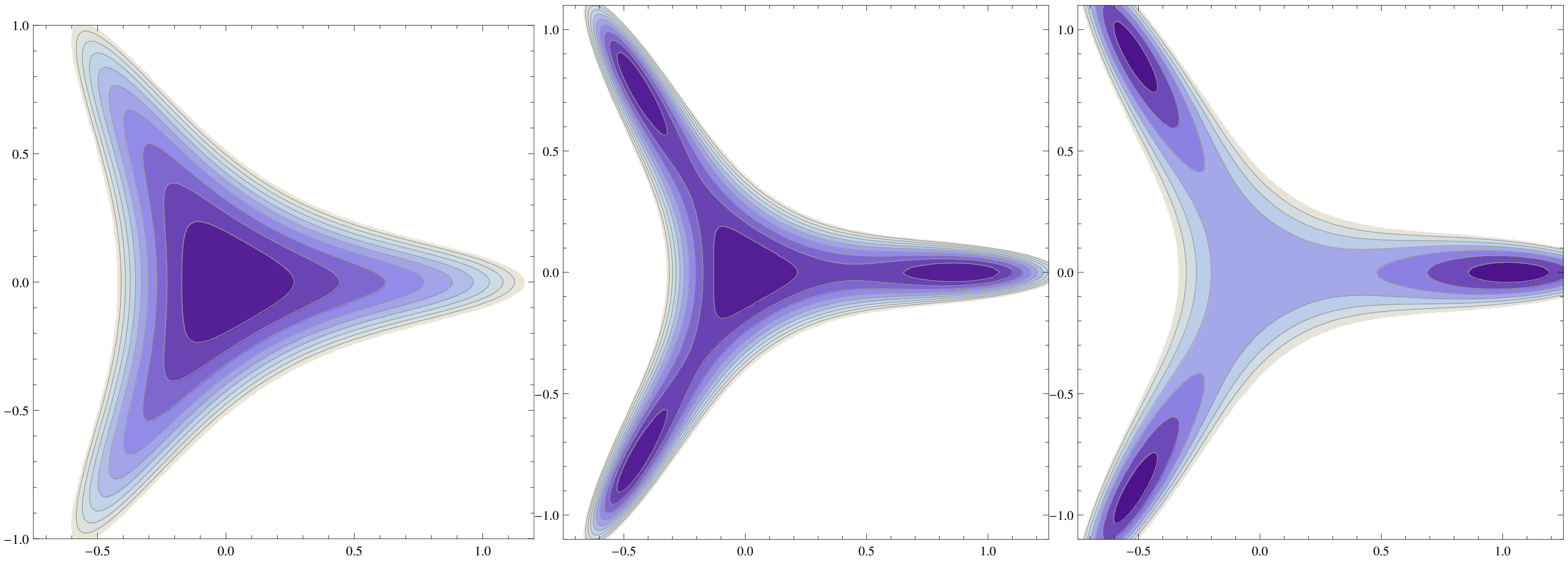
QCD and Instantons at finite Temperature: 45 years later

Thomas Schaefer, North Carolina State University



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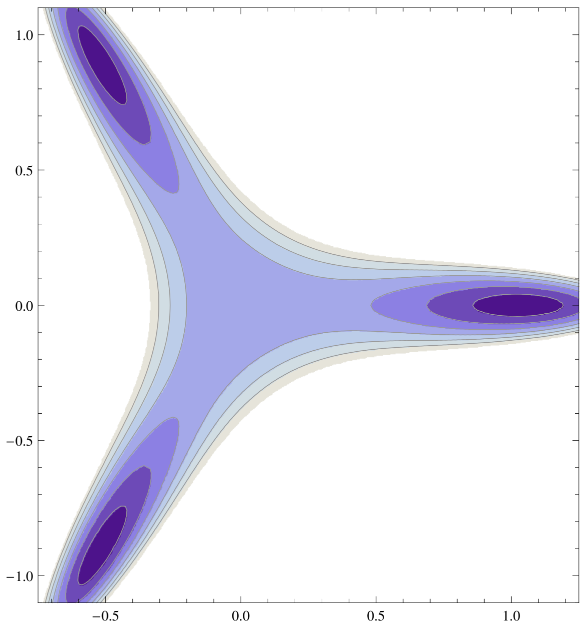
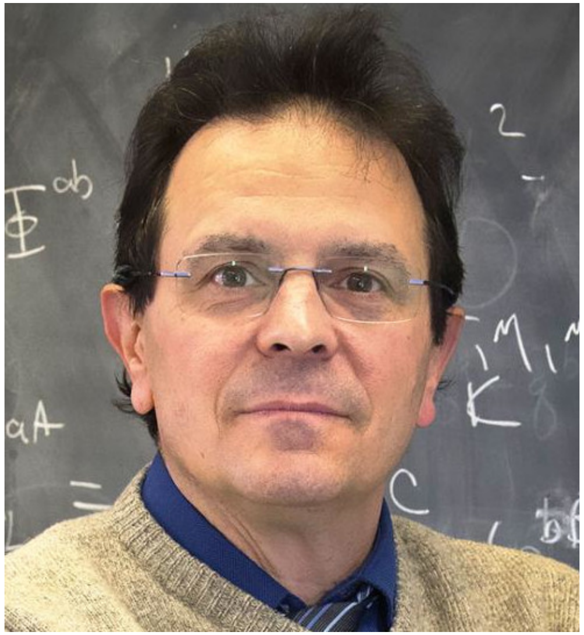




