Confinement and Chiral Symmetry Breaking from Monopoles and Duality

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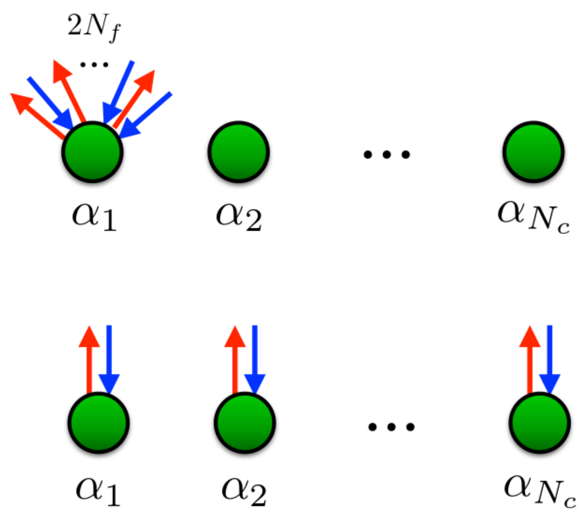
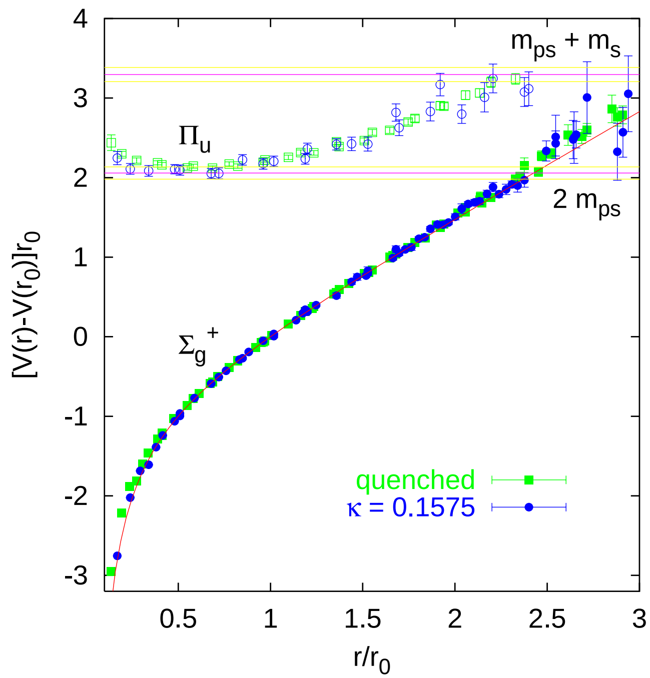


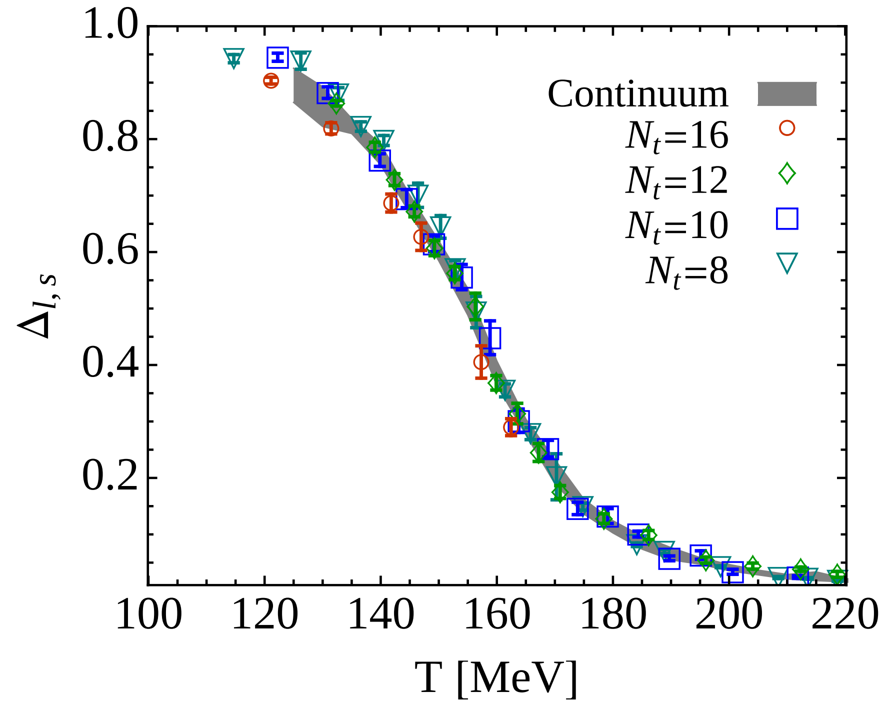
with A. Cherman and M. Unsal, PRL 117 (2016) 081601

