The CFL Phase and m(strange): An Effective Field Theory Approach

Thomas Schaefer

North Carolina State

$\mu \to \infty$: CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, (\delta_i^a \delta_j^b - \delta_j^a \delta_i^b)$$

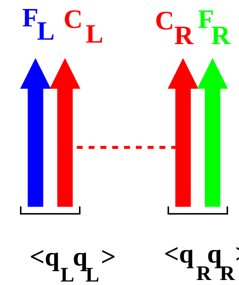
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

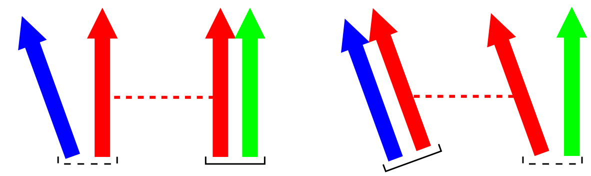
$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

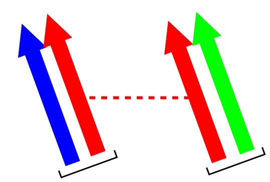
All quarks and gluons acquire a gap





Rotate left flavor

Compensate by rotating color



... have to rotate right flavor also !

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

The Role of the Strange Quark Mass

Include strange quark mass in gap equations

main parameter $x = \frac{m_s^2}{p_F \Delta}$

$x \sim 1$: transition CFL $\rightarrow$ 2SC

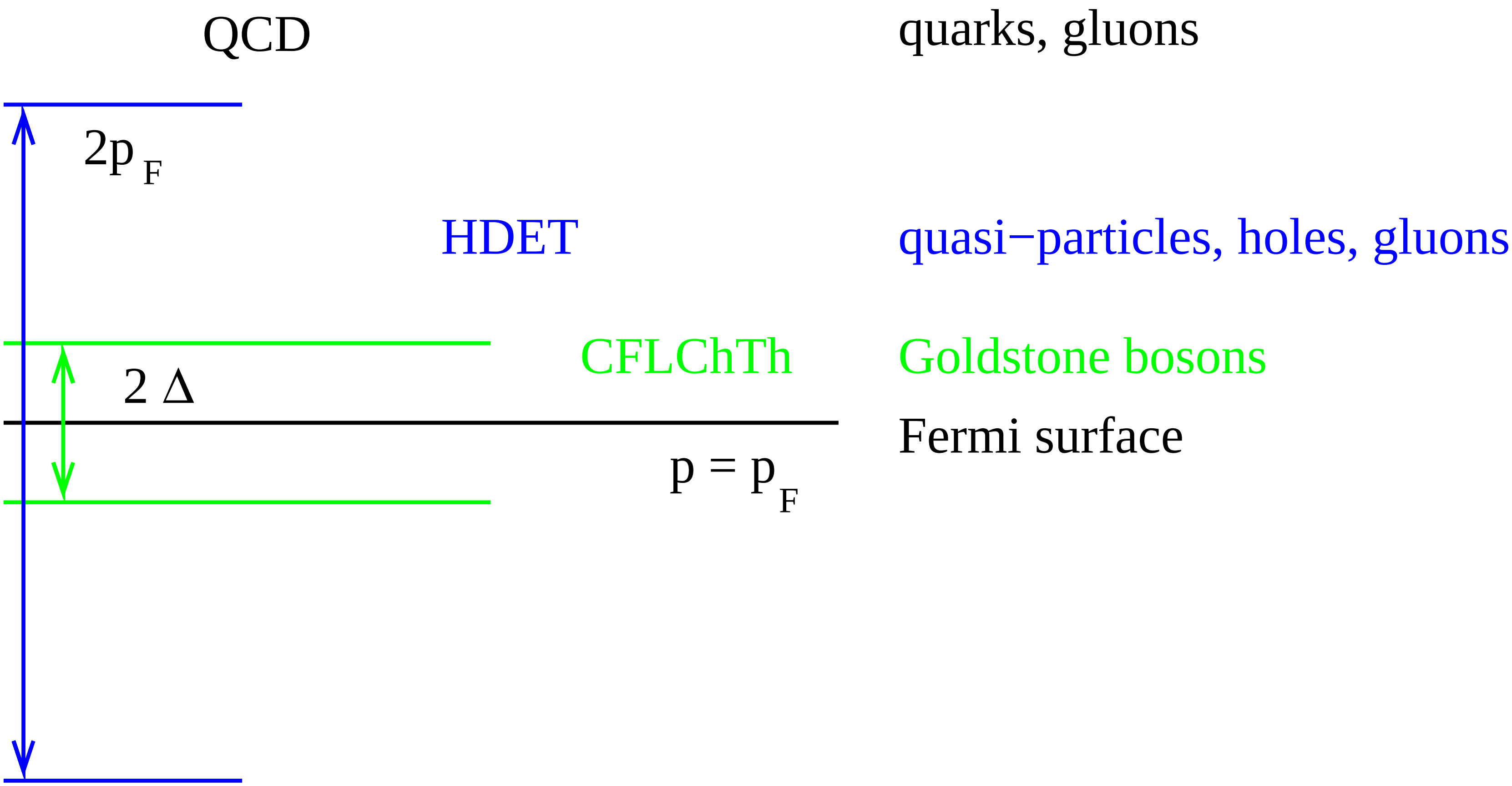
But: Problems turns out to be much more difficult (even if the coupling is weak!)

