A Non-Fermi Liquid EFT for QCD at High Density

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Motivation

There is a successful effective theory of fermionic many body systems

Landau Fermi-Liquid Theory

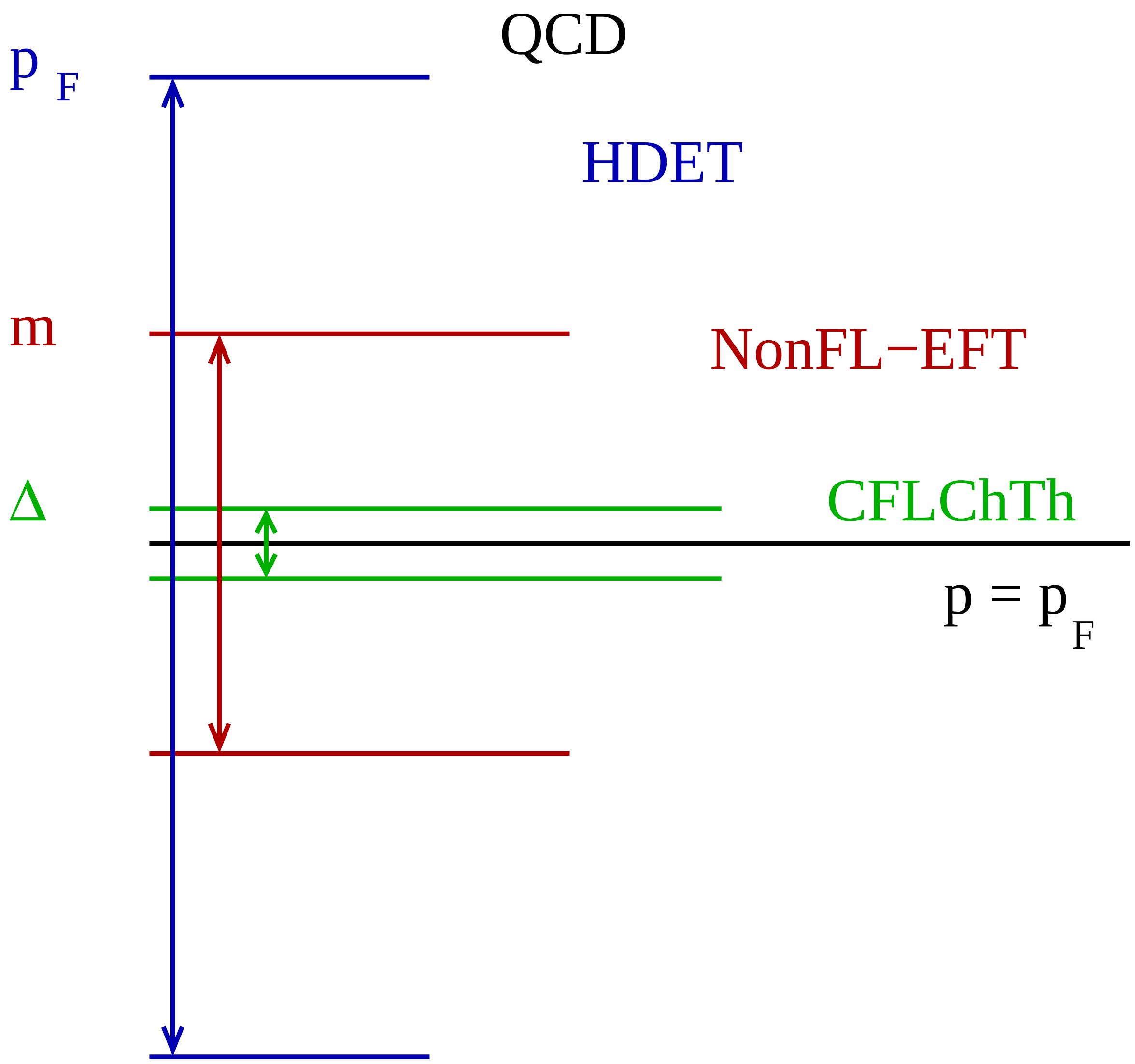
FLT theory: Quasi-particles near the Fermi surface. Interactions characterized by FL parameters. Does not rely on weak coupling.