and T.S., M.U., and E. Poppitz, JHEP 1303 (2013) 087

Motivation

Confinement and chiral symmetry breaking are well established





Goal: Find deformations of QCD, continuously connected to the full theory, that exhibit χSB and confinement in weak coupling.

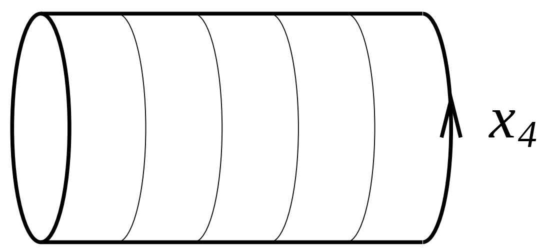
Bali et al. (2000), Borsanyi et al (2011)

Background: Confinement in Weak Coupling

Consider $SU(2)$ gauge theory with $N_f^{ad} = 1$ on $R^3 \times S_1$

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a$$

$$A^a_\mu(0) = A^a_\mu(L)$$
$$\lambda^a_\alpha(0) = \lambda^a_\alpha(L)$$



Large mass limit: Pure YM. Small mass limit: SUSY YM.

Small S_1 and m: Confinement can be studied using semi-classical methods, based on monopole-instantons, instantons, and bions.

Theory abelianizes. Low energy fields: Holonomy b and dual photon σ

Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x \, F\tilde{F}$$

2. Holonomy (eigenvalues q^α of Polyakov line at spatial infinity)

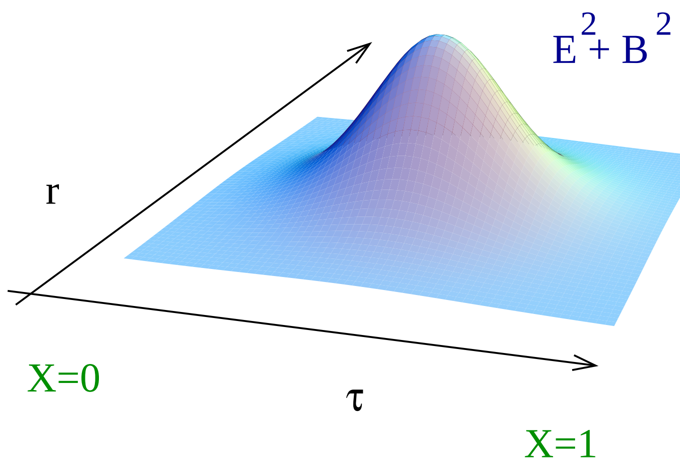
$$\langle \Omega(\vec{x}) \rangle = \left\langle \mathrm{Tr} \exp \left[i \int_0^\beta A_4 dx_4 \right] \right\rangle$$

3. Magnetic charges

$$Q_M^\alpha = \frac{1}{4\pi} \int d^2S \, \mathrm{Tr} \left[P^\alpha B \right]$$

Periodic instantons (calorons)

