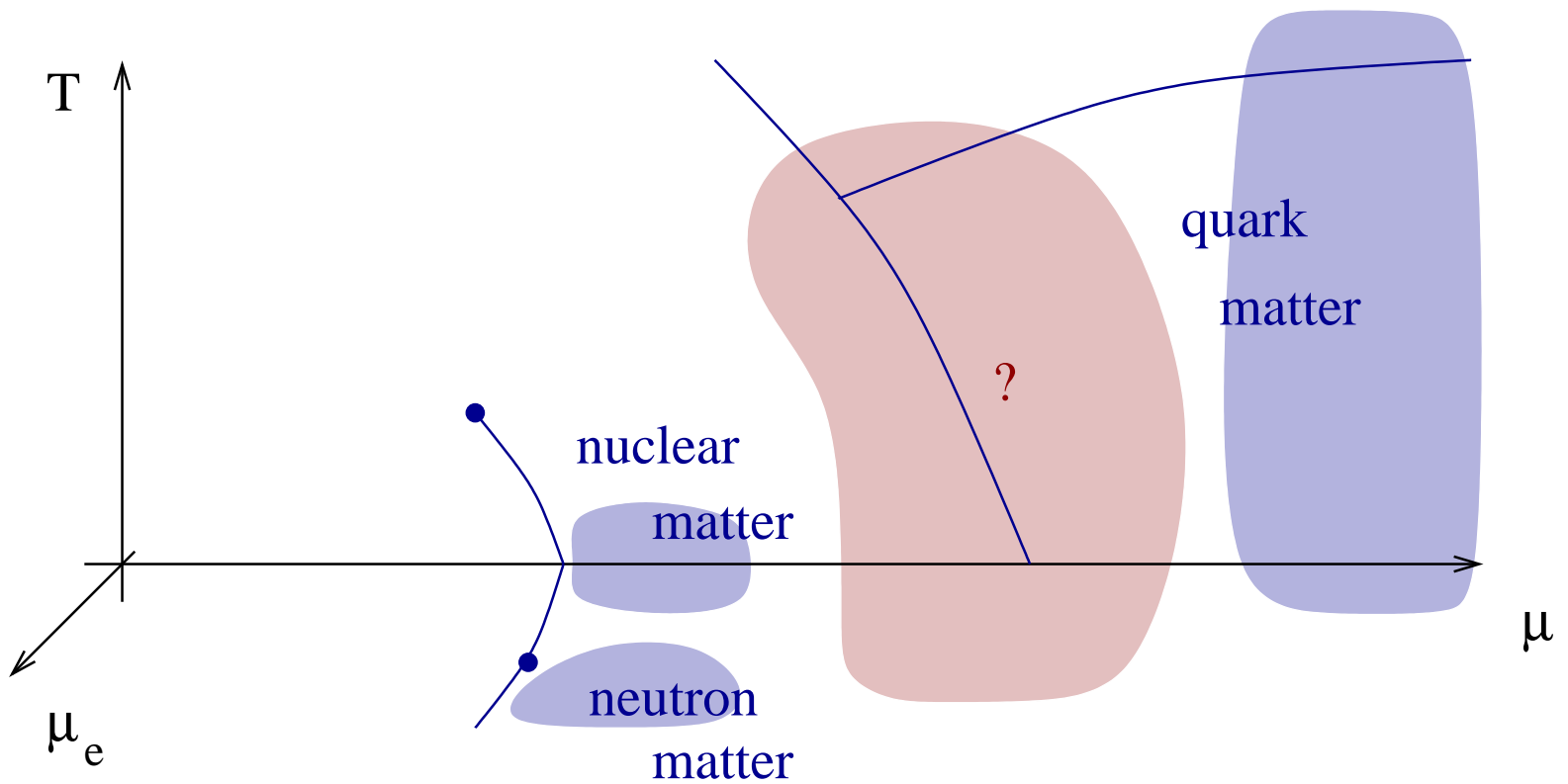# Phase Structure of (Very) Dense Quark Matter

Thomas Schaefer

North Carolina State

Schematic Phase Diagram

T

μ

μ_e

quark

matter

nuclear

matter

?

