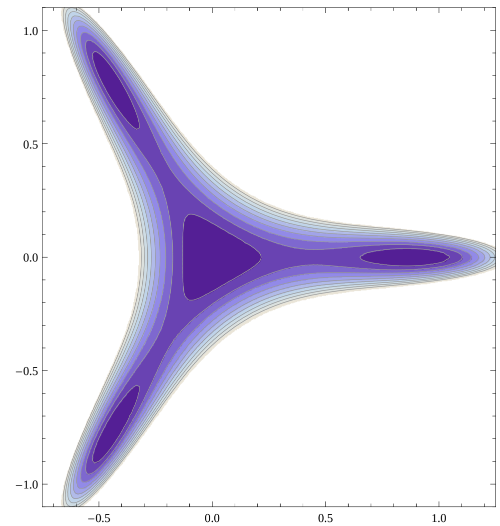
Phase diagram in L-m plane



Outlook: higher rank gauge groups, θ dependence, pure gauge



- $SU(N \geq 3)$: First order transition

$$Z_N \to \emptyset$$

- G_2: First order transition without change of symmetry.

- $\theta \neq 0$: Get $V \sim \cos\left(\frac{2\pi k+\theta}{N}\right)$, $k = 1, \ldots, N-1$.

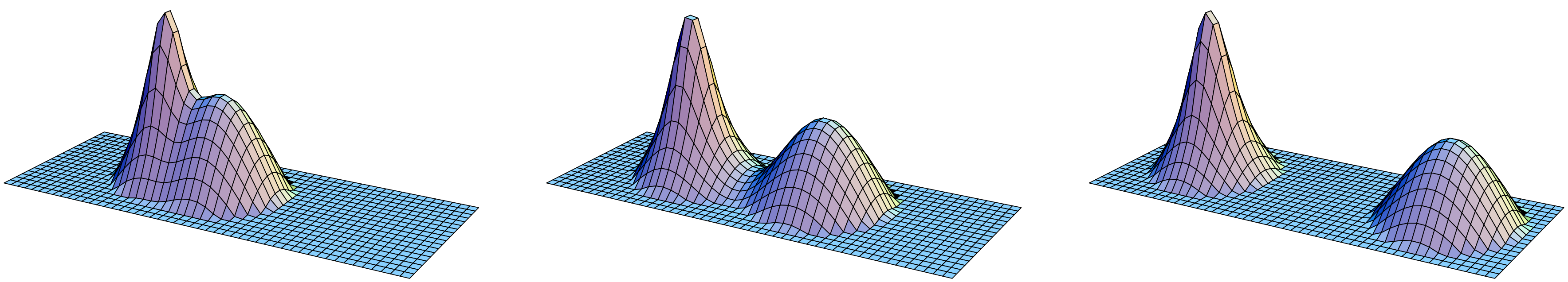
- Pure gauge theory: Find center stabilizing molecules from BZJ.

  But: Semi-classical approximation not reliable.

Extra

Calorons at finite holonomy: monopole constituents

Caloron $=$ instanton on $R^3 \times S_1$. Exists for any value of holonomy Ω. Always $Q_{top} = \pm 1$. Solution has $3+1+1+3 = 8$ zero modes (position in $R^3 \times S_1$, size, $SU(2)$ orientation).



Monopole constituents: Fractional topological charge, $1/2$ at center symmetric point. $2 \times (3+1) = 8$ zero modes (position in R^3, $U(1)$ angle).