Predicts collective modes, thermodynamics, transport, ...

Gauge Theories: Unscreened long range forces

Does a quasi-particle EFT exist?

Effective Field Theories

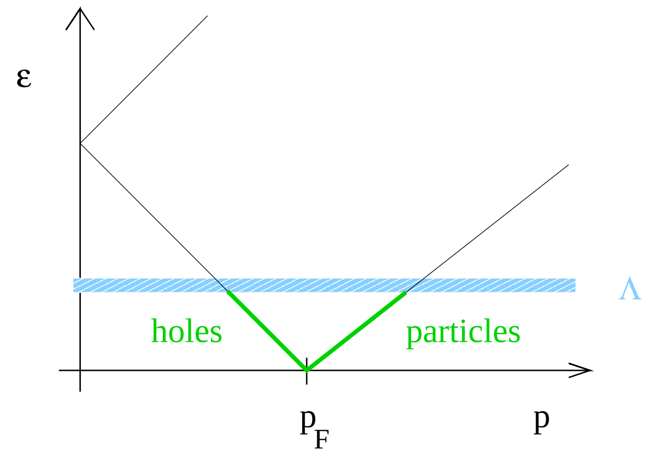


High Density Effective Theory

QCD lagrangian

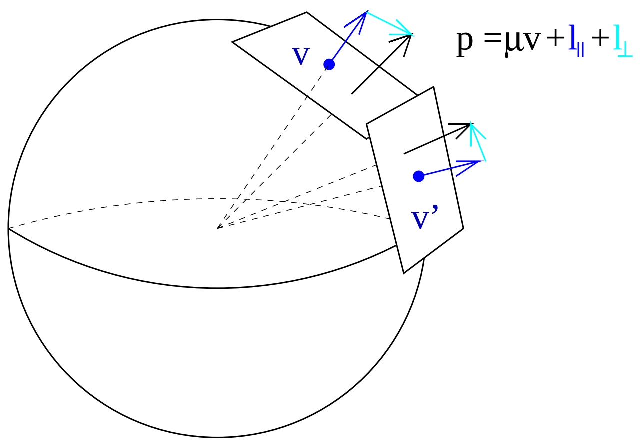
$$\mathcal{L} = \bar{\psi}\left(i\not{D} + \mu\gamma_0 - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

Quasi-particles (holes)



$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

High Density Effective Theory, cont

Insert $\psi_{v\pm}$ in QCD lagrangian

$$\mathcal{L} = \sum_v \Big\{ \psi^\dagger_{v+} (iv\cdot D)\,\psi_{v+} + \psi^\dagger_{v-}(2\mu + i\bar{v}\cdot D)\,\psi_{v-}$$

$$+ \psi^\dagger_{v+}(i\not{D}_\perp)\,\psi_{v-} + \psi^\dagger_{v-}(i\not{D}_\perp)\,\psi_{v+} \Big\}$$

Integrate out ψ_{v-} at tree level

$$\psi_{v-} = \frac{1}{2\mu + i\bar{v}\cdot D}(i\not{D}_\perp)\psi_{v+}$$

Effective lagrangian for ψ_{v+}

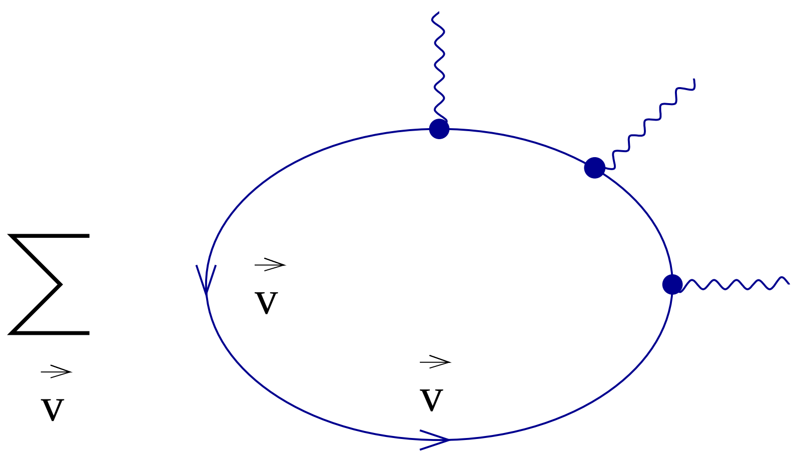
$$\mathcal{L} = \sum_v \psi^\dagger_v \left(iv\cdot D - \frac{D^2_\perp}{2\mu} \right) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots$$