neutron

matter

Dense Baryonic Matter

Low Density

     Equation of state of nuclear/neutron matter

     Neutron/proton superfluidity, pairing gaps

Moderate Density

     Pion/kaon condensation, hyperon matter

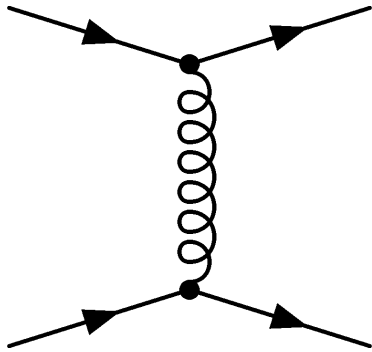   Pairing, equation of state at high density

High Density

     Quark matter

     Color superconductivity, Color-flavor-locking

Pairing in Quark Matter
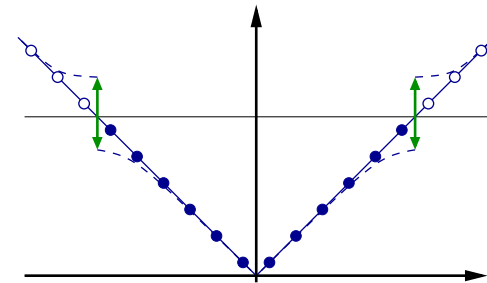
QQ scattering in perturbative QCD

$$(\vec{T})_{ac}(\vec{T})_{bd} = -\frac{1}{3}(\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}) + \frac{1}{6}(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc})$$

$$[3] \quad \times \quad [3] \quad = \quad [\bar{3}] \quad + \quad [6]$$

Fermi surface: pairing instability in weak coupling

$$\Phi_{ij}^{ab,\alpha\beta} = \langle \psi_i^{a,\alpha} C \psi_j^{b,\beta} \rangle$$

Phase structure in perturbation theory

$$\text{Minimize } \Omega(\Phi_{ij}^{ab,\alpha\beta})$$

In practice: consider residual symmetries

Superconductivity

Thermodynamic potential

$$\Omega \quad = \quad \bigcirc \quad + \quad \bigcirc \quad + \quad \bigcirc \quad + \ldots$$

Variational principle $\delta\Omega/\delta\Phi$ gives gap equation

$$= \frac{g^2}{18\pi^2} \int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0 - q_0|}\right) \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

$\Lambda_{BCS} = c_i 256\pi^4 \mu g^{-5}$ determined by symmetries of order parameter

$$\Omega = \frac{\mu^2}{4\pi^2} \sum_i \Delta_i^2 \qquad \Delta_i = c_i \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

Remarks

Behavior of perturbative expansion

$$\Delta = \mu g^{-5} \exp\left(-\frac{c}{g}\right)\left(c_0 + c_1 g \log(g) + c_2 g + O(g^{n/3}\log^m(g))\right)$$

Also: non-perturbative effects ($\mu \sim 1$ GeV)

$$\Delta \sim \mu \exp\left(-\frac{8\pi^2}{g^2}\right)$$

Magnitude of gap quite uncertain

But: Phase Structure not sensitive to interaction

$N_f = 1$: Color-Spin-Locking

$N_f = 1$, color-anti-symmetric: spin-1 condensate

$$\langle \psi^b C \gamma_i \psi^c \rangle = \Phi_i^a \epsilon^{abc}$$

Ground state: Color-Spin-Locking (CSL): $\Phi_i^a \sim \delta_i^a$

$SU(3)_c \times SO(3) \to SO(3)$: rotational symmetry

$U(1)_B$ broken: massless Goldstone boson

$(s=3/2) + (s=1/2)$ gapped fermions

$N_f = 2$: 2SC Phase

$N_f = 2$, color-anti-symmetric: spin-0 BCS condensate

$$\langle \psi_i^b C \gamma_5 \psi_j^c \rangle = \Phi^a \epsilon^{abc} \epsilon_{ij}$$

Order parameter $\phi^a \sim \delta^{a3}$ breaks $SU(3)_c \to SU(2)$

$SU(2)_L \times SU(2)_R$ unbroken

4 gapped, 2 (almost) gapless fermions

light $U(1)_A$ Goldstone boson

$SU(2)$ confined $(\Lambda_{conf} \ll \Delta)$

$N_f = 3$: CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \; \epsilon^{abI} \epsilon_{ijI}$$

