Non-perturbative Methods in QCD at finite μ and T

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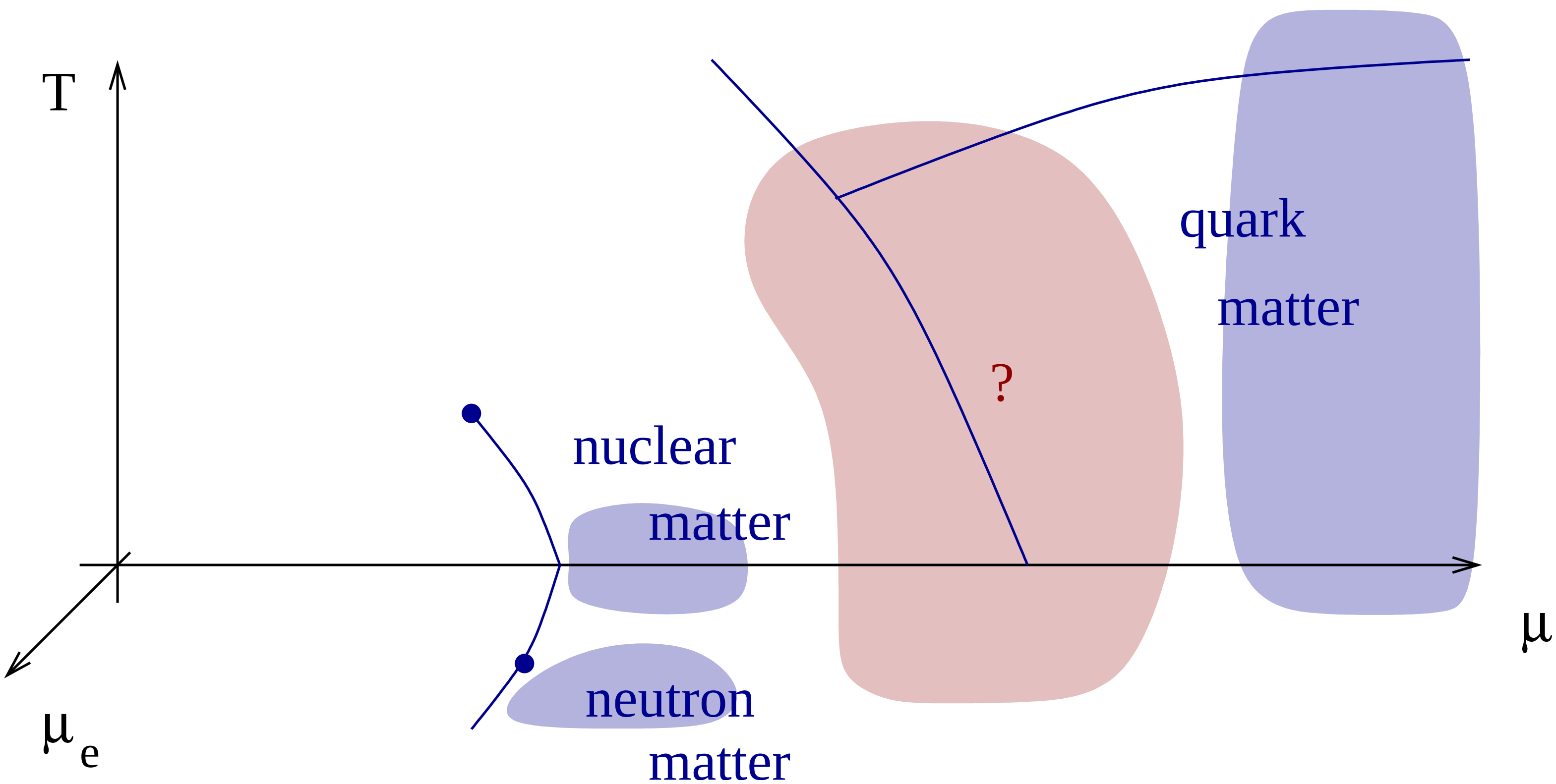
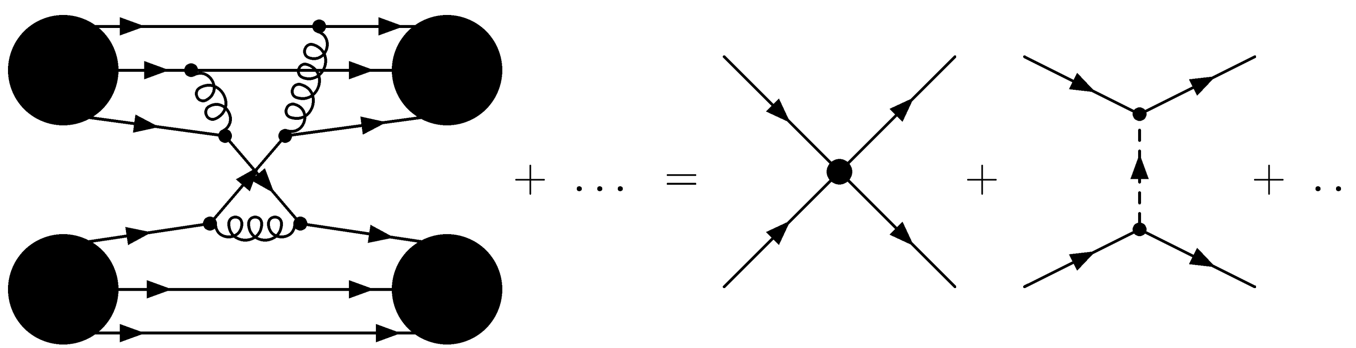
More modest: Systematic approaches to dense matter