additional scales

electric neutrality, gauge invariance

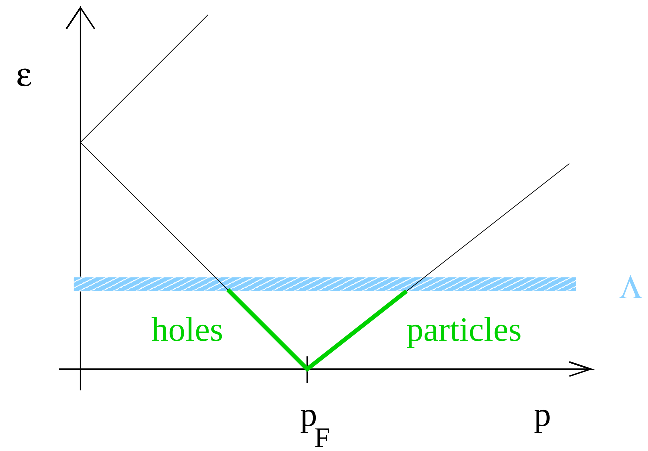
many gap parameters, how to find the right ansatz

Effective Field Theories



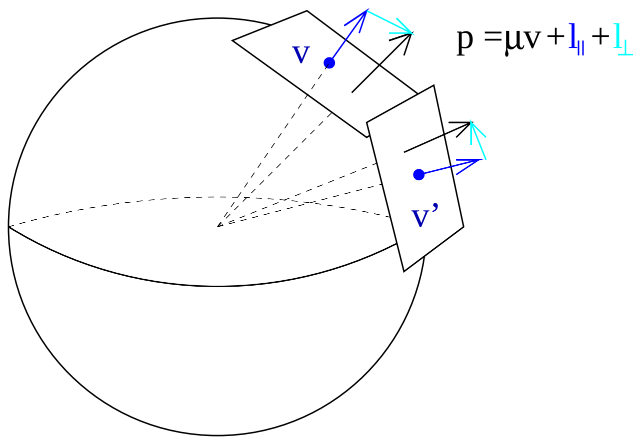
High Density Effective Theory

Quasi-particles (holes)



$$E_{\pm} = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches



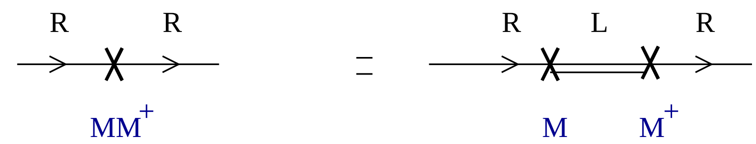
$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

Effective lagrangian for ψ_{v+}

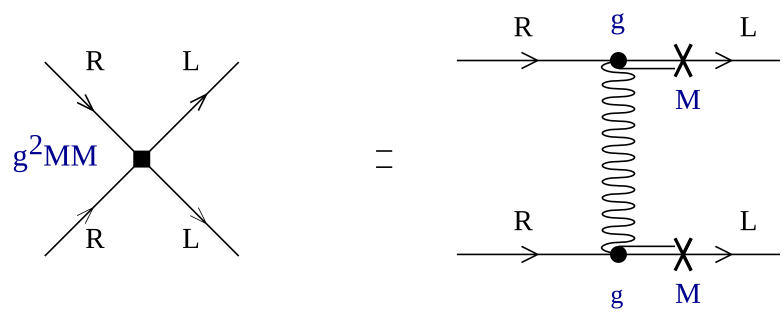
$$\mathcal{L} = \sum_v \psi_v^\dagger (iv\cdot D)\psi_v - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + O(1/\mu)$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$



$$+ \frac{C}{\mu^2}(\psi_R^\dagger M\lambda^a\psi_L)(\psi_R^\dagger M\lambda^a\psi_L)$$



mass corrections to FL parameters $\hat{\mu}, v_F$ and V_0^{++--}

EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\}$$

$$+ (L\leftrightarrow R, X\leftrightarrow Y)$$

$$\psi_L \to L\psi_L C^T,\ X \to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = \mathbb{1}$$