Pearls of wisdom from GPY (1981)

Topological classification on $R^3 \times S^1$

In Sec. III we give a topological classification of the smooth, finite-energy gauge fields which may contribute to the functional integral. A complete classification of periodic gauge fields, satisfying a weak asymptotic condition that ensures finite energy, is given in terms of three sets of invariants. Two of these are the familiar Pontryagin index ν and the values of the quantized magnetic charges q_α. The third is related to the asymptotic spatial behavior of the observable

$$\Omega(\mathbf{x}) = P \exp \int_0^\beta dt\, A_0(t, \mathbf{x}), \quad \beta = 1/kT$$

which may be thought of as a closed, periodic timelike Wilson loop. Its eigenvalues are gauge invariant and at spatial infinity approach constant values, λ_α^∞. Any finite-energy gauge field is classified by λ_α^∞, q_α, and ν, in terms of which the topological charge

$$Q = \frac{1}{32\pi^2} \int_0^\beta dt \int d^3x\, \hat{\mathrm{t}}\mathrm{r} F_{\mu\nu} \tilde{F}_{\mu\nu}$$

is given by

$$Q = \nu + \sum_\alpha q_\alpha (\ln \lambda_\alpha^\infty)/2\pi i\,.$$

Pearls of wisdom from GPY (1981)

Effective potential for holonomy (GPY-potential)

$$\ln \det{}_+(-D^2_{\mathrm{adj}}) = -\pi^2 \frac{V}{\beta^3}\left[\sum_{j,k=1}^{N}\left(\frac{1}{45}-\frac{1}{24}(1-[q^j-q^k]^2_+)^2\right)-\frac{1}{45}\right]$$

$$= -\pi^2 \frac{V}{\beta^3}\left(\frac{N^2-1}{45}-\frac{1}{6}\,\mathrm{tr}[(\ln\Omega^{adj}/\pi i) \times(1-\ln\Omega^{\mathrm{adj}}/2\pi i)]^2\right). \quad \text{(D5)}$$

Trivial holonomy preferred

Therefore, we conclude that for sufficiently high temperature only fields with $\Omega(\infty)=1$ contribute to the functional integral. The topological charge Q is "dynamically" quantized due to the screening behavior of the thermal fluctuations. Consequently, for sufficiently high temperature, the theory will be periodic in θ with period 2π.

Pearls of wisdom from GPY (1981)

Instanton density at $T \gg T_c$

$$n(\rho, T) = \frac{C_N}{\rho^5}(4\pi^2/g^2)^{2N}\left(\prod_{i=1}^{N_f} \xi\rho m_i\right) \exp(-\{8\pi^2/g^2 + \tfrac{1}{3}\lambda^2(2N+N_f) + 12A(\lambda)[1+\tfrac{1}{6}(N-N_f)]\})$$

$$= n(\rho, 0)\exp(-\{\tfrac{1}{3}\lambda^2(2N+N_f) + 12A(\lambda)[1+\tfrac{1}{6}(N-N_f)]\}).$$