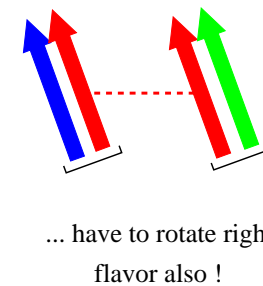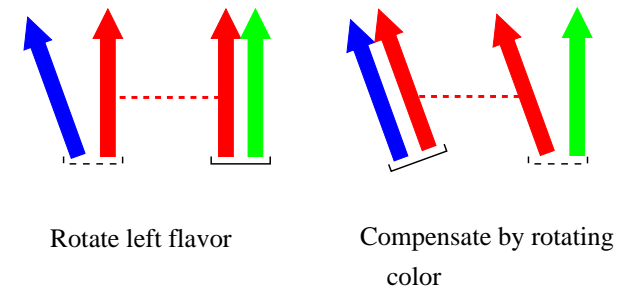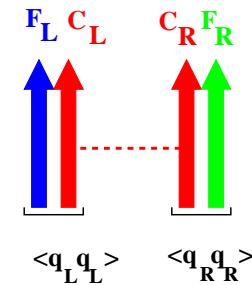$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C$$
$$\times \, U(1) \rightarrow SU(3)_{C+F}$$

All quarks and gluons acquire a gap

$[8] + [1]$ fermions, Q integer

$\langle q_L q_L \rangle$ $\langle q_R q_R \rangle$

Rotate left flavor   Compensate by rotating color

... have to rotate right flavor also !

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$
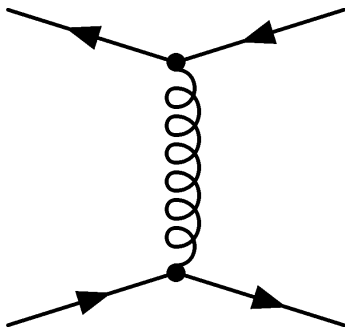
QCD with many flavors/colors

QCD with $N_f > 3$ flavors: CFL-like, fully gapped, phases

$N_f = 4 \quad SU(4)_L \times SU(4)_R \times U(1)_V \rightarrow [SU(2)_V]^2 \times SU(2)_A$

$N_f = 5 \quad SU(5)_L \times SU(5)_R \times U(1)_V \rightarrow SU(2)_V$

$N_f = 6 \quad SU(6)_L \times SU(6)_R \times U(1)_V \rightarrow SU(3)_V \times U(1)_V \times U(1)_A$

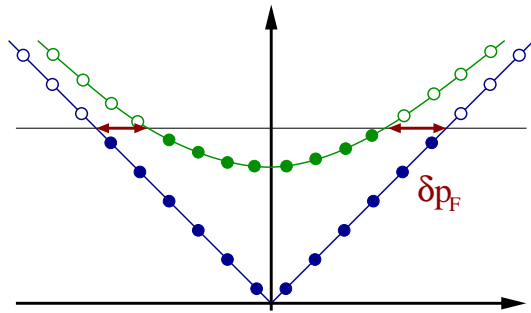$N_c \rightarrow \infty$: qq pairing suppressed, qq^{-1} chiral density wave

