(Quasi) Quarks and (Quasi) Baryons in QCD at High Density

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Motivation

There is a successful effective theory of fermionic many body systems

Landau Fermi-Liquid Theory

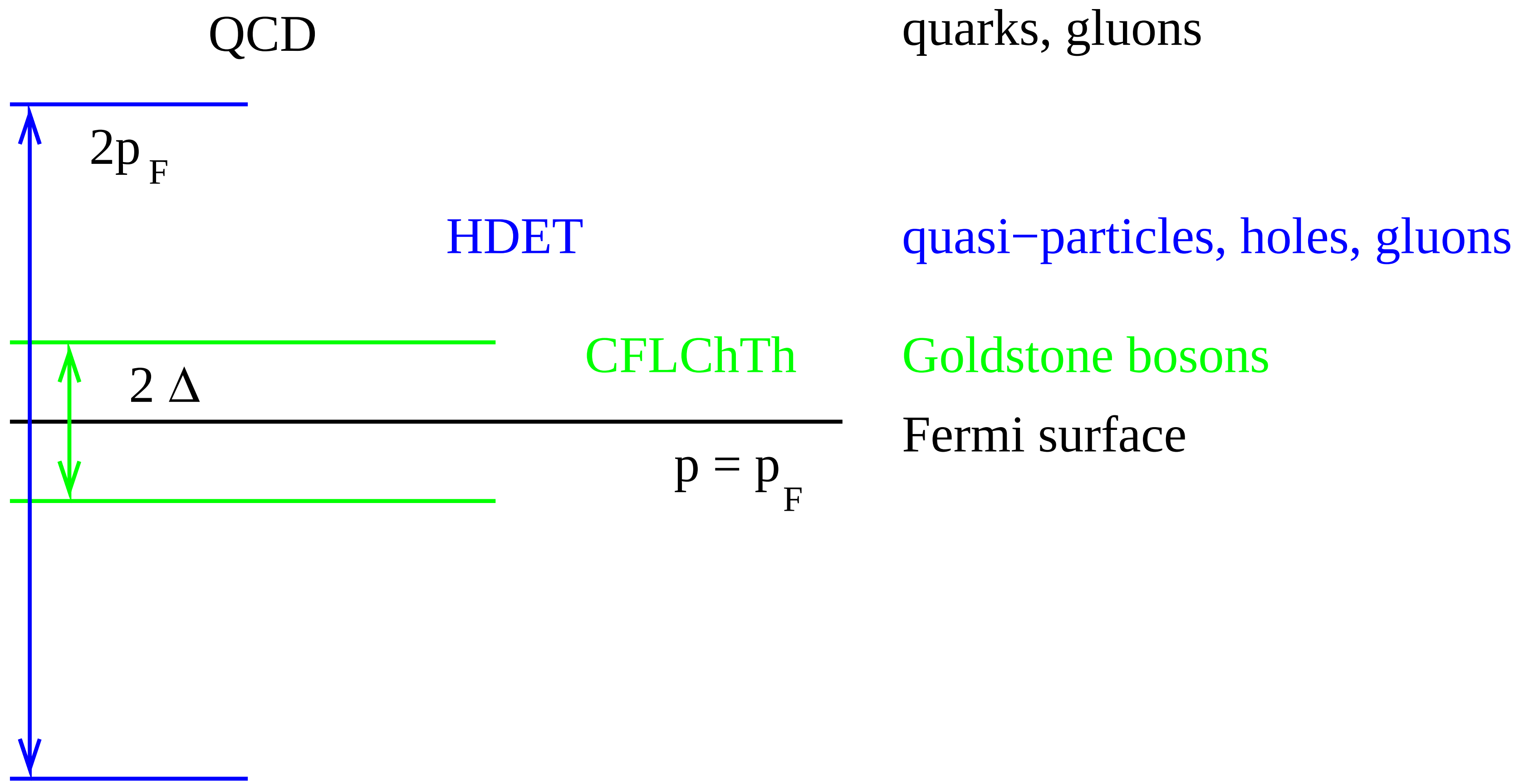
FLT theory: Quasi-particles near the Fermi surface. Interactions characterized by Fermi-liquid parameters.