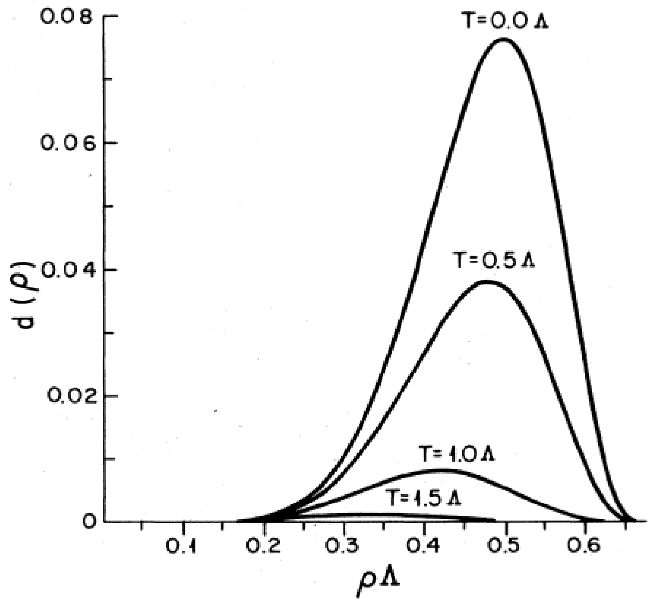
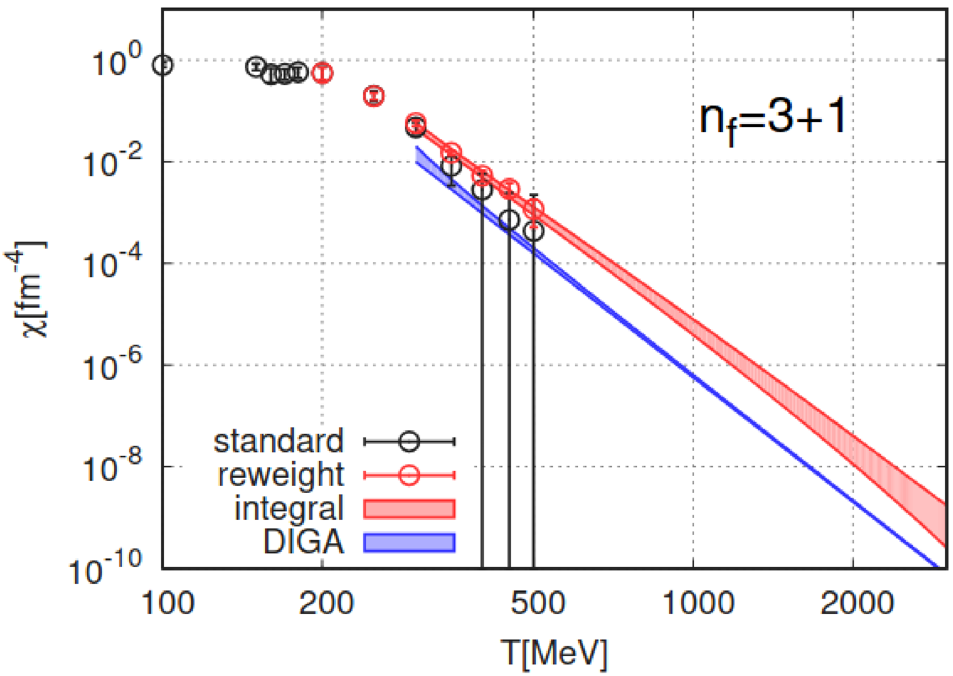


FIG. 4. Instanton density $d(\rho) \equiv \rho^5 n(\rho)$ for $N=2$ and $N_f = 0$.

Not just correct – works better than it should!



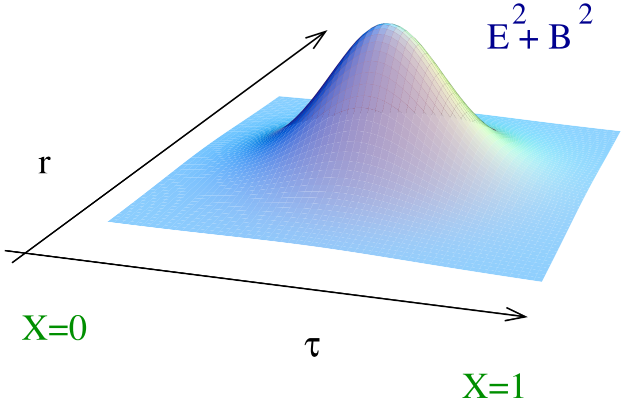
Topological susceptibility for $T > T_c$ from Borsanyi et al [1606.07494]

What about the solutions with non-trivial holonomy?

That question was studied by Kraan and van Baal [hep-th/9805168] as well as Lee and Liu [hep-th/9802108] in 1997.

Periodic instantons (calorons)