Power Counting

Naive power counting

$$\mathcal{L} = \hat{\mathcal{L}}\left(\psi, \psi^\dagger, \frac{D_{||}}{\mu}, \frac{D_\perp}{\mu}, \frac{\bar{D}_{||}}{\mu}, \frac{m}{\mu}\right)$$

Problem: hard loops (large $N_{\vec{v}}$ graphs)



$$\frac{1}{2\pi}\sum_{\vec{v}}\int \frac{d^2 l_\perp}{(2\pi)^2} = \frac{\mu^2}{2\pi^2}\int \frac{d\Omega}{4\pi}.$$

Have to sum large $N_{\vec{v}}$ graphs

Effective Theory for $l < m$

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}$$

Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\frac{\pi}{2} m^2 \frac{k_0}{|\vec{k}|}},$$

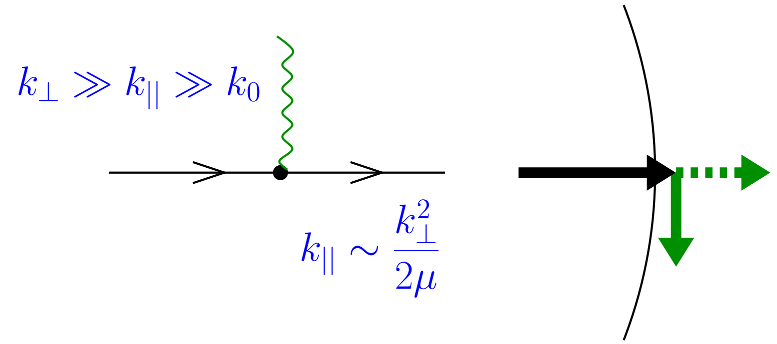
Scaling of gluon momenta

$$|\vec{k}| \sim k_0^{1/3} m^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Non-Fermi Liquid Effective Theory

Gluons very spacelike $|\vec{k}| \gg |k_0|$. Quark kinematics?

$$k_0 \simeq k_{||} + \frac{k_\perp^2}{2\mu}$$



Scaling relations

$$k_\perp \sim m^{2/3} k_0^{1/3}, \qquad k_{||} \sim m^{4/3} k_0^{2/3}/\mu$$

Propagators

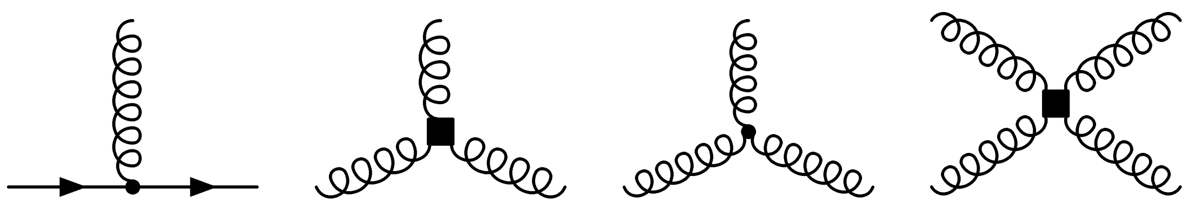
$$S_{\alpha\beta} = \frac{i\delta_{\alpha\beta}}{p_0 - p_{||} - \frac{p_\perp^2}{2\mu} + i\epsilon sgn(p_0)} \qquad D_{ij} = \frac{-i\delta_{ij}}{k_\perp^2 - i\frac{\pi}{2} m^2 \frac{k_0}{k_\perp}},$$