Predicts collective modes, thermodynamics, transport, ...

Gauge Theories: Unscreened long range forces

Does a quasi-particle EFT exist?

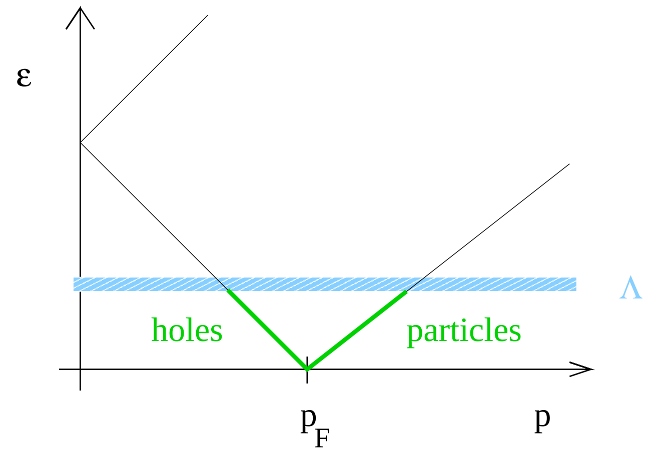
Effective Field Theories



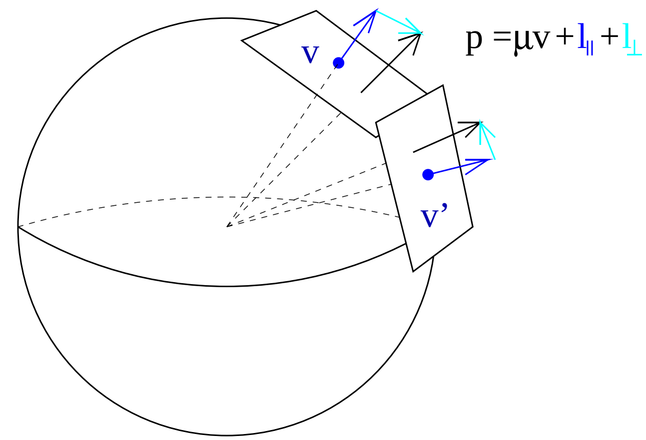
High Density Effective Theory

Quasi-particles (holes)

$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$



Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v \cdot x} \left(\frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$