Instanton solution in R^4 can be extended to solution on $R^3 \times S^1$



$$Q_{top} = \pm 1$$

$$\Omega_\infty = 1 \quad Q_M^\alpha = 0$$

$SU(2)$ solution has $1+3+1+3=8$ bosonic zero modes

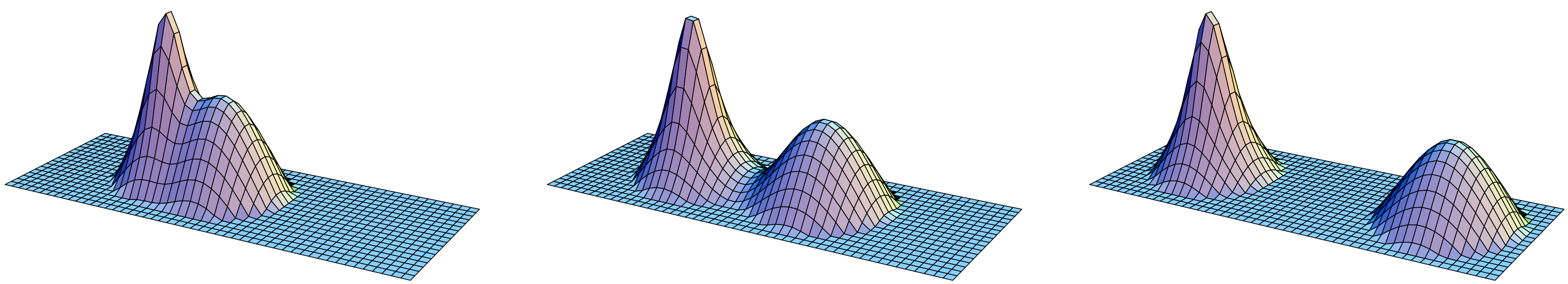
$$\int \frac{d\rho}{\rho^5} \int d^3x\, dx_4 \int dU\, e^{-2S_0} \qquad 2S_0 = \frac{8\pi^2}{g^2}$$

$4n_{adj}$ fermionic zero modes

$$\int d^2\zeta d^2\xi$$

Calorons at finite holonomy: monopole constituents

KvBLL (1998) construct calorons with non-trivial holonomy



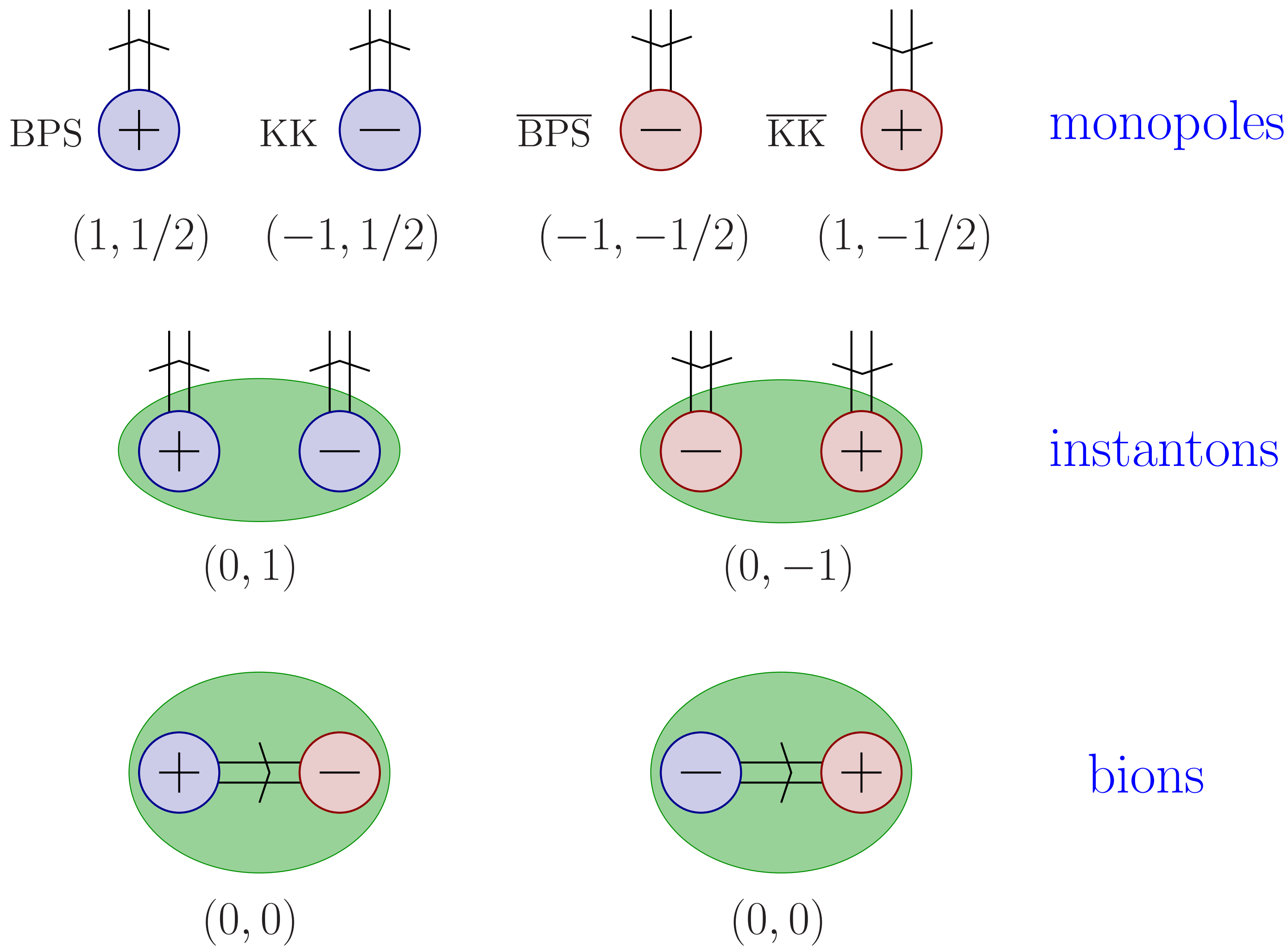
BPS and KK monopole constituents. Fractional topological charge, $1/2$ at center symmetric point.