Low Density: Nuclear Effective Field Theory

	Nucleons are point particles
Low Energy Nucleons:	Interactions are local
	Long range part: pions



	Systematically improvable
Advantages:	Symmetries manifest (Chiral, gauge, ...)
	Connection to lattice QCD

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \ldots$$

Simplifications: neutrons only, no pions (very low energy)

Effective range expansion

$$p \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} \Lambda^2 \sum_n r_n \left(\frac{p^2}{\Lambda^2} \right)^{n+1}$$

Coupling constants

$$C_0 = \frac{4\pi a}{M}, \qquad C_2 = \frac{4\pi a^2}{M} \frac{r}{2}, \quad \ldots \qquad a = -18\,\text{fm}, \; r = 2.8\,\text{fm}$$

Toy Problem (Neutron Matter)

Consider limiting case ("Bertsch" problem)

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5}\left(\frac{k_F^2}{2M}\right)$$

No Expansion Parameters!

How to find ξ?

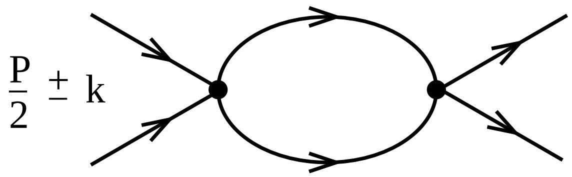
Numerical Simulations

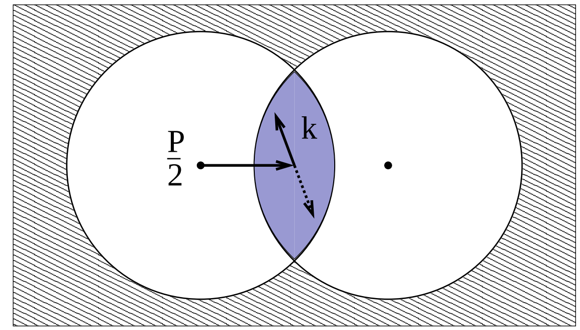
Experiments with trapped fermions

Analytic Approaches

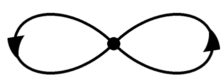
Large d Limit

In medium scattering strongly restricted by phase space

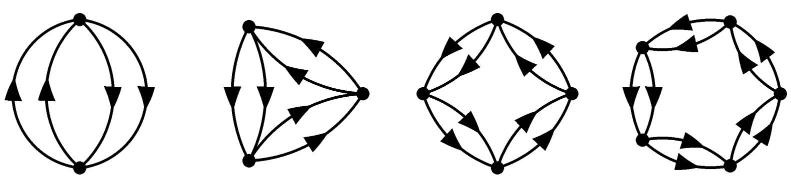




Find limit in which ladders are leading order



$(C_0/d) \cdot 1/d$



$(C_0/d)^k \cdot 1/d$

$$\lambda \equiv \left[\frac{\Omega_d C_0 k_F^{d-2} M}{d(2\pi)^d} \right]$$

$$\lambda = const \;\; (d \to \infty)$$

$$\xi = \frac{1}{2} + O(1/d)$$

Epsilon Expansion

Bound state wave function $\psi \sim 1/r^{d-4}$. For $d \geq 4$

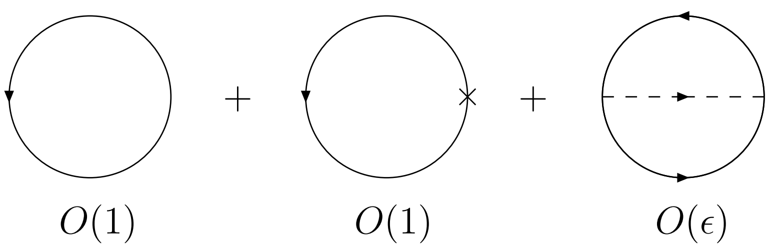
Non-interacting bosons $\xi(d=4)=0$ Nussinov & Nussinov

Effective lagrangian for atoms $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$ and dimers ϕ

$$\mathcal{L} = \Psi^\dagger \left(i\partial_0 + \frac{\sigma_3 \nabla^2}{2m} \right) \Psi + \mu \Psi^\dagger \sigma_3 \Psi - \frac{1}{c_0} \phi^* \phi + \Psi^\dagger \sigma_+ \Psi \phi + h.c.$$