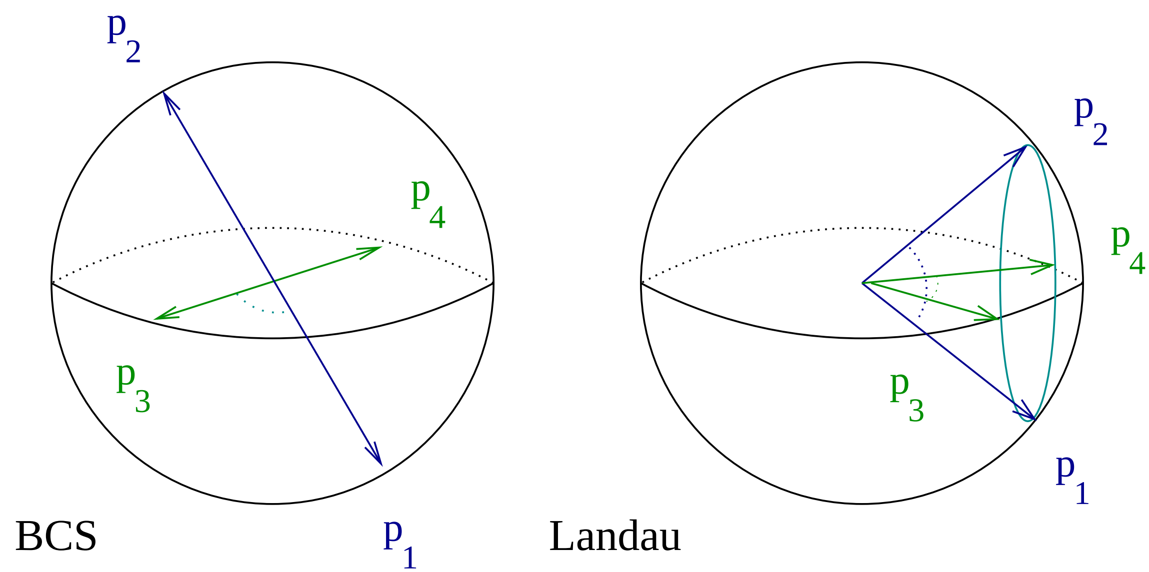
Effective lagrangian for ψ_{v+}

$$\mathcal{L} = \sum_v \psi_v^\dagger (iv \cdot D) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + O(1/\mu)$$

Four Quark Operators

quark-quark scattering

$(v_1, v_2) \to (v_3, v_4)$



$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'}(\vec{v}\cdot\vec{v}')(\psi_v \Gamma \psi_{-v})(\psi^\dagger_{v'} \Gamma' \psi^\dagger_{-v'}),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'}(\vec{v}\cdot\vec{v}')(\psi_v \Gamma \psi_{v'})(\psi^\dagger_{\tilde{v}} \Gamma' \psi^\dagger_{\tilde{v}'})$$

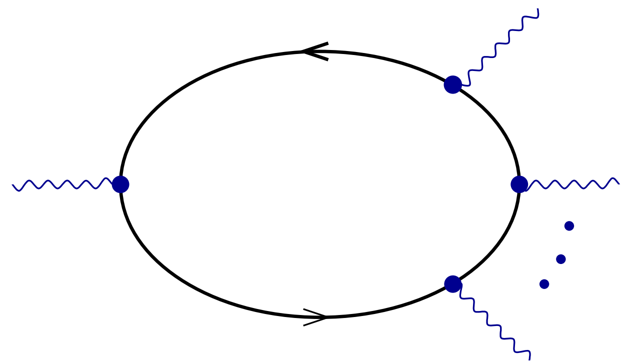
perturbative matching: $f_0^s = \frac{C_F}{4N_cN_f}\frac{g^2}{p_F^2}, \quad f_i^s = 0 \ \ (i>1)$

Effective Theory for $l \sim g\mu$

Integrate out hard dense loops

$$\mathcal{L} = \psi_v^\dagger (iv \cdot D)\psi_v + f_0^s (\psi_v^\dagger \psi_v)(\psi_{v'}^\dagger \psi_{v'}) - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G_{\mu\alpha}^a \frac{v^\alpha v^\beta}{(v \cdot D)^2} G_{\mu\beta}^b$$



Transverse gauge boson propagator

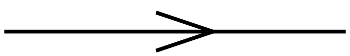
$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0| / |\vec{k}|},$$

Scaling of gluon momenta

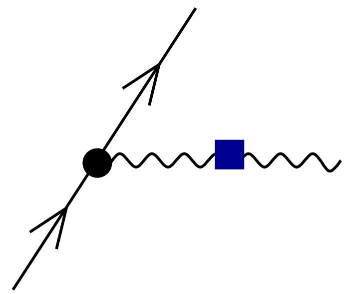
$$|\vec{k}| \sim k_0^{1/3} \eta^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Effective Theory for $l < g\mu$

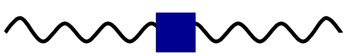
Scaling: $(p_0, p_{||}, p_\perp) \sim (l, l^{1/3}, l^{2/3})$