Instanton solution in R^4 can be extended to solution on $R^3 \times S^1$



$$Q_{top} = \pm 1$$

$$\Omega_\infty = 1 \quad Q_M^\alpha = 0$$

$SU(2)$ solution has $1+3+1+3=8$ bosonic zero modes

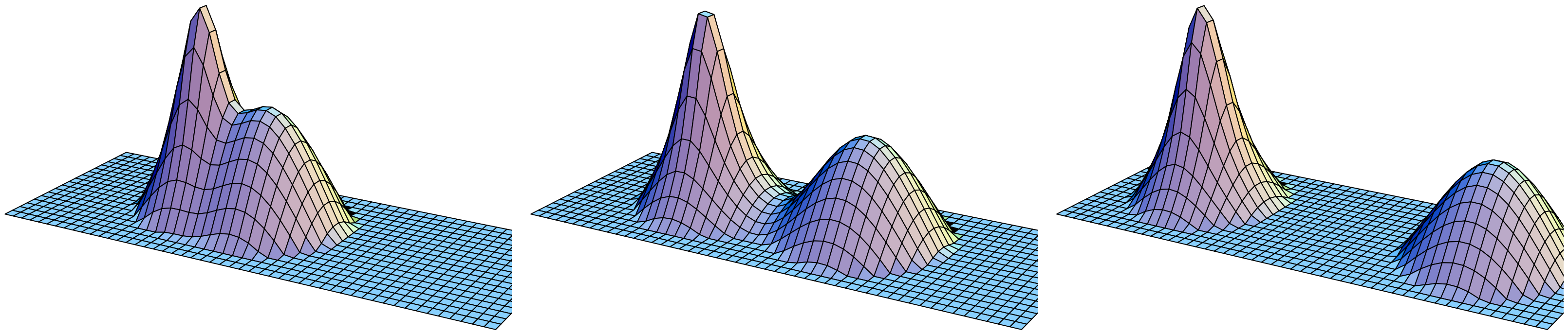
$$\int \frac{d\rho}{\rho^5} \int d^3x\, dx_4 \int dU\, e^{-2S_0} \qquad 2S_0 = \frac{8\pi^2}{g^2}$$

$4n_{adj}$ fermionic zero modes

$$\int d^2\zeta d^2\xi$$

Calorons at finite holonomy: monopole constituents

KvBLL (1998) construct calorons with non-trivial holonomy (here: $SU(2)$)



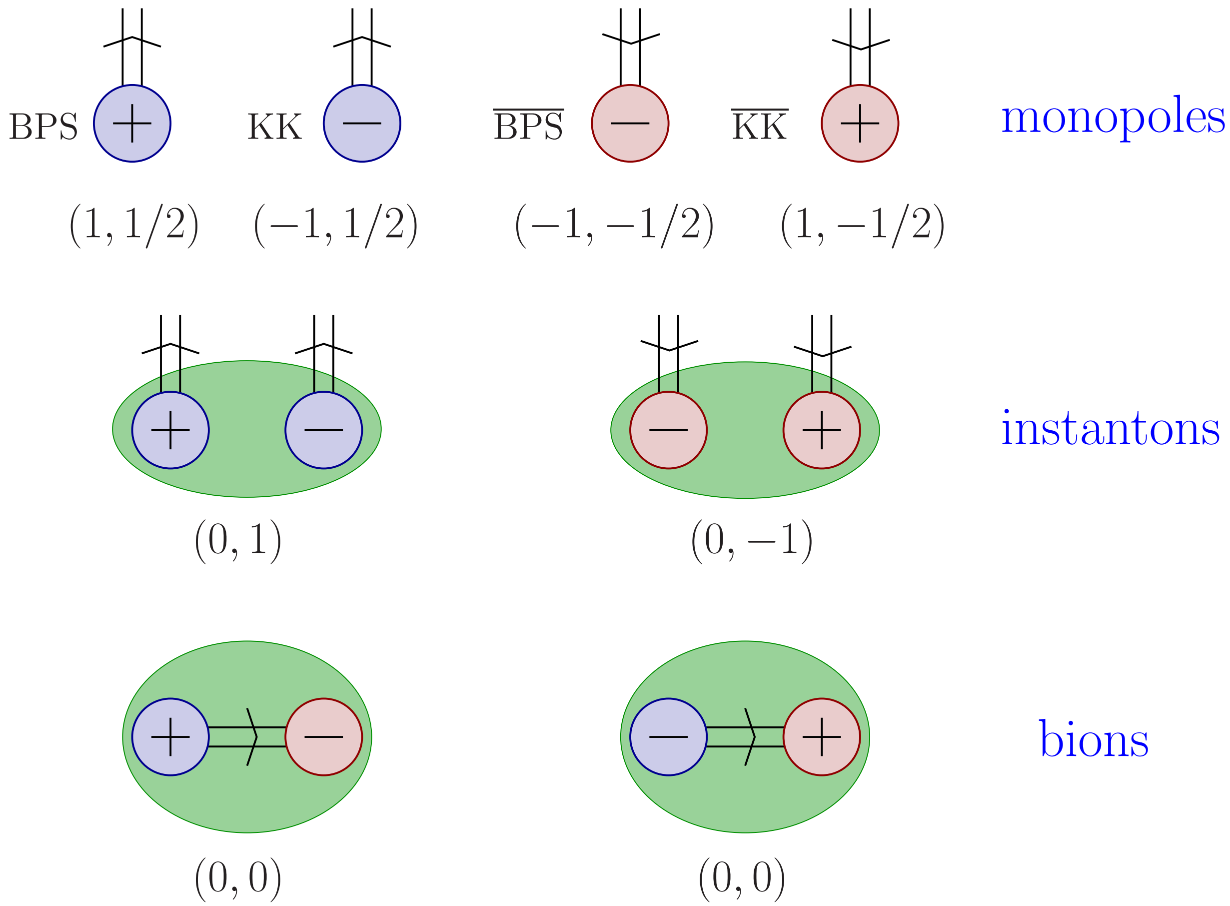
BPS and KK monopole constituents. Fractional topological charge, $1/2$ at center symmetric point.