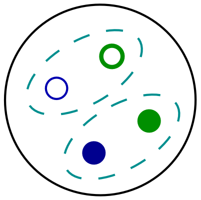
Quark loops generate a kinetic term for X, Y

$$\mathcal{L} = -\frac{f_\pi^2}{2}\left\{\mathrm{Tr}\left((X^\dagger D_0 X)^2 + (Y^\dagger D_0 Y)^2\right)\right\} + \ldots$$

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

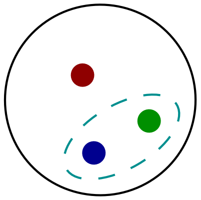
$$\Sigma = XY^\dagger$$

[8]+[1] GBs



$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons



Effective chiral theory

$$\begin{aligned}
\mathcal{L} &= \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2\mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\
&\quad + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left\{\mathcal{A}_\mu, N\right\}\right) \\
&\quad - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

$$\begin{aligned}
\mathcal{V}_\mu &= -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \\
\mathcal{A}_\mu &= -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi
\end{aligned}$$

$$f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}$$

Mass Terms: Match HDET to CFLχTh

Kinetic term: $\psi_L^\dagger X_L \psi_L + \psi_R^\dagger X_R \psi_R$

$$\begin{aligned} D_0 N &= \partial_0 N + i[\Gamma_0, N], \qquad \Gamma_0 = \mathcal{V}_0 + \frac{1}{2}\left(\xi X_R \xi^\dagger + \xi^\dagger X_L \xi\right) \\ \nabla_0 \Sigma &= \partial_0 \Sigma + i X_L \Sigma - i \Sigma X_R \end{aligned}$$

vector (axial) potentials

Contact term: $(\psi_R^\dagger M \psi_L)(\psi_R^\dagger M \psi_L)$

$$\mathcal{L} = \frac{3\Delta^2}{4\pi^2}\left\{[\mathrm{Tr}(M\Sigma)]^2 - \mathrm{Tr}(M\Sigma M\Sigma)\right\}$$

meson mass terms

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2} \mathrm{Tr}\left(X_L \Sigma X_R \Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1 \left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