Non-Fermi Liquid Expansion

Scale momenta $(k_0, k_{||}, k_\perp) \to (sk_0, s^{2/3}k_{||}, s^{1/3}k_\perp)$

$$[\psi] = 5/6 \qquad [A_i] = 5/6 \qquad [S] = [D] = 0$$

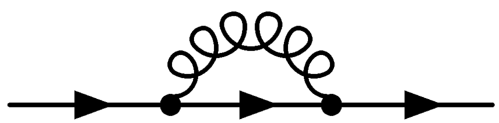
Scaling behavior of vertices



$s^{1/6}$ $\quad s^{1/2}$ $\quad s^{5/6}$ $\quad s$

Systematic expansion in $\epsilon^{1/3} \equiv (\omega/m)^{1/3}$

Loop Corrections: Quark Self Energy



$$= g^2 C_F \int \frac{dk_0}{2\pi} \int \frac{dk_\perp^2}{(2\pi)^2} \, \frac{k_\perp}{k_\perp^3 + i\eta k_0}$$

$$\times \int \frac{dk_{||}}{2\pi} \, \frac{\Theta(p_0 + k_0)}{k_{||} + p_{||} - \frac{(k_\perp + p_\perp)^2}{2\mu} + i\epsilon}$$

Transverse momentum integral logarithmic

$$\int \frac{dk_\perp^3}{k_\perp^3 + i\eta k_0} \sim \log\left(\frac{\Lambda}{k_0}\right)$$

Quark self energy

$$\Sigma(p) = \frac{g^2}{9\pi^2} p_0 \log\left(\frac{\Lambda}{|p_0|}\right)$$

Quark Self Energy, cont

Higher order corrections?

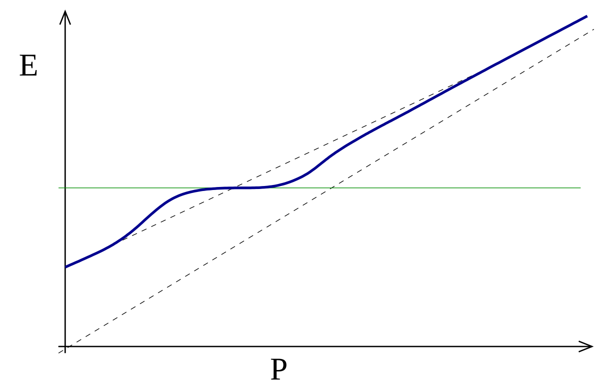
$$\Sigma(p) = \frac{g^2}{9\pi^2}\left(p_0 \log\left(\frac{2^{5/2}m}{\pi|p_0|}\right) + i\frac{\pi}{2}p_0\right) + O\left(\epsilon^{5/3}\right)$$

Scale determined by electric gluon exchange

No $p_0[\alpha_s \log(p_0)]^n$ terms

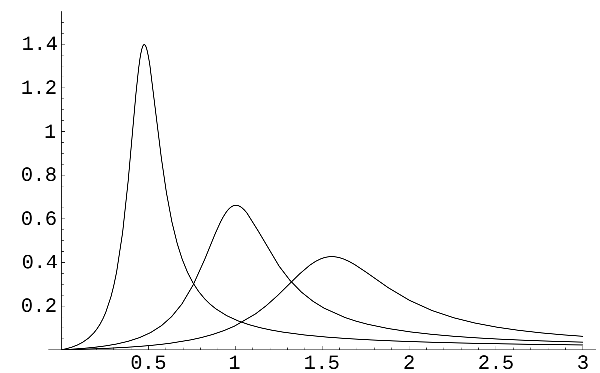
quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$