$2\times(3+1)=8$ bosonic zero modes, 2×2 fermionic ZM.

$$\int d\phi_1 \int d^3x_1 \int d^2\zeta\, e^{-S_1} \int d\phi_2 \int d^3x_2 \int d^2\xi\, e^{-S_2}$$

Topological objects

$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

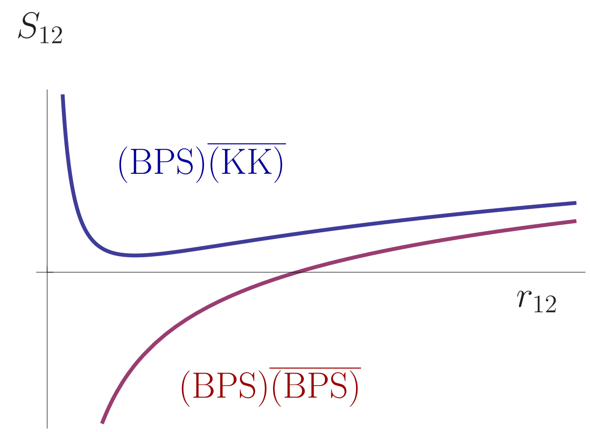
Effective potential

Instantons and monopoles: Exact solutions, but $V(b,\sigma) = 0$.

Bions: Approximate solutions

$$V_{BPS,\overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r \, e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r}(q_m^1 q_m^2 - q_b^1 q_b^2) + 4\log(r)$$



Saddle point integral after analytic continuation $g^2 \to -g^2$ (BZJ)