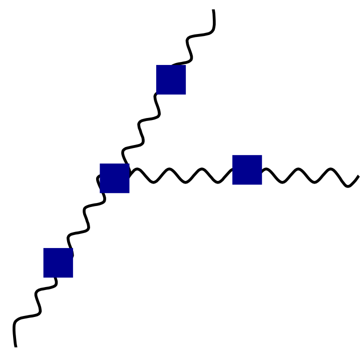
$$\frac{i}{p_{||} + \frac{p_\perp^2}{2\mu} + i\epsilon sgn(p_0)}$$



$$igv_i$$



$$\frac{i\delta_{ij}k_\perp}{k_\perp^3 - i\eta k_0}$$



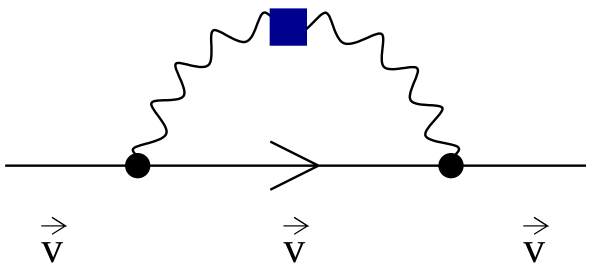
$$ig^3\Gamma_{ijk}$$

Low energy expansion contains fractional powers $\omega^{1/3}$

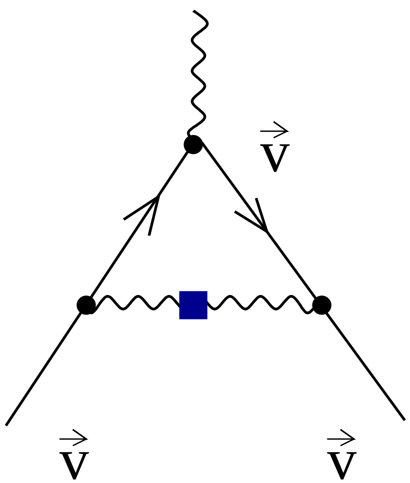
Quasi-Quarks at Large Density:

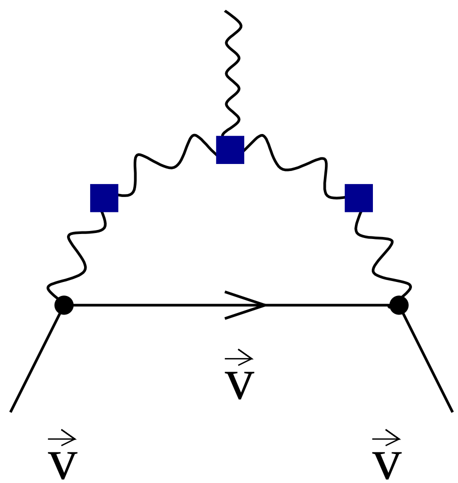
Non-Fermi Liquid Effects

Loop Corrections



$$\Sigma(\omega) \simeq \frac{g^2 C_F}{12\pi^2}\,\omega \log\left(\frac{\Lambda}{\omega}\right)$$





$$\Gamma_\alpha = \frac{g^3 C_F v_\alpha}{12\pi^2} \log\left(\frac{\Lambda}{\omega}\right)$$ time like

$$\Gamma_\alpha = O(g^3)$$ space like

"Migdal's Theorem" for QCD

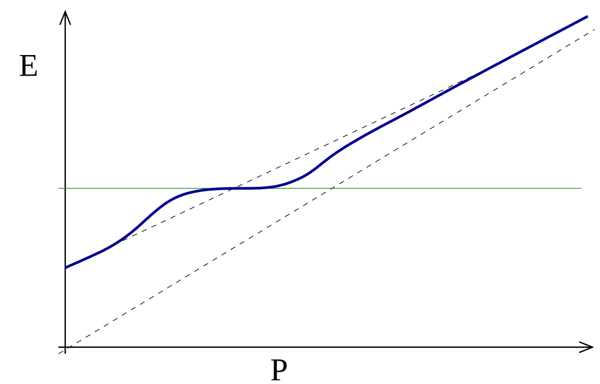
self energy non-perturbative for $\omega \sim \Lambda \exp(-9\pi^2/g^2)$?