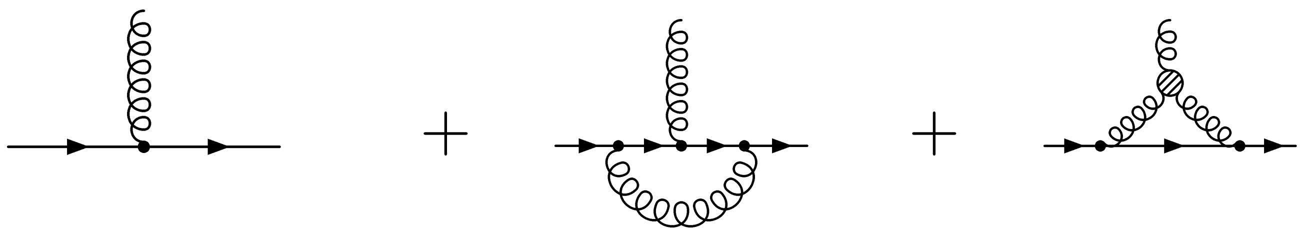


anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$

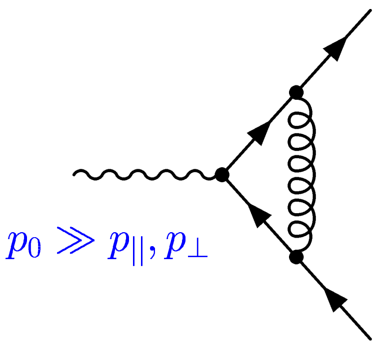


Vertex Corrections, Migdal's Theorem



$$\sim \quad gv(1+O(\epsilon^{1/3}))$$

Can this fail? Yes, if external momenta fail to satisfy $p_\perp \gg p_0$



$$= g^2 e C_F v_\mu \int \frac{dk_0}{2\pi} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 + \frac{\pi}{2} m^2 \frac{k_0}{k_\perp}}$$

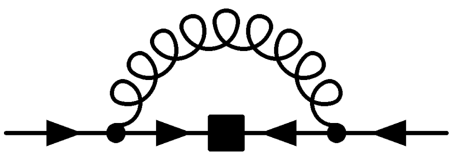
$$\times \int \frac{dk_{||}}{2\pi} \frac{1}{[p_{1,0} - k_{||} - \frac{k_\perp^2}{2\mu}][p_{2,0} - k_{||} - \frac{k_\perp^2}{2\mu}]}$$

Dominant terms in quark propagator cancel. Find

$$\Gamma_\mu(p_1, p_2) = \frac{eg^2}{9\pi^2} v_\mu \log\left(\frac{\Lambda}{p_0}\right).$$

Superconductivity

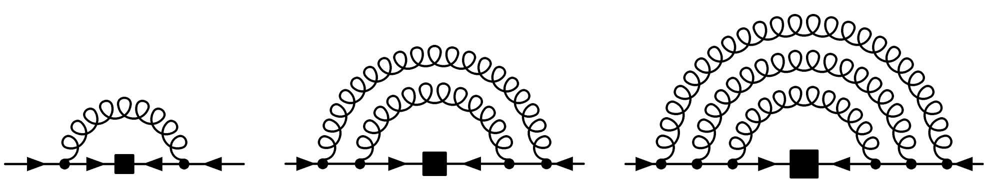
Same phenomenon occurs in anomalous self energy



$$= \frac{g^2}{18\pi^2}\int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0-q_0|}\right)\frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}$$

$\Lambda_{BCS} = 256\pi^4 g^{-5}\mu$ determined by electric exchanges

Have to sum all planar diagrams, non-planar suppressed by $\epsilon^{1/3}$



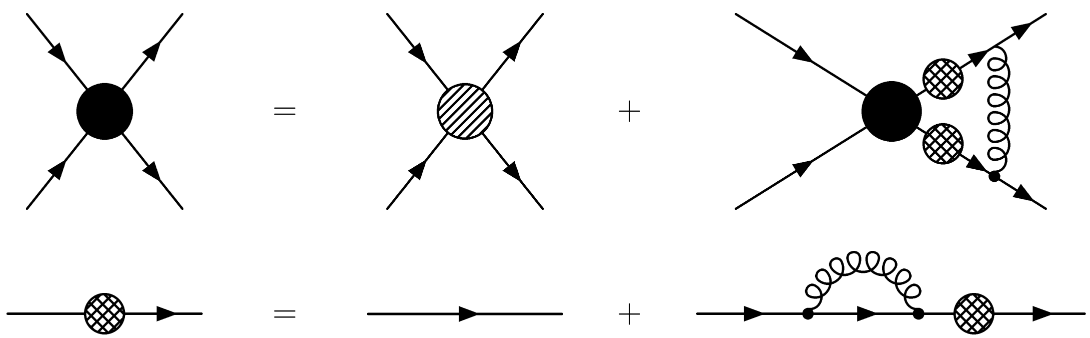
Solution at next-to-leading order (includes normal self energy)