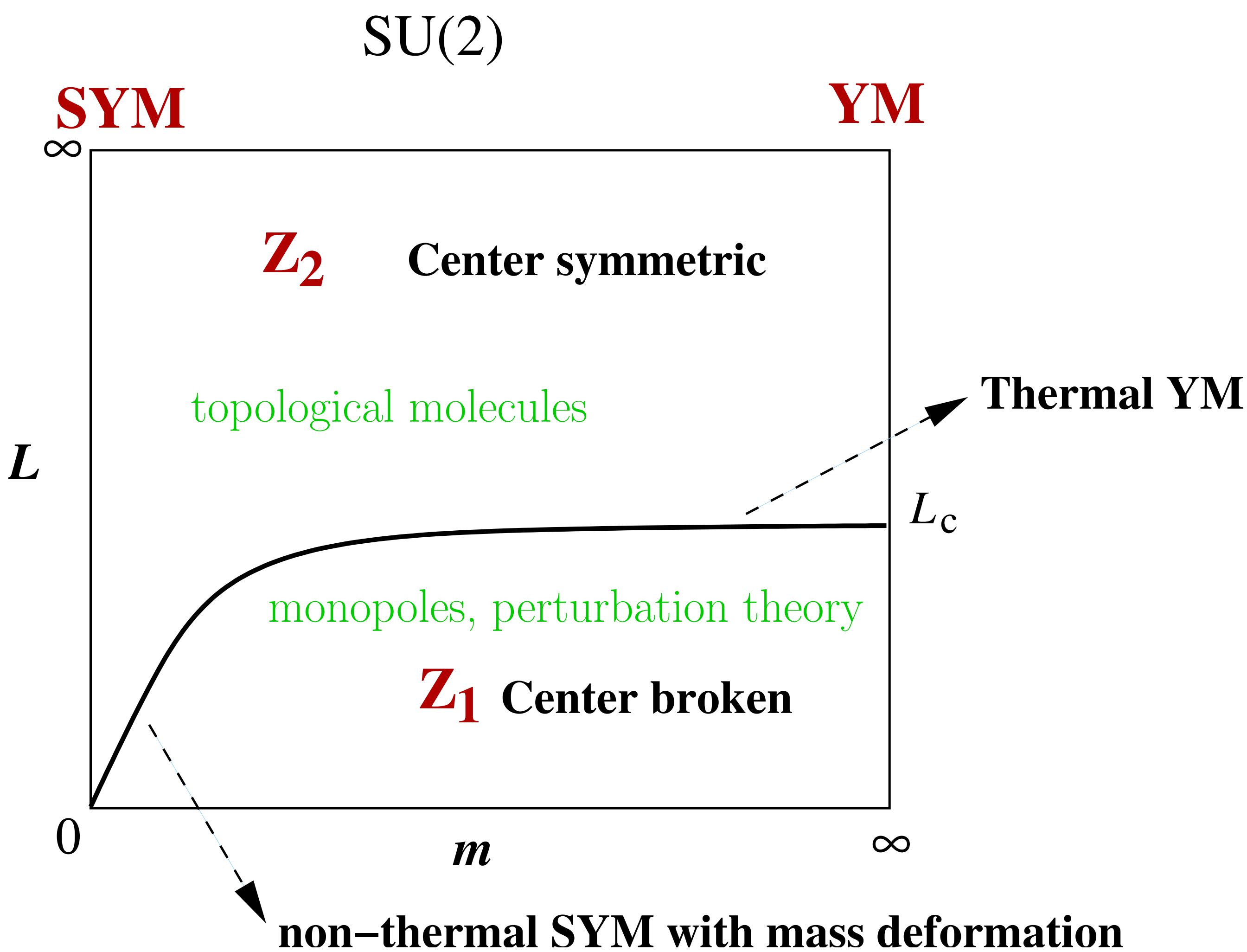
$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \left[\cosh\left(2\left(b - b_0\right)\right) - \cos(2\sigma)\right]$$

Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\to$ confinement)

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L-m plane



Direct calculation at $m = \infty$: See Shuryak's talk.

What about chiral symmetry breaking?

Original setup: One adjoint fermion, chiral symmetry is discrete.

$$\langle \bar{\lambda}\lambda \rangle \neq 0 \qquad Z_{2N_c} \to Z_2$$

Light fundamental fermions: Need strong coupling.

$$\mathcal{L} \sim G \det_{N_f}(\bar{\psi}_L \psi_R) + \mathrm{h.c.}$$

Heavy fundamental fermions: Study explicit breaking of Z_N center symmetry.

Role of Boundary Conditions

Consider flavor twisted boundary conditions

$$\psi(\tau+\beta) = \Omega_F \psi(\tau) \qquad \Omega_F = \mathrm{diag}(1, e^{2\pi i/N_f}, \ldots, e^{2\pi i(N_f-1)/N_f})$$

Flavor holonomy Ω_F has several interesting properties:

1. $N_f = N_c$: Respects Z_{N_c} center symmetry.

2. Large L: Breaks flavor symmetry, but in a controlled fashion.

3. Small L: New semi-classical picture of chiral symmetry breaking: Distributed zero modes and color-flavor transmutation.

Large L expectations

Flavor holonomy corresponds imaginary flavor (isospin) chemical potential $\tilde{\mu}_F \sim i/L$.

Can be studied using chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\nabla_\mu U \nabla^\mu U^\dagger\right] - B\mathrm{Tr}\left[MU + h.c\right]$$

with $\nabla_\mu U = \partial_\mu U + i[\tilde{\mu}_F T_F, U]$.

Consider $N_f = 2$ (isospin chemical potential)

$$m_{\pi^0}^2 = m_\pi^2 \qquad m_{\pi^\pm}^2 = m_\pi^2 + \tilde{\mu}_I^2$$

$N_f - 1$ exact Goldstone modes (m=0), others acquire gaps.

Small L theory: Perturbation theory

Consider center symmetric gauge holonomy (add double trace deformation). For $LN_c \lesssim \Lambda^{-1}$ theory abelianizes

$$SU(N_c) \to [U(1)]^{N_c-1}$$

Gapless (Cartan) gluons described by dual photon $\vec{\sigma}$

$$S = \frac{g^2}{8\pi^2 L} \int d^3x \, (\partial_\mu \vec{\sigma})^2$$

with $F^i_{\mu\nu} = \frac{g^2}{2\pi L} \epsilon_{\mu\nu\alpha} \partial^\alpha \sigma^i$.