$2 \times (3+1) = 8$ bosonic zero modes, 2×2 fermionic ZM.

$$\int d\phi_1 \int d^3x_1 \int d^2\zeta \, e^{-S_1} \int d\phi_2 \int d^3x_2 \int d^2\xi \, e^{-S_2}$$

Topological objects

$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

What are these solutions good for?

The instanton liquid model (Shuryak, 1982; Schaefer, Shuryak, hep-ph/9610451) has many successes, but there are some questions it does not address.

Confinement?

Large N_c?

Reliable semi-classics? IA pairs?

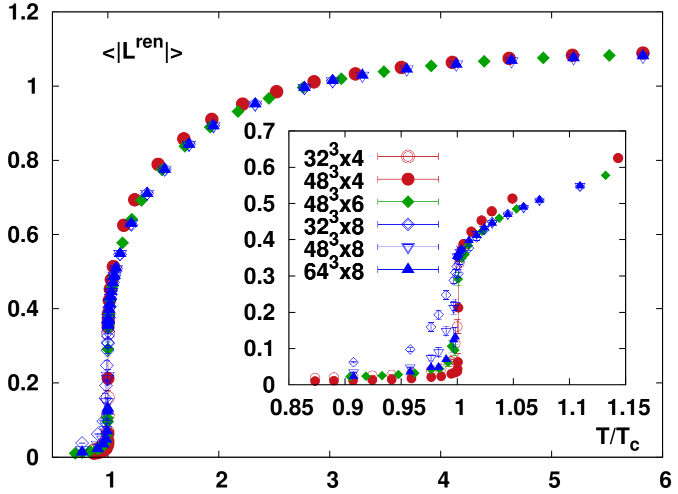
What are these solutions good for?

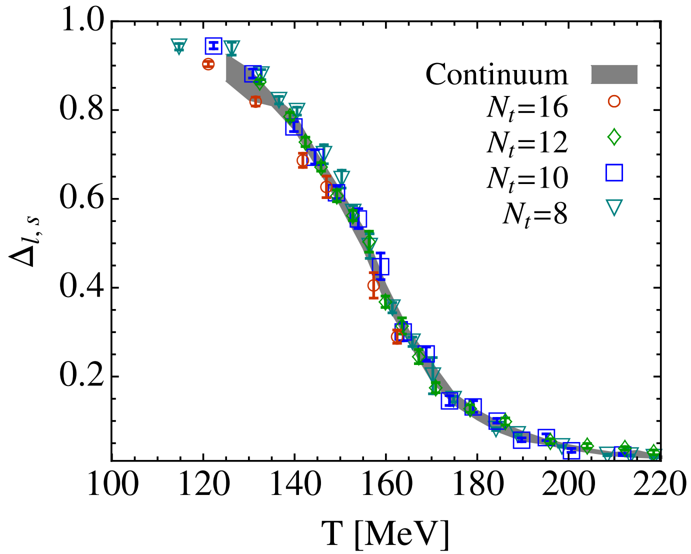
The instanton liquid model (Shuryak, 1982; Schaefer, Shuryak, hep-ph/9610451) has many successes, but there are some questions it does not address.

Confinement?	Self-dual monopoles.
Large N_c?	Fractional topological charge.
Reliable semi-classics? IA pairs?	Deformed QCD, resurgence.

Deformed QCD

Confinement and chiral symmetry breaking are well established





Goal: Find deformations of QCD, **continuously** connected to the full theory, that exhibit χSB and confinement in weak coupling.

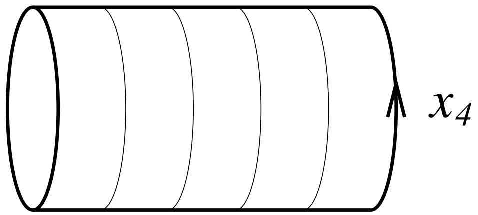
Bali et al. (2000), Borsanyi et al (2011)

Semi-classical Confinement

Consider $SU(2)$ gauge theory with $N_f^{ad} = 1$ on $R^3 \times S_1$

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a$$

$$A^a_\mu(0) = A^a_\mu(L)$$
$$\lambda^a_\alpha(0) = \lambda^a_\alpha(L)$$



Large m: Thermal pure YM Z_β. Small m: Twisted SUSY YM $\tilde{Z}_\beta$.