Nishida & Son (2006)

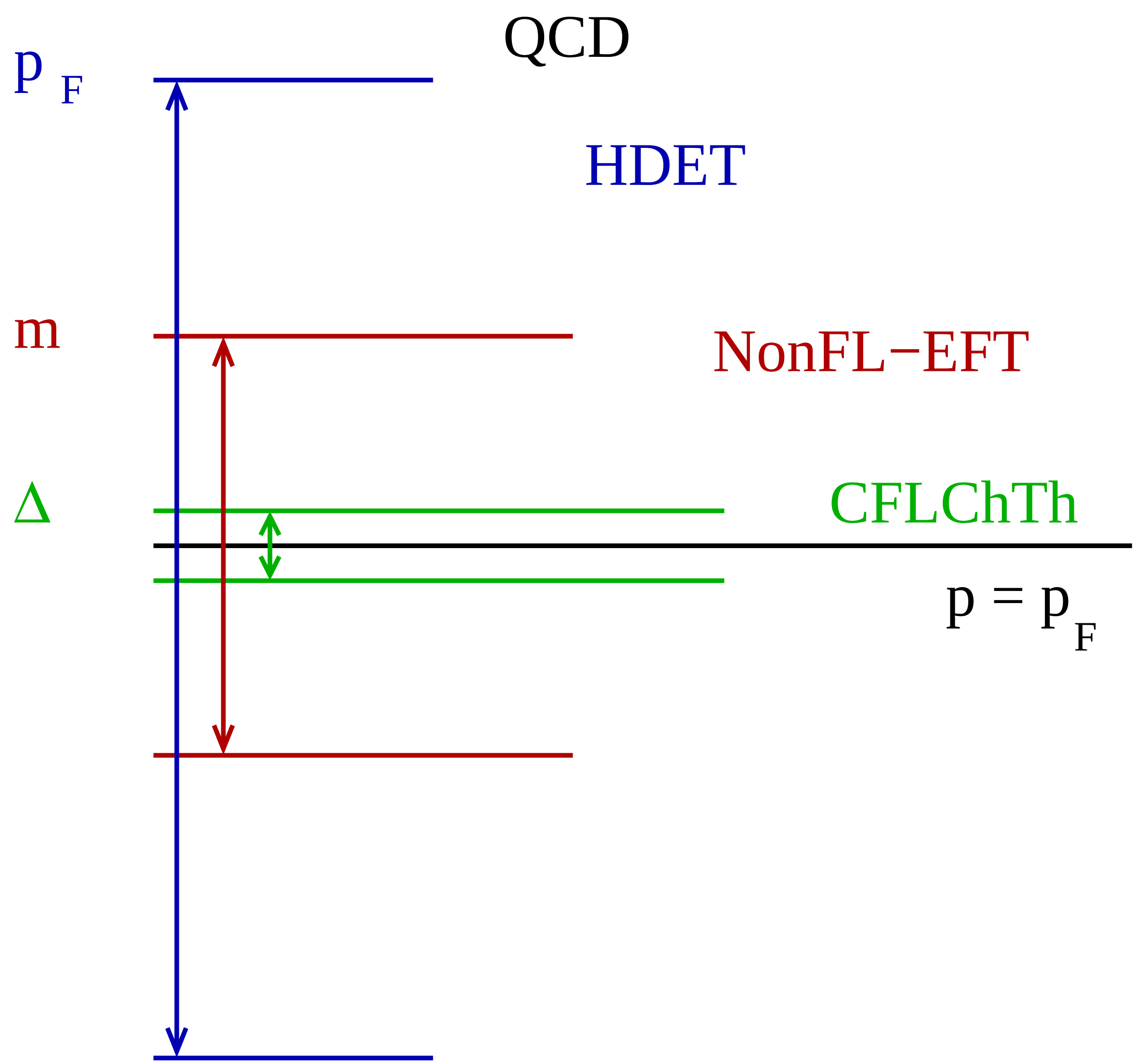
Unitary limit $c_0 \to \infty$. Effective potential



$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2} \ln \epsilon - 0.0246\, \epsilon^{5/2} + \ldots$$