$\gamma = (4\alpha/)(9\pi) \ll 1$: RG equation can be solved exactly

$$S^{-1}(\omega, l) = \omega \left(1 + \gamma \log\left(\frac{\Lambda}{\omega}\right)\right) - v_F l \quad \text{no } \alpha^2 \log^2(\omega), .. \text{ terms}$$

quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$

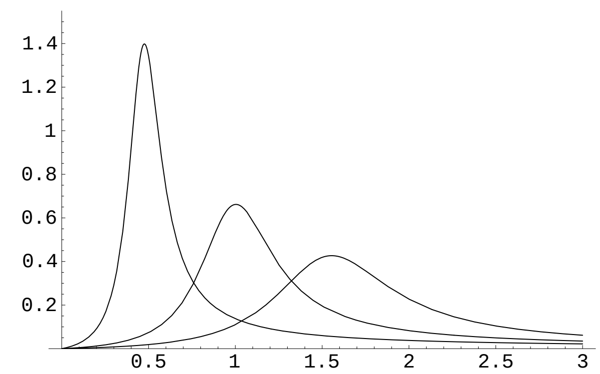


anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$

enhanced corrections to the gap

$$\log(\mu/\Delta) = \log(\mu/\Delta_0)(1 - O(\gamma g))$$



Remnants of Fermi Liquid Behavior

Perturbative corrections to matching QCD→HDET

$$v_F = 1 - \frac{C_F \alpha_s}{2\pi}, \qquad \delta\mu = \frac{C_F \alpha_s}{\pi}\mu.$$

These coefficients satisfy Fermi-liquid relations (Baym&Chin)

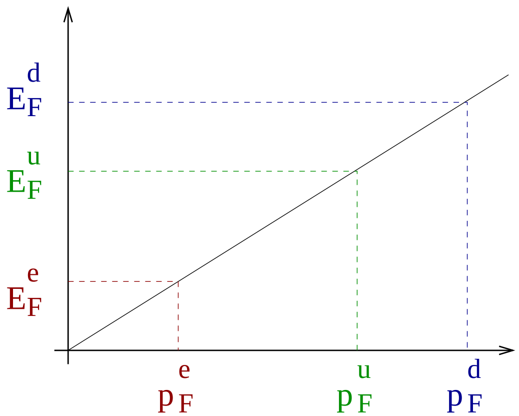
$$v_F = \frac{p_F}{\mu} - \frac{N f_1^s}{3} \qquad p_F = \mu\left(1 - \frac{N f_0^s}{2}\right)$$

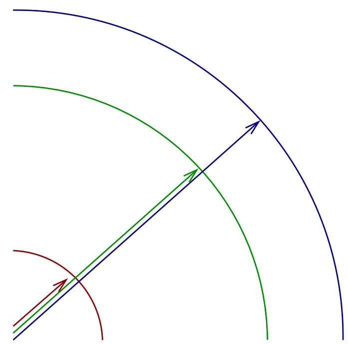
Fermi-liquid parameters

$$f_0^s = \frac{C_F}{4 N_c N_f}\frac{g^2}{p_F^2} \qquad f_1^s = 0$$

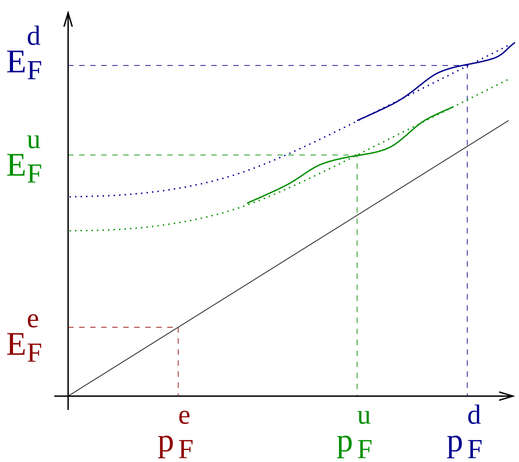
Application: Neutrino Emission

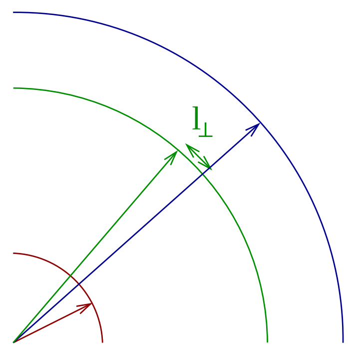
Quark Direct URCA: $d \to u + e^- + \bar{\nu},\ u + e^- \to d + \bar{\nu}$





$$\epsilon \sim G_F T^7$$





$$\epsilon \sim G_F \alpha_s^3 T^6 \log^2(T)$$

Quasi-Baryons at Large Density:

CFL Phase

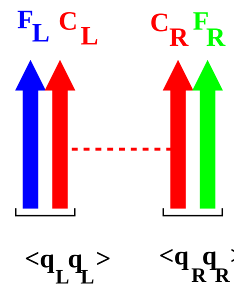
CFL Phase

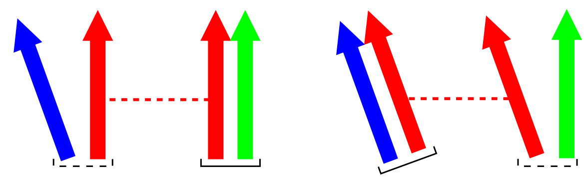
Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$



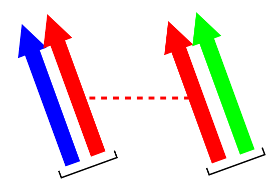


Rotate left flavor

Compensate by rotating color

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$



... have to rotate right flavor also !

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

All quarks and gluons acquire a gap

EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\} + (L\leftrightarrow R, X\leftrightarrow Y)$$

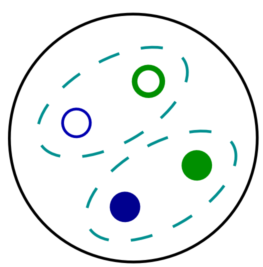
$$\psi_L \to L\psi_L C^T,\ X\to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = 1\!\!1$$

Quark loops generate a kinetic term for X, Y

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

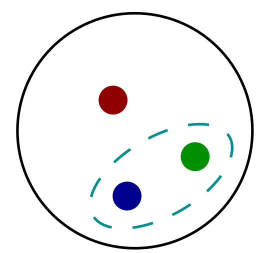
$$\Sigma = XY^\dagger$$

[8]+[1] GBs



$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons



Effective theory: (CFL) baryon chiral perturbation theory

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2 \mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\ &\quad + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left\{\mathcal{A}_\mu, N\right\}\right) \\ &\quad - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}\end{aligned}$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

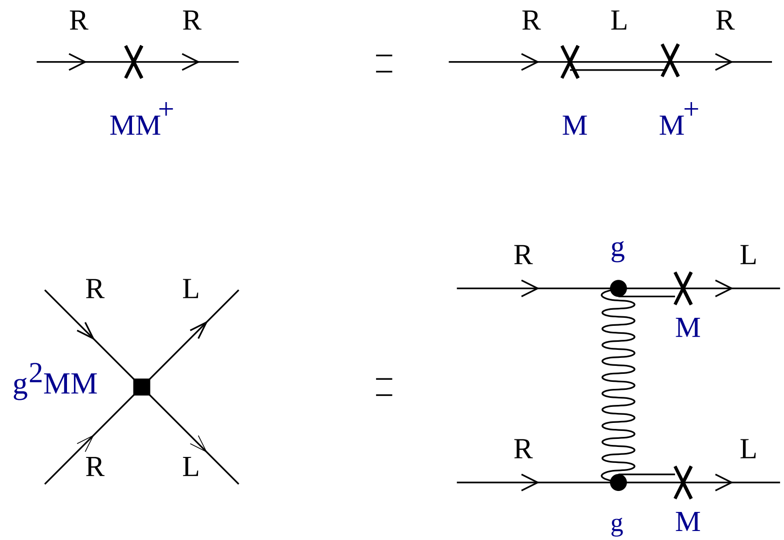
$$\begin{aligned}\mathcal{V}_\mu &= -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \\ \mathcal{A}_\mu &= -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi\end{aligned}$$

$$f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L$$

$$+ \frac{C}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L)(\psi_R^\dagger M \lambda^a \psi_L)$$



mass corrections to FL parameters $\hat{\mu}$ and $F^0(++ \to --)$

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(X_L\Sigma X_R\Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