Small S_1 and m: Confinement can be studied using semi-classical methods, based on monopole-instantons, instantons, and bions.

Small S_1: Effective Theory

Consider small S_1: Effective theory in 3d

$\Omega \neq 1$: A_4^3 is a Higgs field, theory abelianizes $SU(2) \to U(1)$.

Light bosonic modes: (dual) "photon" σ and holonomy b

$$\mathcal{L} = \frac{g^2}{32\pi^2 L} \left[(\partial_i b)^2 + (\partial_i \sigma)^2 \right] + V(\sigma, b)$$

$$\Omega = \begin{pmatrix} e^{i\Delta\theta/2} & 0 \\ 0 & e^{-i\Delta\theta/2} \end{pmatrix} \quad b = \frac{4\pi}{g^2}\Delta\theta \qquad \epsilon_{ijk}\partial_k\sigma = \frac{4\pi L}{g^2} F_{ij}$$

holonomy b — dual photon σ

Note: $m = 0$ effective theory can be super-symmetrized

$$B = b + i\sigma + \sqrt{2}\theta^\alpha \lambda^\alpha$$

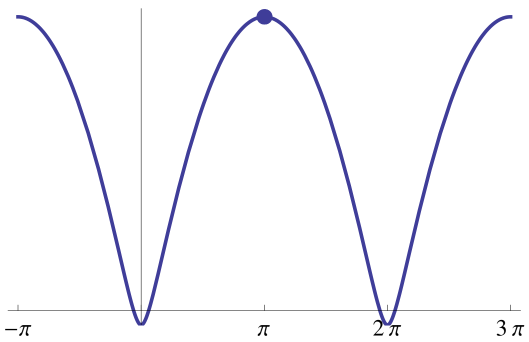
Perturbation Theory

Perturbative potential for holonomy (Gross, Pisarski, Yaffe, 1981)

$$V(\Omega) = -\frac{m^2}{2\pi^2 L^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \, |\mathrm{tr}\,\Omega^n|^2 = -\frac{m^2}{L^2} B_2\left(\frac{\Delta\theta}{2\pi}\right)$$

$m = 0$: Bosonic and fermionic terms cancel.

$m \neq 0$: Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ unstable.



Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x\, F\tilde{F}$$

2. Holonomy (eigenvalues q^α of Polyakov line at spatial infinity)

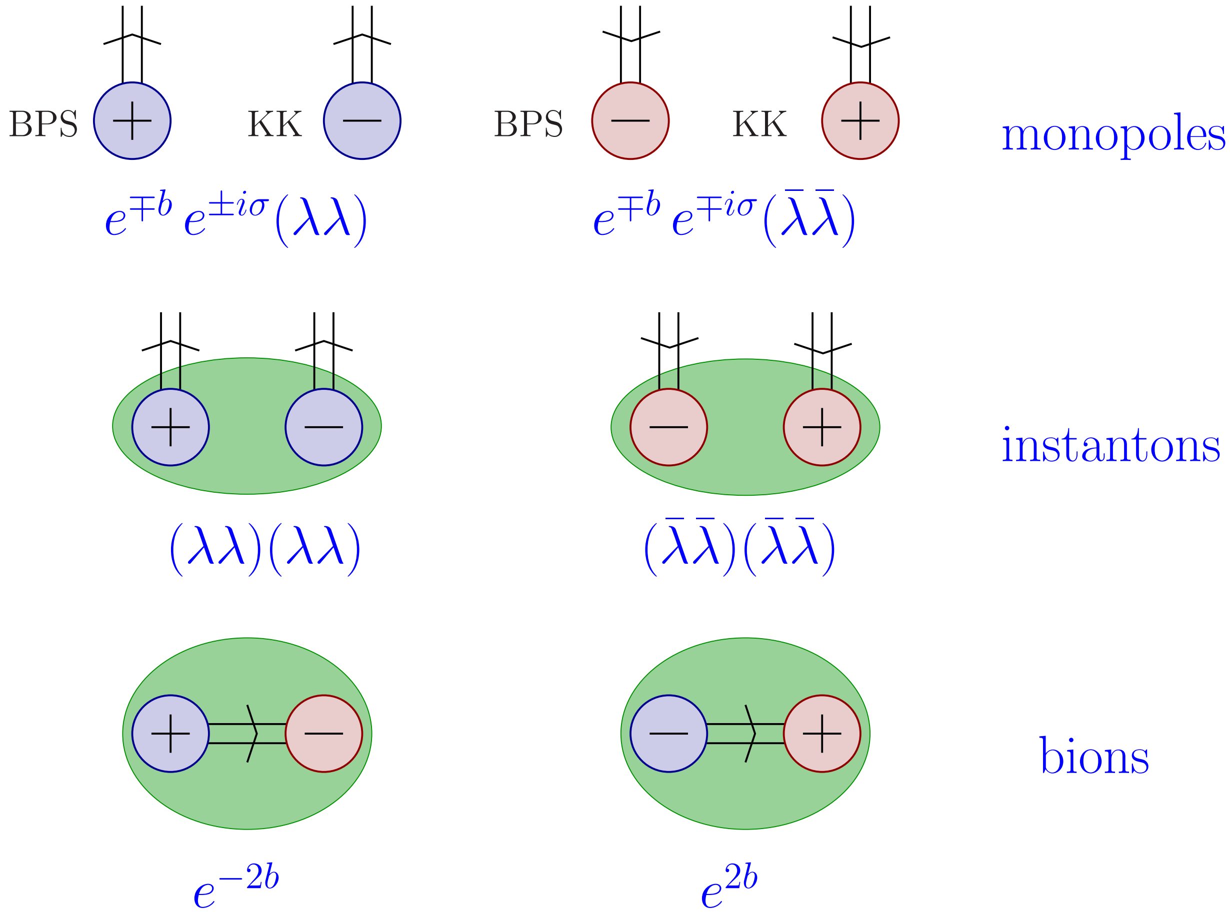
$$\langle \Omega(\vec{x}) \rangle = \left\langle \mathrm{Tr}\exp\left[i \int_0^\beta A_4 dx_4 \right] \right\rangle$$