$$\xi(\epsilon = 1) = 0.475$$

Very Dense Matter: Effective Field Theories

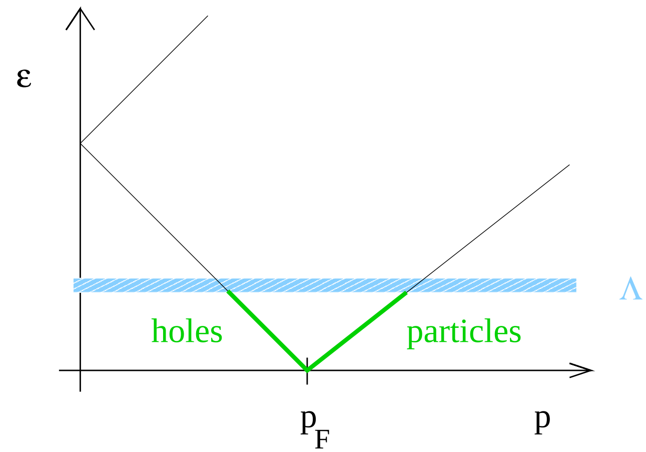


High Density Effective Theory

QCD lagrangian

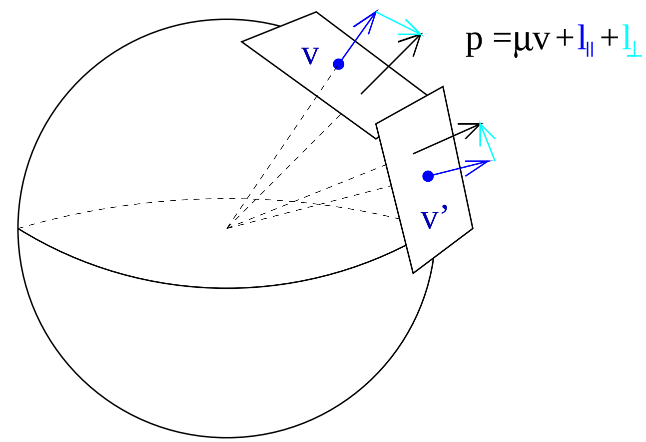
$$\mathcal{L} = \bar{\psi}\left(i\not{D} + \mu\gamma_0 - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

Quasi-particles (holes)



$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

High Density Effective Theory, cont

Effective lagrangian for ψ_{v+}

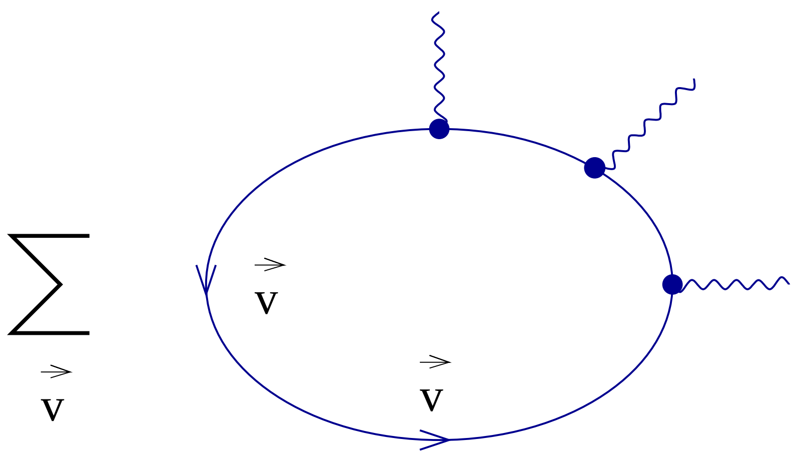
$$\mathcal{L} = \sum_v \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots$$

Power Counting

Naive power counting

$$\mathcal{L} = \hat{\mathcal{L}}\left(\psi, \psi^\dagger, \frac{D_{||}}{\mu}, \frac{D_\perp}{\mu}, \frac{\bar{D}_{||}}{\mu}, \frac{m}{\mu}\right)$$

Problem: hard loops (large $N_{\vec{v}}$ graphs)



$$\frac{1}{2\pi}\sum_{\vec{v}}\int\frac{d^2 l_\perp}{(2\pi)^2} = \frac{\mu^2}{2\pi^2}\int\frac{d\Omega}{4\pi}.$$

Have to sum large $N_{\vec{v}}$ graphs

Effective Theory for $l < m$

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}$$

Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\frac{\pi}{2} m^2 \frac{k_0}{|\vec{k}|}},$$

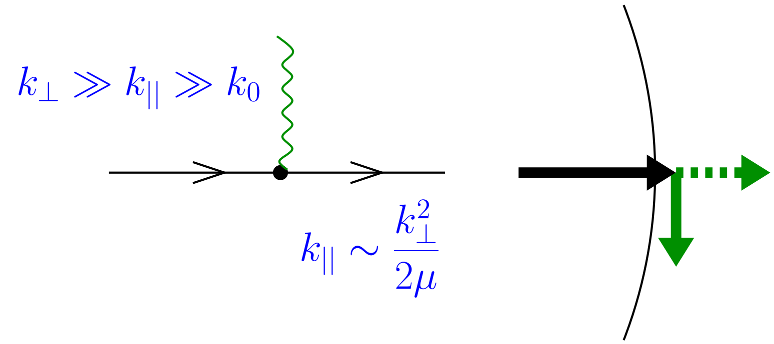
Scaling of gluon momenta

$$|\vec{k}| \sim k_0^{1/3} m^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Non-Fermi Liquid Effective Theory

Gluons very spacelike $|\vec{k}| \gg |k_0|$. Quark kinematics?