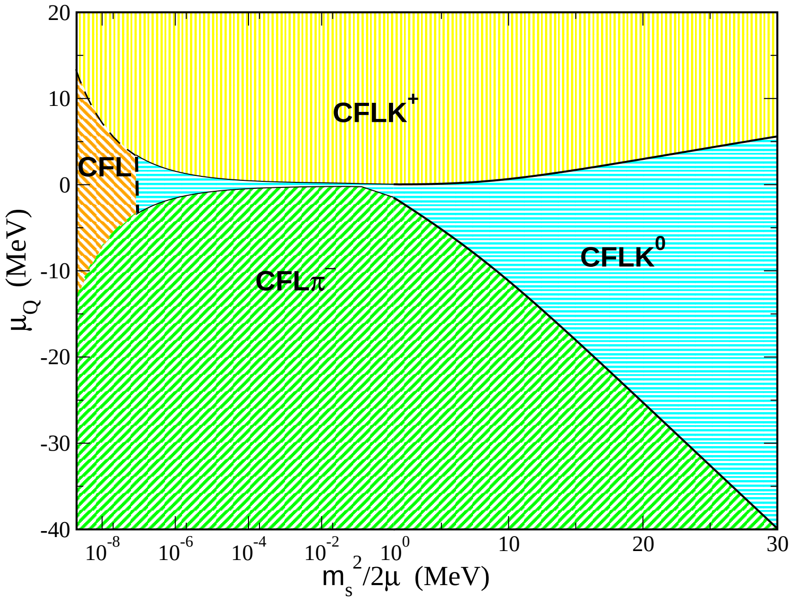
$$V(\Sigma_0) \equiv min$$

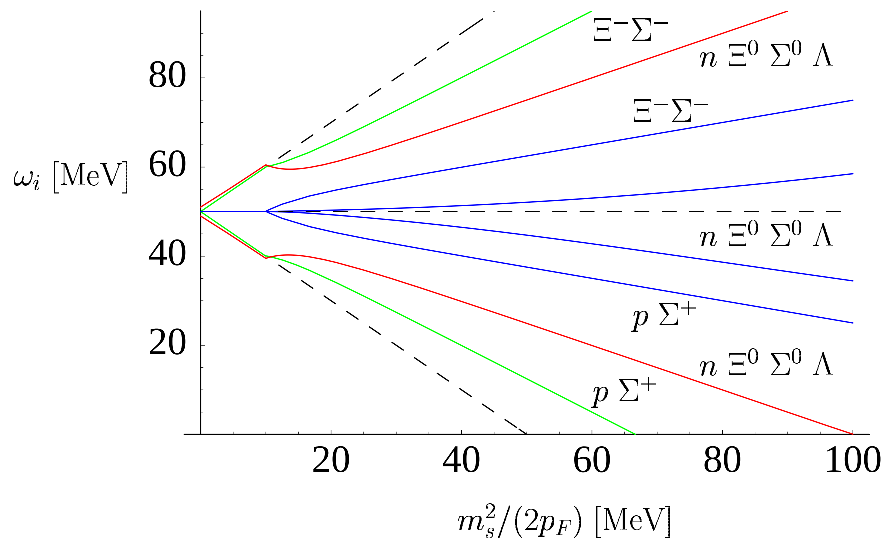
Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi \frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger \frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





meson condensation: CFLK

$$m_s(crit) \sim m_u^{1/3}\Delta^{2/3}$$

gapless modes? (gCFLK)

$$\mu_s(crit) \sim \frac{4\Delta}{3}$$

Figures: Kaplan & Reddy (2002)

Kryjevski & Schäfer (2005)

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$

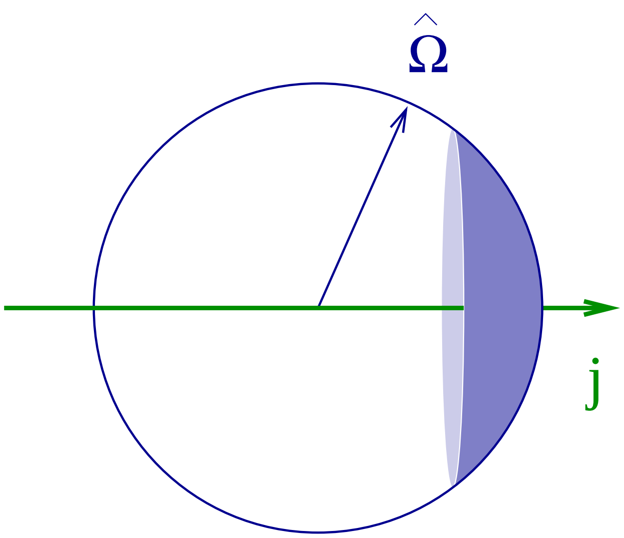
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2}v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$