3. Magnetic charges

$$Q_M^\alpha = \frac{1}{4\pi} \int d^2S\, \mathrm{Tr}\left[P^\alpha B \right]$$

Topological objects: Coupling to low energy fields

$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Kraan, van Baal (1998); Lee, Lu (1998); Unsal (2008)

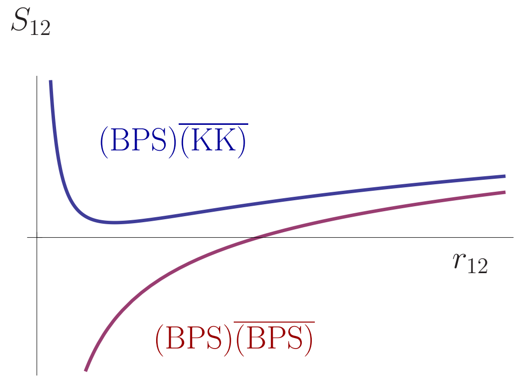
Effective potential

Instantons and monopoles: Exact solutions, but $V(b,\sigma)=0$.

Bions: Approximate solutions

$$V_{BPS,\overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r\, e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r}(q_m^1 q_m^2 - q_b^1 q_b^2) + 4\log(r)$$



Saddle point integral after resurgent cancellations.