$$k_0 \simeq k_{||} + \frac{k_\perp^2}{2\mu}$$



Scaling relations

$$k_\perp \sim m^{2/3} k_0^{1/3}, \qquad k_{||} \sim m^{4/3} k_0^{2/3}/\mu$$

Propagators

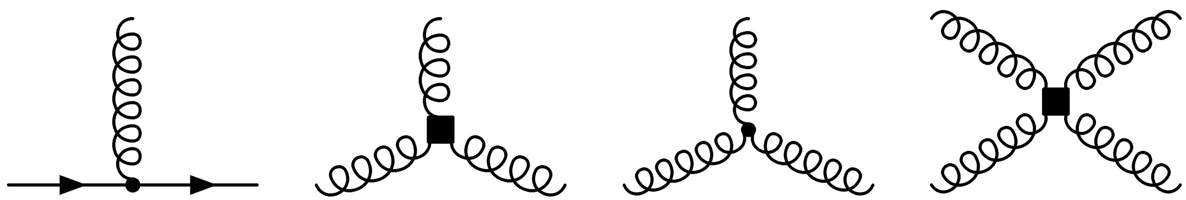
$$S_{\alpha\beta} = \frac{-i\delta_{\alpha\beta}}{p_{||} + \frac{p_\perp^2}{2\mu} - i\epsilon sgn(p_0)} \qquad D_{ij} = \frac{-i\delta_{ij}}{k_\perp^2 - i\frac{\pi}{2} m^2 \frac{k_0}{k_\perp}},$$

Non-Fermi Liquid Expansion

Scale momenta $(k_0, k_{||}, k_\perp) \to (sk_0, s^{2/3}k_{||}, s^{1/3}k_\perp)$

$$[\psi] = 5/6 \qquad [A_i] = 5/6 \qquad [S] = [D] = 0$$

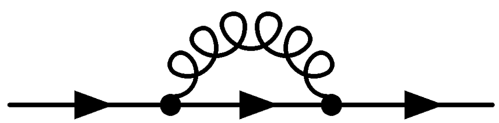
Scaling behavior of vertices



$s^{1/6}$ $\qquad$ $s^{1/2}$ $\qquad$ $s^{5/6}$ $\qquad$ s

Systematic expansion in $\epsilon^{1/3} \equiv (\omega/m)^{1/3}$

Loop Corrections: Quark Self Energy



$$= g^2 C_F \int \frac{dk_0}{2\pi} \int \frac{dk_\perp^2}{(2\pi)^2} \frac{k_\perp}{k_\perp^3 + i\eta k_0}$$

$$\times \int \frac{dk_{||}}{2\pi} \frac{\Theta(p_0 + k_0)}{k_{||} + p_{||} - \frac{(k_\perp + p_\perp)^2}{2\mu} + i\epsilon}$$

Transverse momentum integral logarithmic

$$\int \frac{dk_\perp^3}{k_\perp^3 + i\eta k_0} \sim \log\left(\frac{\Lambda}{k_0}\right)$$

Quark self energy

$$\Sigma(p) = \frac{g^2}{9\pi^2} p_0 \log\left(\frac{\Lambda}{|p_0|}\right)$$

Quark Self Energy, cont

Higher order corrections?

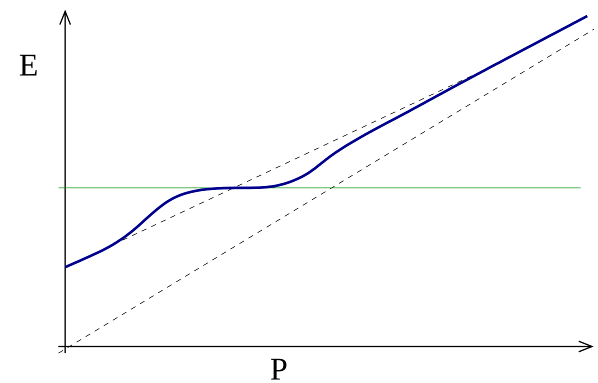
$$\Sigma(p) = \frac{g^2}{9\pi^2}\left(p_0 \log\left(\frac{2^{5/2}m}{\pi|p_0|}\right) + i\frac{\pi}{2}p_0\right) + O\left(\epsilon^{5/3}\right)$$

Scale determined by electric gluon exchange

No $p_0[\alpha_s \log(p_0)]^n$ terms

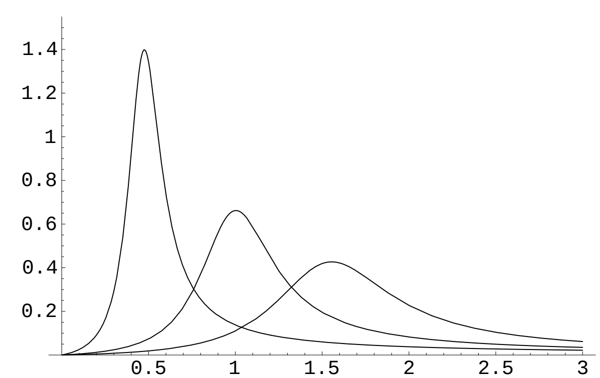
quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$