$$\langle \bar{q}(x) q(x) \rangle \sim \Sigma \, e^{2i\vec{q}_F \cdot \vec{x}}$$

Requires very large $N_c > 1000$

Towards the real world: Non-zero strange quark mass

Have $m_s > m_u, m_d$: Unequal Fermi surfaces

$$\delta p_F \simeq \frac{m_s^2}{2p_F}$$

Also: If $p_F^s < p_F^{u,d}$ have unequal densities

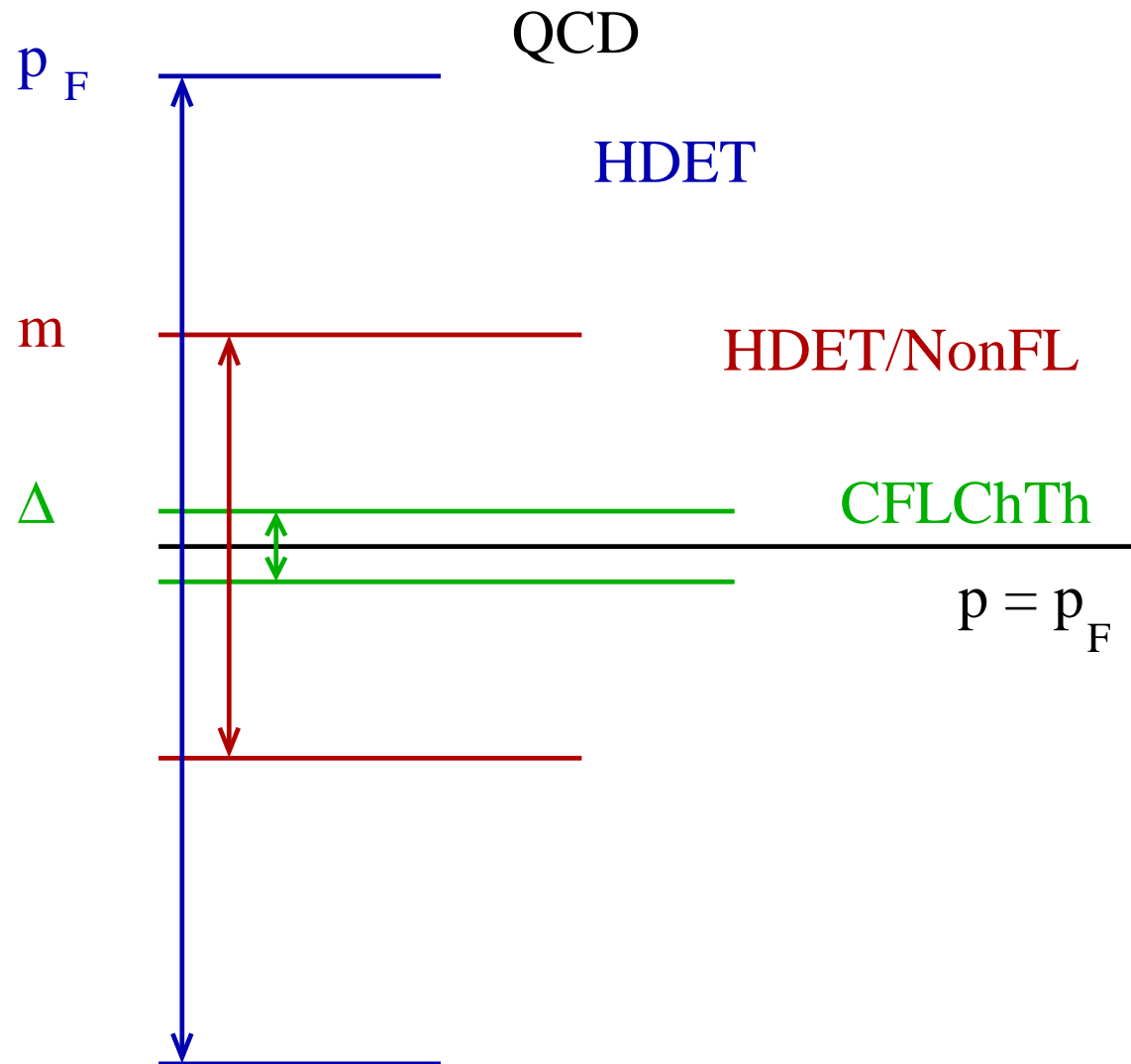Charge neutrality not automatic

Strategy

Consider $N_f = 3$ at $\mu \gg \Lambda_{QCD}$ (CFL phase)

Study response to $m_s \neq 0$

Constrained by chiral symmetry

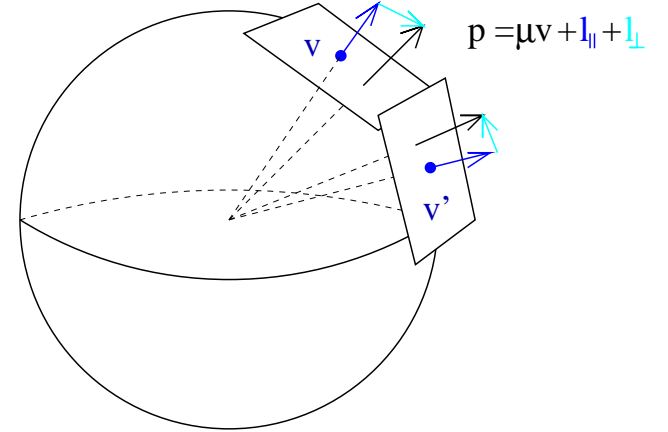# Very Dense Matter: Effective Field Theories