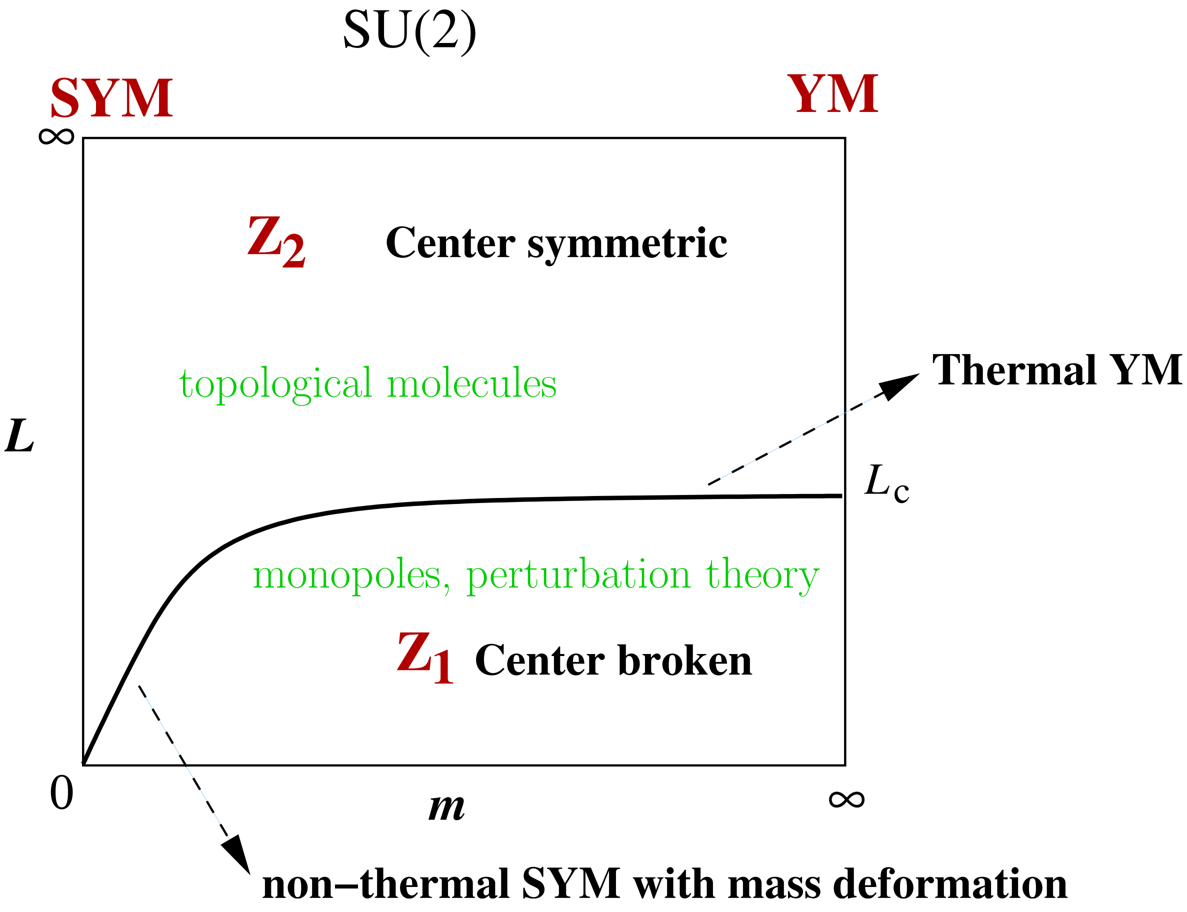
$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6}\left[\cosh\left(2\left(b-b_0\right)\right) - \cos(2\sigma)\right]$$

Center symmetric vacuum $\mathrm{tr}(\Omega)=0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\rightarrow$ confinement)

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L-m plane



Poppitz, T.S., Unsal, [1205.0290[hep-th]].

Bonus: Grand unified picture of topology in QCD

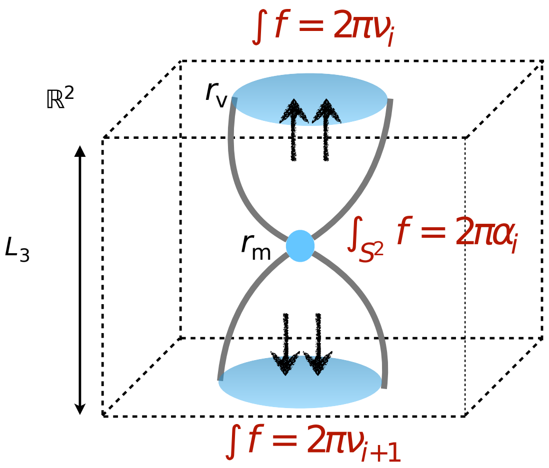
We used to discuss possible "mechanisms" for confinement in QCD:

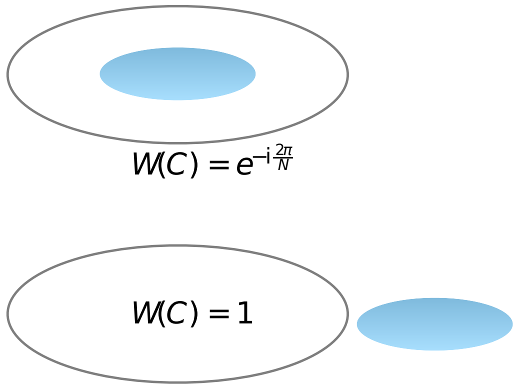
Instantons, monopoles, vortices, or none of the above.

Can we realize these pictures using different deformations of QCD?

Deformed QCD on $R^2 \times S^1 \times S^1$

Consider QCD on $R^2 \times S^1 \times S^1$ with $L_3 > L_4$ and 't Hooft flux $n_{34} = 1$.
This corresponds to $P_4 = C$, $P_3 = S$.





Monopole flux collimated into tubes. Confinement due to vortex gas.

Vortex arises like the caloron, as a linear periodic array (with period N) of self-dual monopoles.

Guvendik, T.S., Unsal, [2405.13696[hep-th]].

Conclusions and Outlook

Topology remains a fertile play ground in QCD, and not only the GPY classification, but also the GPY potential play a central role.

Calculable mechanism for chiral symmetry breaking and confinement in compactified versions of QCD.

Can realize monopole and vortex picture in deformed QCD. Both objects are descended from instantons.

Happy Birthday, Rob!