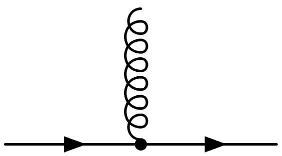
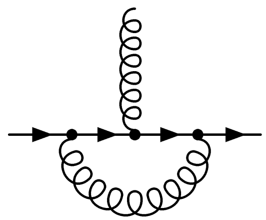
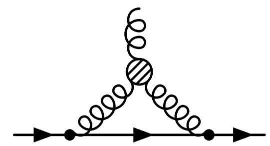
anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$



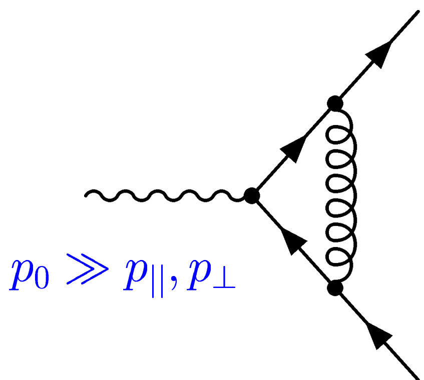
Vertex Corrections, Migdal's Theorem

Corrections to quark gluon vertex

 $+$  $+$  $\sim \quad gv(1+O(\epsilon^{1/3}))$

Analogous to electron-phonon coupling

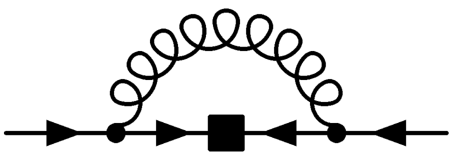
Can this fail? Yes, if external momenta fail to satisfy $p_\perp \gg p_0$



$$= \quad \frac{eg^2}{9\pi^2} v_\mu \log(\epsilon)$$

Superconductivity

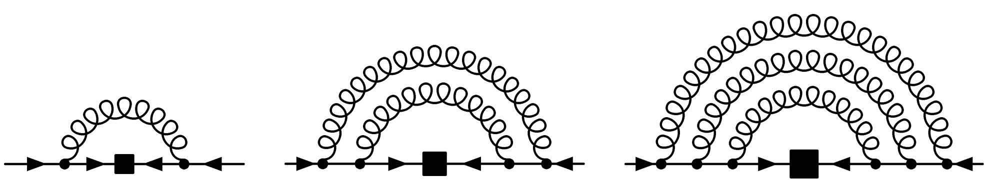
Same phenomenon occurs in anomalous self energy



$$= \frac{g^2}{18\pi^2}\int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0-q_0|}\right)\frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}$$

$\Lambda_{BCS} = 256\pi^4 g^{-5}\mu$ determined by electric exchanges

Have to sum all planar diagrams, non-planar suppressed by $\epsilon^{1/3}$



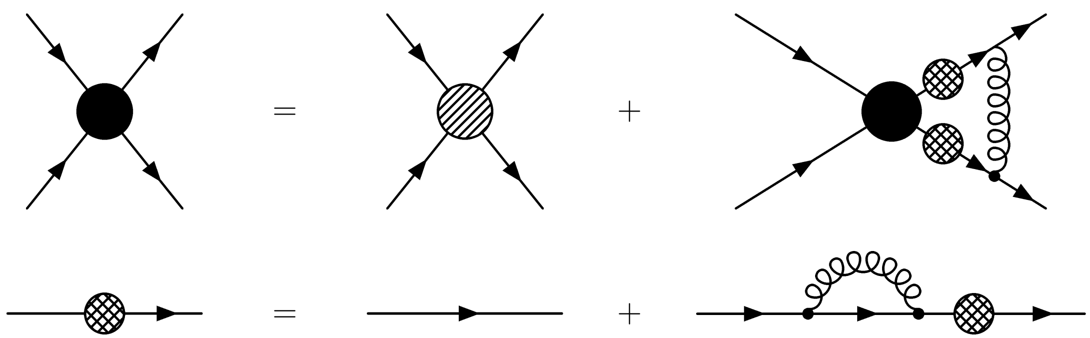
Solution at next-to-leading order (includes normal self energy)

$$\Delta_0 = 2\Lambda_{BCS}\exp\left(-\frac{\pi^2+4}{8}\right)\exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \qquad \Delta_0 \sim 50\,\mathrm{MeV}$$

Summary

Systematic low energy expansion in $(\omega/m)^{1/3}$ and $\log(\omega/m)$

Standard FL channels (BCS, ZS, ZS'): Ladder diagrams have to be summed, kernel has perturbative expansion



CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$

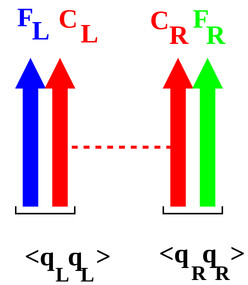
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

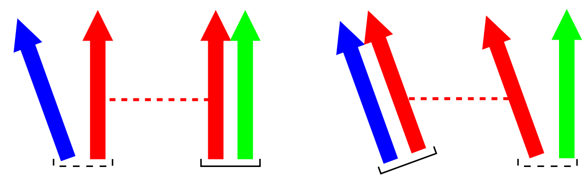
$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

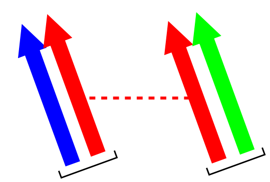
All quarks and gluons acquire a gap





Rotate left flavor

Compensate by rotating color



... have to rotate right flavor also !