Fermion spectrum

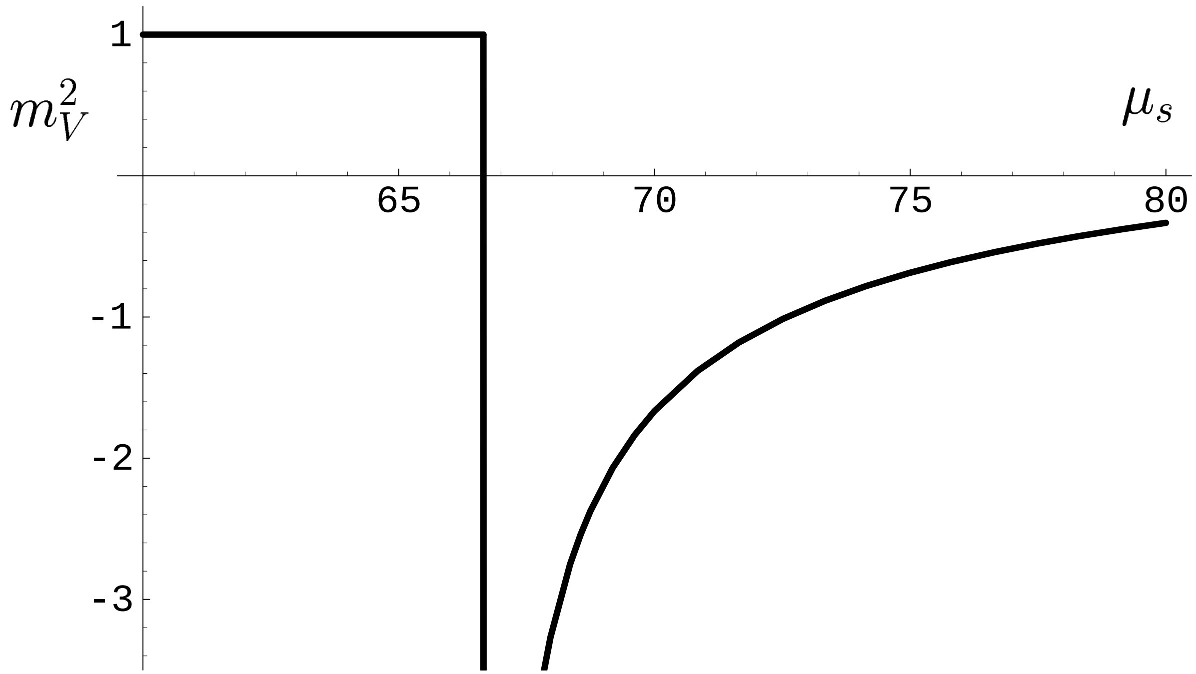
$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\;\omega_l\Theta(-\omega_l)$$



Stability lost

$$m_V^2 = \left.\frac{\partial^2 \mathcal{E}}{\partial \jmath^2}\right|_{\jmath=0}$$

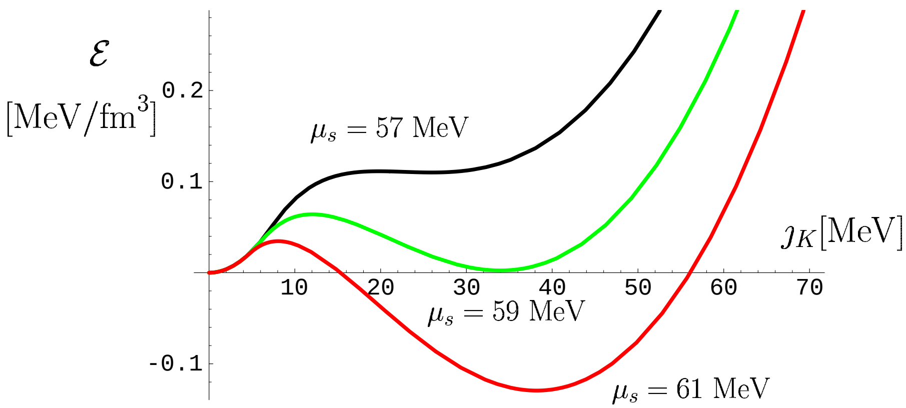


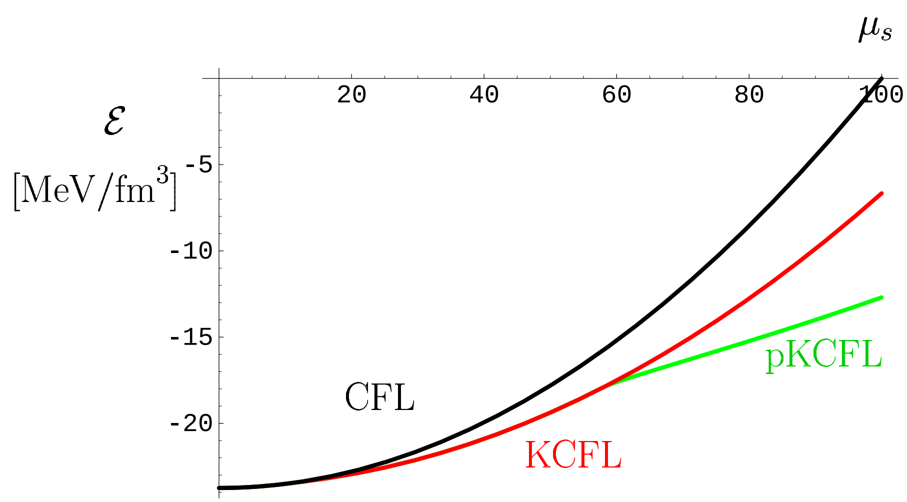
$$\mathcal{E} = C f_h(x) \qquad x = \frac{j_k}{a\Delta} \quad h = \frac{3\mu_s - 4\Delta}{a\Delta}$$