High Density Effective Theory

Effective field theory on v-patches

$$\psi_{v\pm} = e^{-i\mu v \cdot x} \left(\frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$
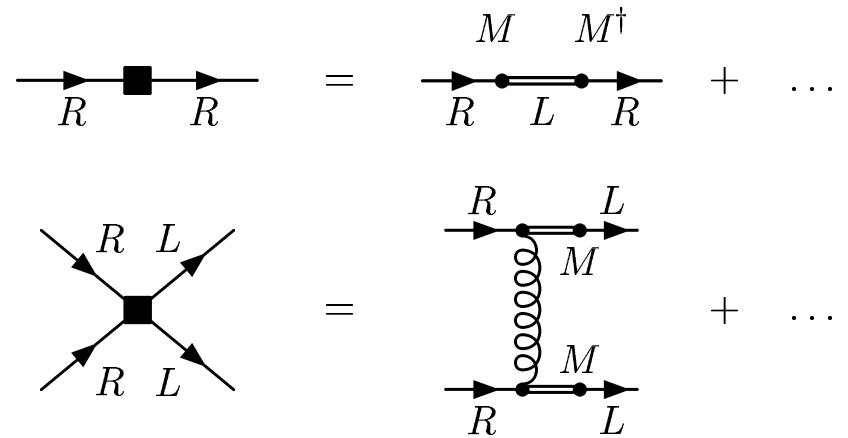
Effective lagrangian for $p_0 < m$

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G_{\mu\alpha}^a \frac{v^\alpha v^\beta}{(v \cdot D)^2} G_{\mu\beta}^b$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L$$

$$+ \frac{V_M^0}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L)(\psi_R^\dagger M \lambda^a \psi_L)$$

mass corrections to FL parameters $\hat{\mu}_{L,R}$ and $V^0(RR \to LL)$

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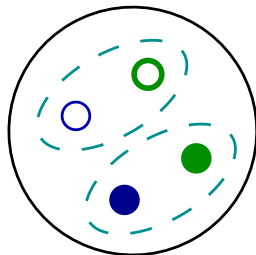
EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv \cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger \psi_L X^\dagger \psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger \psi_L\right)\right]^2\right\}$$

$$+ (L \leftrightarrow R, X \leftrightarrow Y)$$

$$\psi_L \to L\psi_L C^T,\ X \to LXC^T,\quad \langle X \rangle = \langle Y \rangle = \mathbb{1}$$
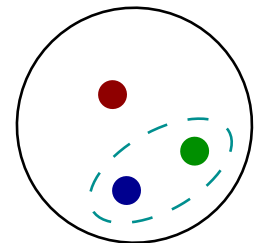
Quark loops generate a kinetic term for X, Y

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

$$\Sigma = XY^\dagger$$

[8]+[1] GBs

$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons

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Effective theory: (CFL) baryon chiral perturbation theory

$$
\begin{aligned}
\mathcal{L} = \ & \frac{f_\pi^2}{4} \left\{ \mathrm{Tr}\left(\nabla_0 \Sigma \nabla_0 \Sigma^\dagger\right) - v_\pi^2 \mathrm{Tr}\left(\nabla_i \Sigma \nabla_i \Sigma^\dagger\right) \right\} \\
& + A \left\{ \left[\mathrm{Tr}\left(M\Sigma\right)\right]^2 - \mathrm{Tr}\left(M\Sigma M\Sigma\right) + h.c. \right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \{\mathcal{A}_\mu, N\}\right) \\
& - F\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 [\mathcal{A}_\mu, N]\right) + \frac{\Delta}{2}\left\{ \mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2 \right\}
\end{aligned}
$$

$$
\nabla_0 \Sigma = \partial_0 \Sigma + i\hat{\mu}_L \Sigma - i\Sigma\hat{\mu}_R
$$

$$
D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]
$$

$$
f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \quad v_\pi^2 = \frac{1}{3} \quad A = \frac{3\Delta^2}{4\pi^2} \quad D = F = \frac{1}{2}
$$