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

EFT in the CFL Phase

Consider HDET with a CFL gap term

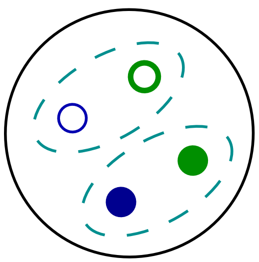
$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\} + (L\leftrightarrow R, X\leftrightarrow Y)$$

$$\psi_L \to L\psi_L C^T,\ X\to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = \mathbb{1}$$

Quark loops generate a kinetic term for X, Y

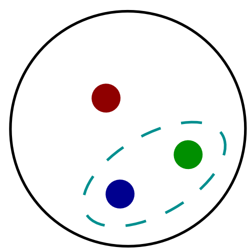
Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

$$\Sigma = XY^\dagger$$



[8]+[1] GBs

$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$



[8]+[1] Baryons

Effective theory: (CFL) baryon chiral perturbation theory

$$\begin{aligned}
\mathcal{L} &= \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2\mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\
&\quad + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5\left\{\mathcal{A}_\mu, N\right\}\right) \\
&\quad - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5\left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

$$\begin{aligned}
\mathcal{V}_\mu &= -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \\
\mathcal{A}_\mu &= -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi
\end{aligned}$$

$$f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}$$

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(X_L \Sigma X_R \Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

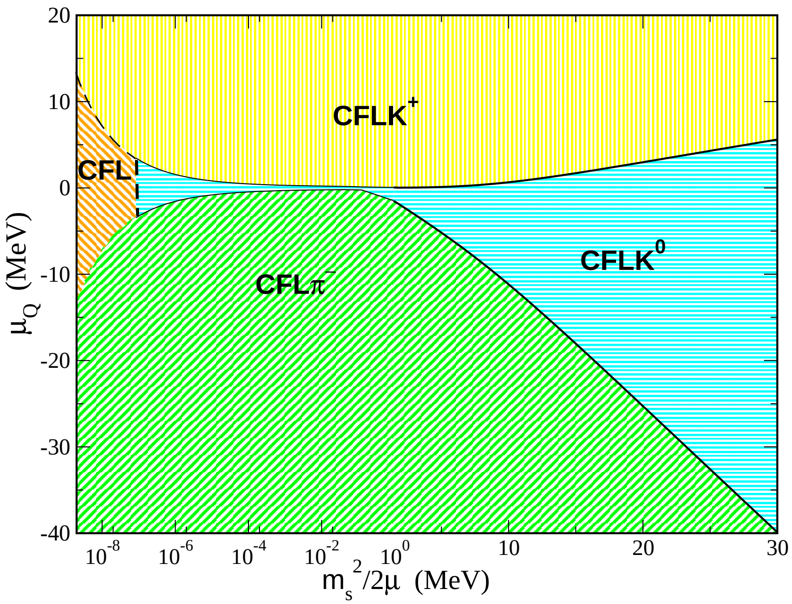
$$V(\Sigma_0) \equiv min$$

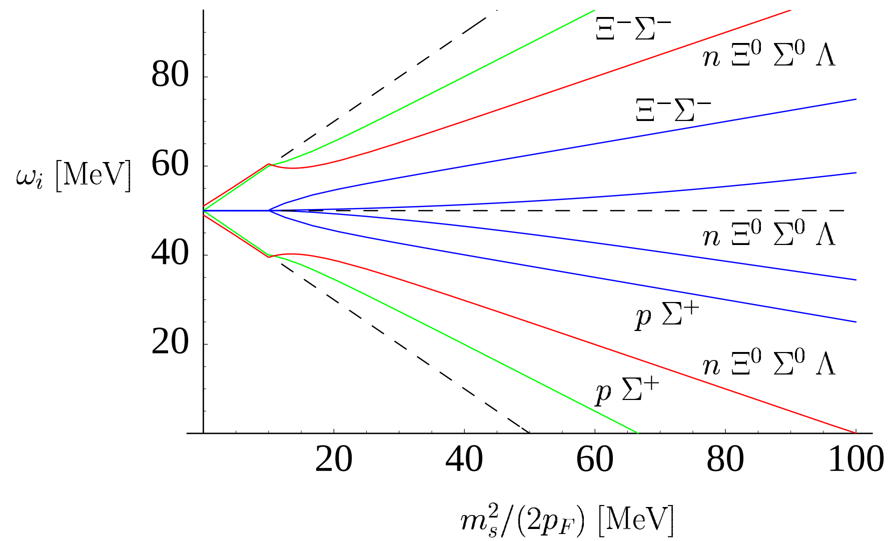
Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi\frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger\frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