$$f_h(x) = x^2 - \frac{1}{x}\left[(h+x)^{5/2}\Theta(h+x) - (h-x)^{5/2}\Theta(h-x)\right]$$

see also: Son & Stephanov cond-mat/0507586, Kryjevski hep-ph/0508180

Energy Functional

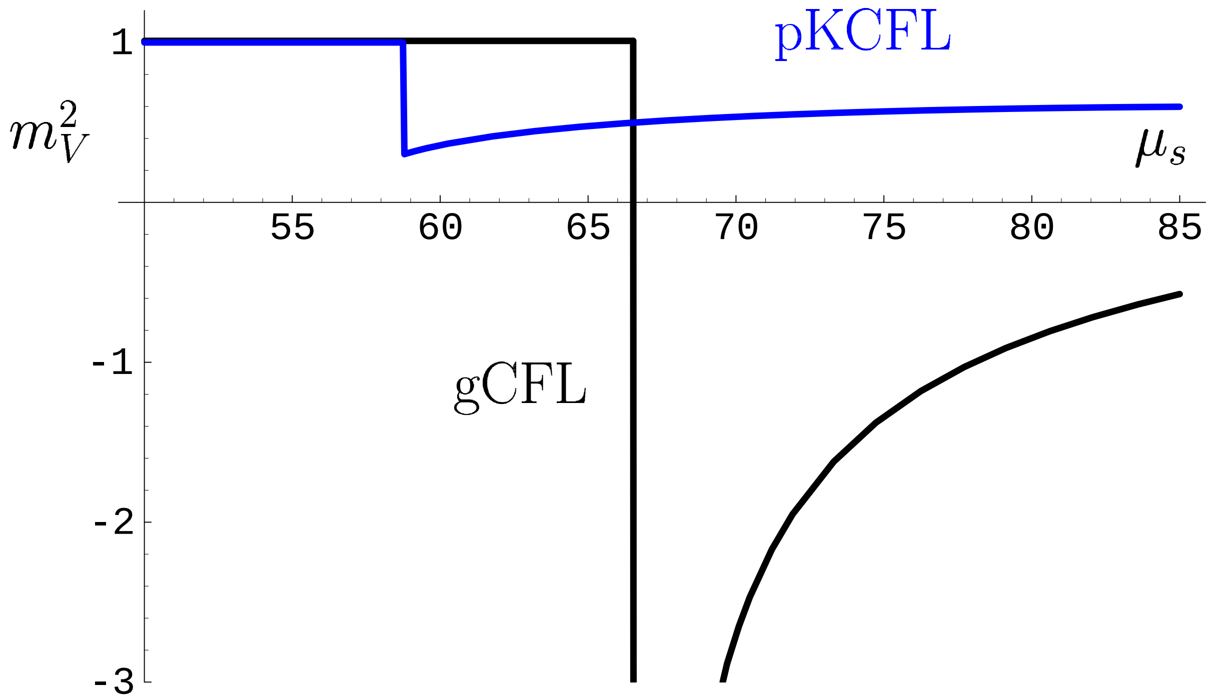




$$\left.\frac{3\mu_s - 4\Delta}{\Delta}\right|_{crit} = a h_{crit} \qquad h_{crit} = -0.067 \qquad a = \frac{2}{15^2 c_\pi^2 v_\pi^4}$$

[Figures include baryon current $j_B = \alpha_B/\alpha_K j_K$]

Stability found



$$m_V^2 = \left.\frac{\partial^2 \mathcal{E}}{\partial \jmath^2}\right|_{\jmath_0}$$

$$m_V^2 = \left.\frac{\partial^2 \mathcal{E}}{\partial \jmath^2}\right|_{\jmath=0}$$

$$\mathcal{E} = C f_h(x) \qquad x = \frac{j_k}{a\Delta} \qquad h = \frac{3\mu_s - 4\Delta}{a\Delta}.$$

Notes

No net current, meson current canceled by backflow of gapless modes

$$(\delta\mathcal{E})/(\delta\nabla\phi) = 0$$

Instability related to “chromomagnetic instability”

CFL phase: gluons carry $SU(3)_F$ quantum numbers

Meson current equivalent to a color gauge field

P-wave meson condensate continuously connected to LOFF?

Additional currents?

Higher order corrections to $\mu_s|_{crit} = (4 + ah_{crit})\Delta/3$?