Remain gapless to all orders in perturbation theory due to emergent shift symmetry $\vec{\sigma} \to \vec{\sigma} + \vec{\epsilon}$.

Small L theory: Semiclassical objects

Center symmetric background, no fermions: Instanton fractionalize into N_c constituents

$$\mathcal{M}_i \sim e^{-S_0} e^{i\vec{\alpha}_i \cdot \vec{\sigma}} \qquad S_0 = \frac{8\pi^2}{g^2 N_c} \qquad \vec{\alpha}_i \;\; SU(N_c) \text{ root vectors}$$

In the ground state these objects proliferate: The monopole-anti-monopole gas.

$$V(\vec{\sigma}) \sim m_W^3 e^{-S_0} \sum_i \cos(\vec{\alpha}_i \cdot \vec{\sigma})$$

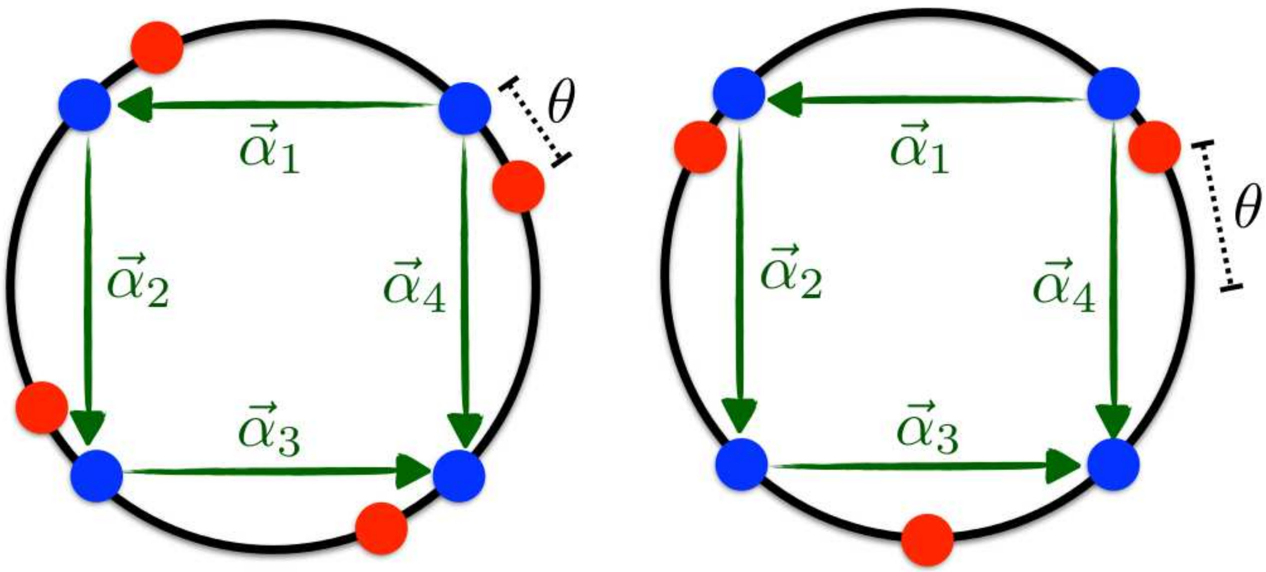
Mass gap for the dual photon, continuous shift symmetry broken.

Massless fermions: Take into account fermion zero modes.

Small L theory: Fermion zero modes

Many eigenvalue circles: Polyakov line Flavor holonomy

Instanton-monopoles θ flavor singlet twist



$N_c = N_f = 4$ $N_c = 4$ $N_f = 3$

Zero modes localize on monopoles jumping over flavor eigenvalues

Bruckmann, Nogradi, van Baal (2003); Moore et al. (2014)