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2} \mathrm{Tr} \left(\hat{\mu}_L \Sigma \hat{\mu}_R \Sigma^\dagger \right) - A \mathrm{Tr}(M\Sigma^\dagger) - B_1 \left[\mathrm{Tr}(M\Sigma^\dagger) \right]^2 + \ldots$$

$$V(\Sigma_0) \equiv min$$

Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr} \left(N^\dagger i v^\mu D_\mu N \right) + \mathrm{Tr} \left(N^\dagger \gamma_5 \rho_A N \right) + \frac{\Delta}{2} \left\{ \mathrm{Tr} \left(NN \right) - \left[\mathrm{Tr} \left(N \right) \right]^2 \right\},$$

$$\rho_{V,A} = \frac{1}{2} \left\{ \xi \hat{\mu}_L \xi^\dagger \pm \xi^\dagger \hat{\mu}_R \xi \right\} \qquad \xi = \sqrt{\Sigma_0}$$
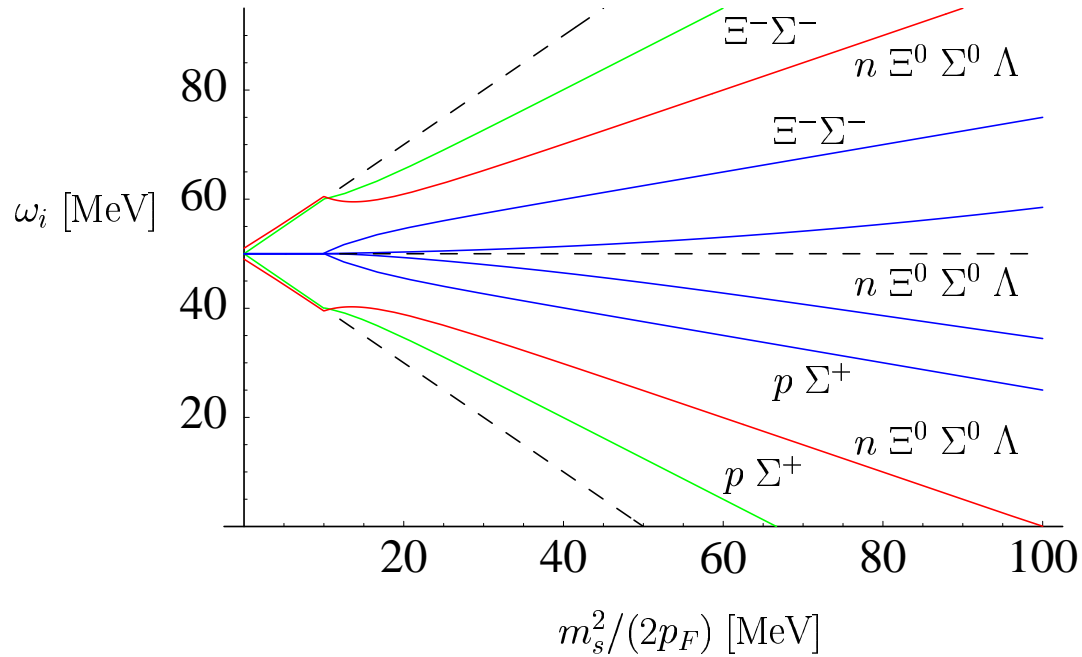
Phase Structure of CFL Phase

$$m_s^{crit} \sim 3.03 \, m_d^{1/3} \Delta^{2/3}$$

$$m^* \sim 0.017 \, \alpha_s^{4/3} \Delta$$

QCD realization of s-wave meson condensation

Driven by strangeness oversaturation of CFL state

Fermion Spectrum

$$m_s^{crit} \sim (8\mu\Delta/3)^{1/2}$$

gapless fermion modes (gCFLK)

(chromomagnetic) instabilities ?