meson condensation: CFLK

s-wave condensate

gapless modes? (gCFLK)

p-wave condensation

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$

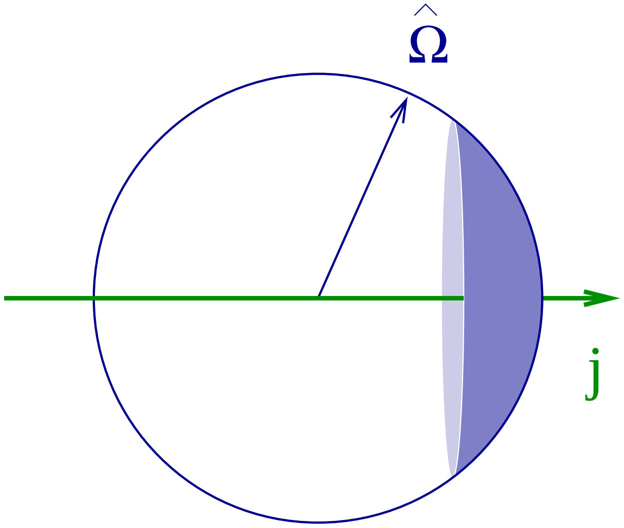
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2} v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$

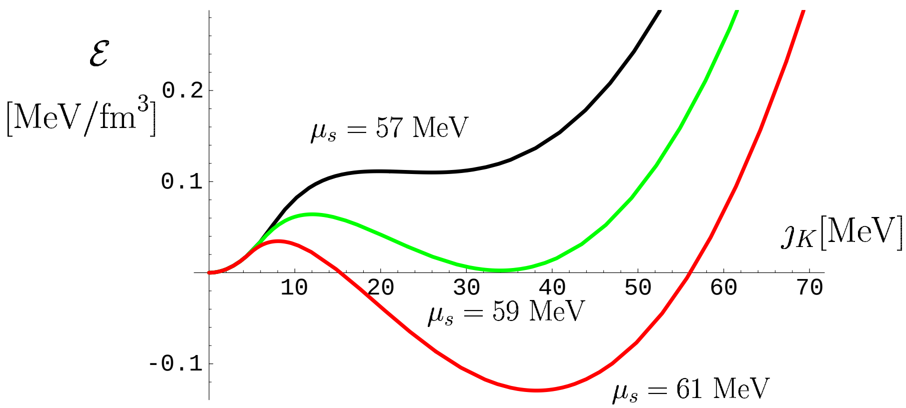
Fermion spectrum

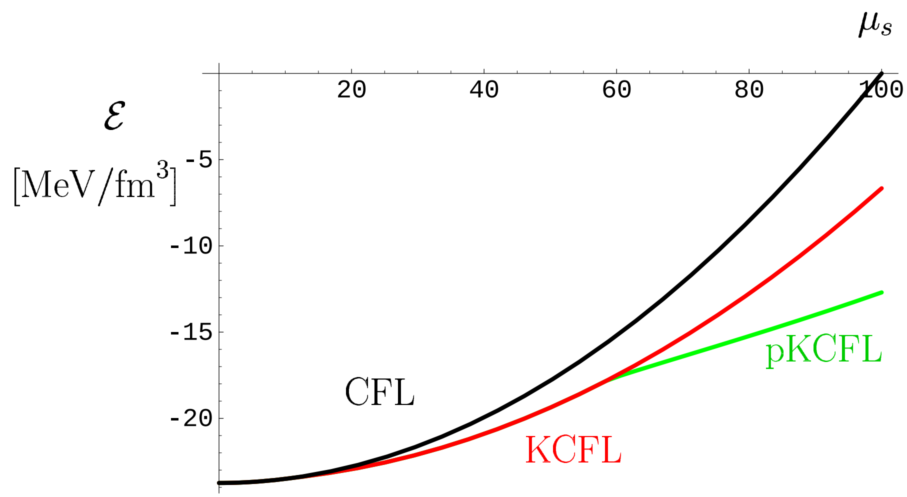
$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\; \omega_l \Theta(-\omega_l)$$



Energy Functional





$$\left.\frac{3\mu_s - 4\Delta}{\Delta}\right|_{crit} = a h_{crit} \qquad h_{crit} = -0.067 \qquad a = \frac{2}{15^2 c_\pi^2 v_\pi^4}$$

[Figures include baryon current $j_B = \alpha_B / \alpha_K j_K$]

Notes

No net current, meson current canceled by backflow of gapless modes

$$(\delta \mathcal{E})/(\delta \nabla \phi) = 0$$

Instability related to "chromomagnetic instability"

CFL phase: gluons carry $SU(3)_F$ quantum numbers

Similar instability exists in polarized cold atomic gases

mixture of atoms and molecules, Goldstone current condensation