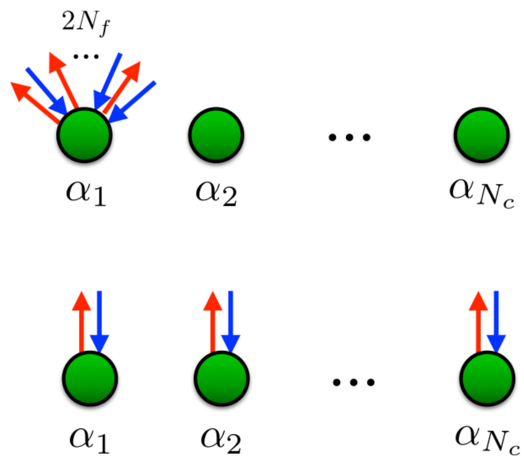
Two basic scenarios ($N_c = N_f$)

No flavor twist: Standard 't Hooft vertex carried by one monopole

$$\mathcal{M}_1 \sim e^{-S_0}\, e^{i\vec{\alpha}_1\cdot\vec{\sigma}}\, \det_F(\bar{\psi}_L^f \psi_R^g) \qquad \mathcal{M}_{i>1} \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}$$

Center symmetric flavor holonomy: Single flavor 't Hooft vertex carried by each monopole

$$\mathcal{M}_i \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\, (\bar{\psi}_L^i \psi_R^i)$$



trivial flavor holonomy

center symmetric holonomy

Spontaneous symmetry breaking

Unbroken symmetries of flavor twisted theory

$$[U(1)_J]^{N_c-1} \times [U(1)_V]^{N_f-1} \times [U(1)_A]^{N_f-1} \times U(1)_Q$$

Shift symmetry Exact flavor symmetry

Symmetries of monopole vertex

$$\mathcal{M}_i \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\, (\bar{\psi}^i_L \psi^i_R)$$

Preserves vectorial symmetry $[U(1)_V]^{N_f-1} \times U(1)_Q$. Breaks axial symmetry

$$[U(1)_A]^{N_f-1}: \qquad (\bar{\psi}^f_L \psi^f_R) \to e^{i\epsilon_i} (\bar{\psi}^i_L \psi^i_R)$$

Spontaneous symmetry breaking, continued

Monopole vertex is invariant provided $[U(1)_A]^{N_f-1}$ is combined with $[U(1)_J]^{N_c-1}$ shift symmetry

$$[\tilde{U}(1)_A]^{N_f-1}: \quad \begin{cases} (\bar{\psi}_L^f \psi_R^f) & \rightarrow \quad e^{i\epsilon_i}(\bar{\psi}_L^i \psi_R^i) \\ e^{i\vec{\alpha}_i\cdot\vec{\sigma}} & \rightarrow \quad e^{-i\epsilon_i} e^{i\vec{\alpha}_i\cdot\vec{\sigma}} \end{cases}$$

Ground state $\langle e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\rangle \rightarrow 1$. Breaks

$$[U(1)_V]^{N_f-1} \times [\tilde{U}(1)_A]^{N_f-1} \rightarrow [U(1)_V]^{N_f-1}$$

For $m=0$ the ground state is degenerate. Massless Goldstone boson

$$S_\sigma = L\int d^3x \left\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] - B\,\mathrm{Tr}\left[M\Sigma + h.c.\right] \right\}$$

Microscopically $\Sigma = e^{i\Pi/f_\pi}$ with $\Pi = \pi^a T^a$ and $\pi^a = \frac{g}{2\pi L}\sigma^a$

Color-flavor transmutation

Chiral Lagrangian

Chiral lagrangian has calculable coefficients

$$S_\sigma = L\int d^3x \left\{ \frac{f_\pi^2}{4}\,\mathrm{Tr}\left[\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right] - B\,\mathrm{Tr}\left[M\Sigma + h.c.\right]\right\}$$

$$f_\pi^2 = \left(\frac{g}{\sqrt{6}\pi L}\right)^2 = \frac{N_c\lambda m_W^2}{24\pi^2}$$

$$B = -\frac{1}{2}\langle\bar\psi\psi\rangle \sim m_W^{-3} e^{-\frac{8\pi^2}{\lambda}}$$

Also note: VEV of monopole operator can be viewed as effective constituent quark mass

$$m_Q \sim m_W e^{-\frac{8\pi^2}{\lambda}}$$

Conclusions and Outlook

Calculable mechanism for chiral symmetry breaking and confinement in compactified versions of QCD.

Results consistent with continuity between large L, m (full QCD) and small L, m theory.

Mechanism based on monopole instantons and color flavor transmutation.

Study: Relation between χSB and confinement? Chiral phase transition?