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$
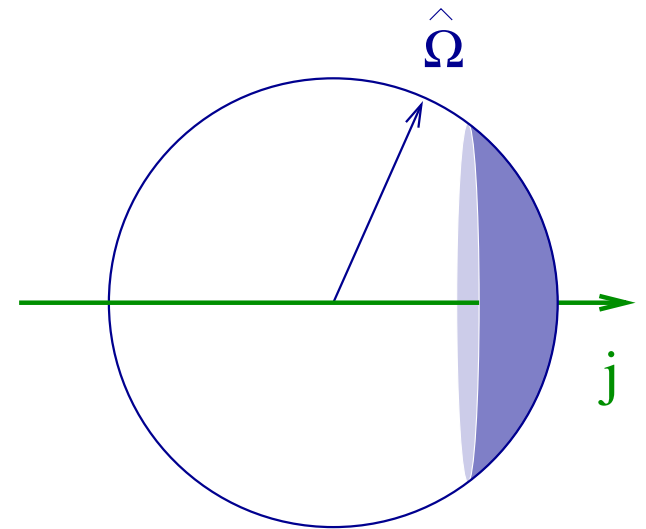
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2}v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$
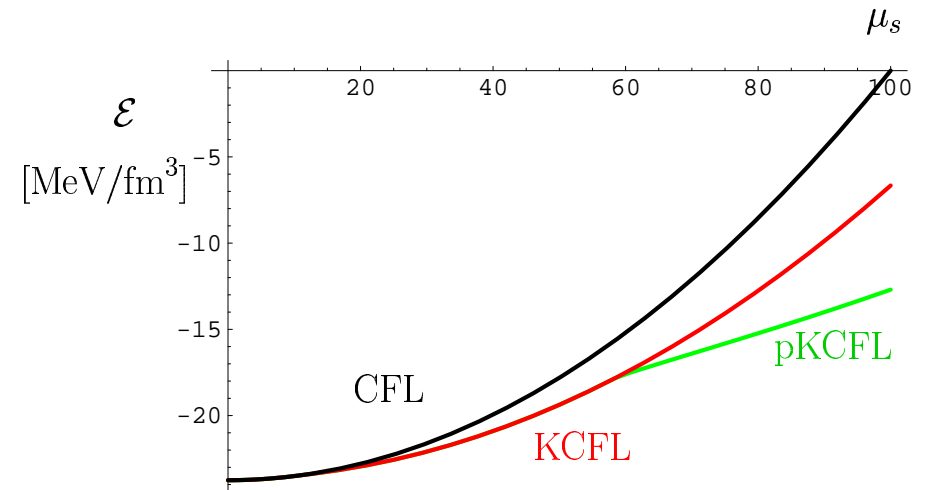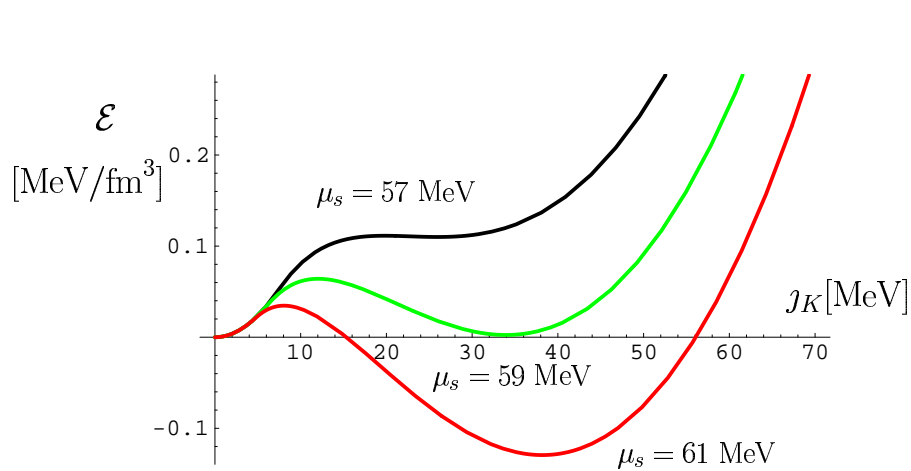
Fermion spectrum

$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\ \omega_l \Theta(-\omega_l)$$

Energy Functional

$$\frac{3\mu_s - 4\Delta}{\Delta}\bigg|_{crit} = a_{crit} \qquad \frac{\jmath_K}{\Delta} = c_{crit}$$

current strongly suppressed by electric charge neutrality

$m_s^2 \sim 2\mu\Delta$: multiple currents? crystaline state?

Summary

Rich, complicated phase diagram even for $\mu \gg \Lambda_{QCD}$

Depends crucially on N_f, N_c and m_q

CFL phase provides

Perturbative realization of s and p-wave meson condensation

Issues not covered in this talk: Transition to nuclear matter, nuclear exotics, etc.

Constraints from compact star pheneomenology