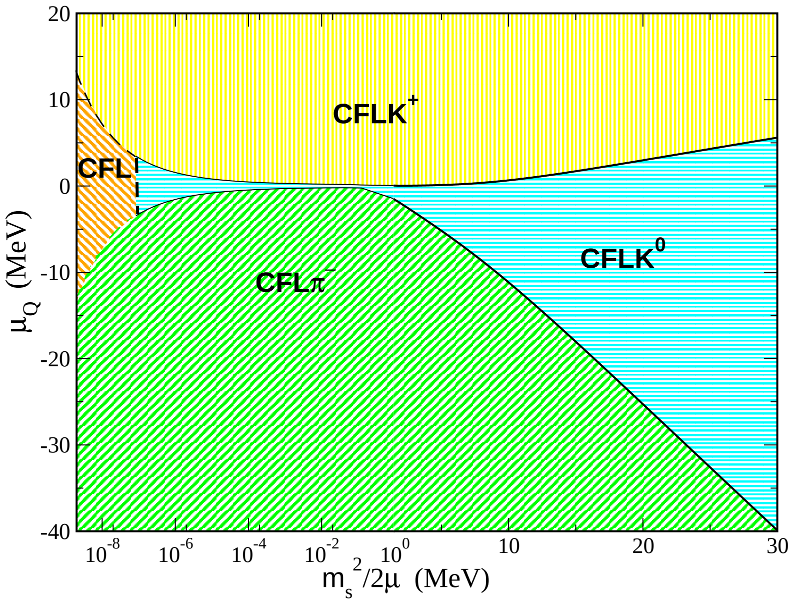
$$V(\Sigma_0) \equiv min$$

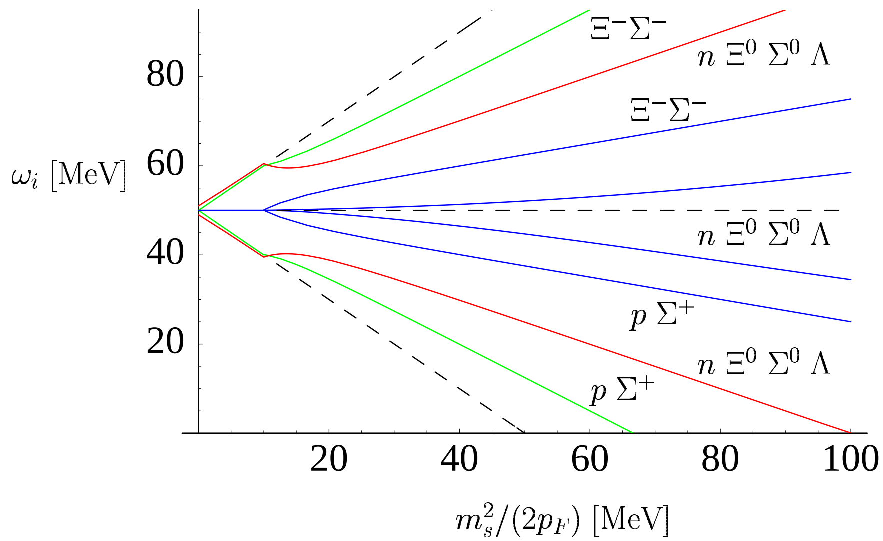
Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger iv^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger\gamma_5\rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi\frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger\frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





meson condensation: CFLK

reliable: yes!

gapless modes? (gCFLK)

reliable: not clear yet

Summary

EFT/RG methods provide powerful tools

phase structure and spectrum at large density

Normal phase: Non-Fermi liquid behavior due to unscreened transverse gauge bosons

modified quasi-particles

but: no break-down of perturbation theory

Superfluid phase: effective chiral theory with calculable coefficients

kaon condensation, possibility of gapless modes