$$\Delta_0 = 2\Lambda_{BCS}\exp\left(-\frac{\pi^2+4}{8}\right)\exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \qquad \Delta_0 \sim 50\,\mathrm{MeV}$$

Summary

Systematic low energy expansion in $(\omega/m)^{1/3}$ and $\log(\omega/m)$

Standard FL channels (BCS, ZS, ZS'): Ladder diagrams have to be summed, kernel has perturbative expansion



Coupling not weak at matching scale: Treat low energy constants v_F, $Z_{||}$, $Z_{\perp}, \ldots$ as low energy parameters, to be determined by lattice, RG methods, etc. (see Kai's poster)

Luttinger Theorem

Luttinger: Relationship between density and Fermi momentum unchanged in interacting system.

$$\frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

Main assumption:

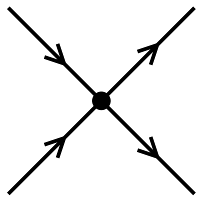
$$i \int_{-\infty}^{0} \frac{d\omega}{2\pi} \frac{\partial}{\partial \omega} \log\left(\frac{S(k,\omega)}{S_{ret}(k,\omega)}\right) = -\left[\varphi(0,k) - \varphi(-\infty,k)\right]$$

Gauge theory: $\Sigma(\omega) = c_1 \omega \log(\omega) + c_2 \omega^{4/3} + \ldots$

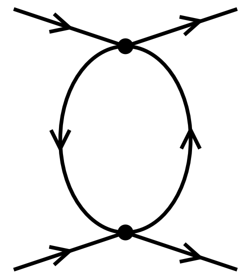
Kohn-Luttinger satisfied

Kohn-Luttinger Effect

Short range forces: Induced interaction gives attraction



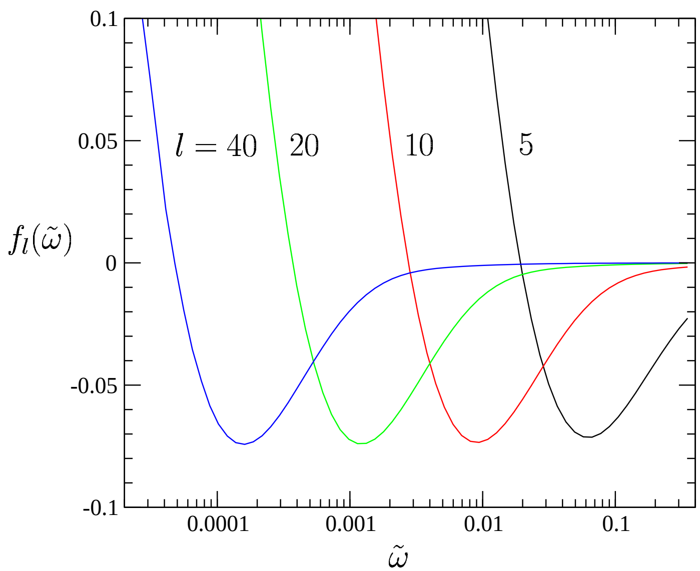
$V_l \sim e^{-l}$



$$\delta V_l \sim \frac{V(\pi)^2}{l^4}$$

Superfluidity (in large l) even if all V_l repulsive

Gauge theory: Attraction in large partial waves



$$\Delta = \frac{g^2 C_R}{12\pi^2} \int \frac{d\omega \Delta(\omega)}{\sqrt{\omega^2 + \Delta^2}} \, f_l(\omega)$$

No superfluidity in repulsive channels