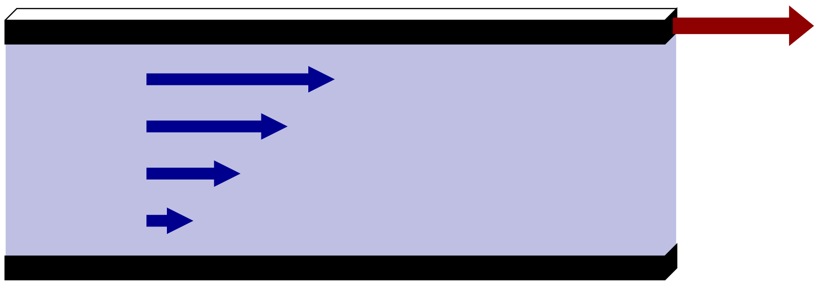
In Search of the Perfect Fluid

Thomas Schaefer, North Carolina State University

See T. Schäfer, D. Teaney, “Perfect Fluidity” [arXiv:0904.3107]

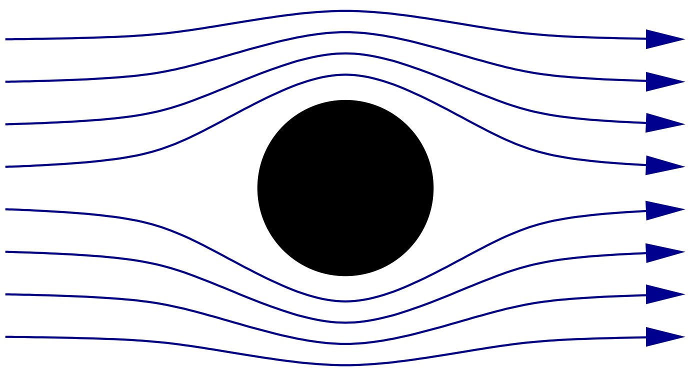
Measures of Perfection

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Dimensionless measure of shear stress: Reynolds number



$$Re = \frac{n}{\eta} \times mvr$$

fluid property   flow property

- $[\eta/n] = \hbar$
- Relativistic systems $Re = \dfrac{s}{\eta} \times \tau T$

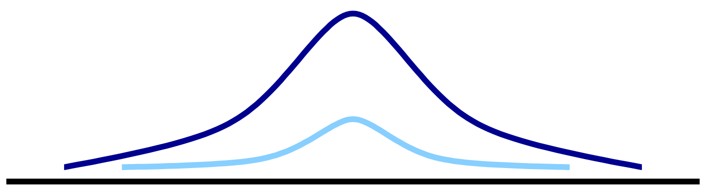
Other sources of dissipation (thermal conductivity, bulk viscosity, ...) vanish for certain fluids, but shear viscosity is always non-zero.

There are reasons to believe that η is bounded from below by a constant times $\hbar s/k_B$. In a large class of theories $\eta/s \geq \hbar/(4\pi k_B)$.

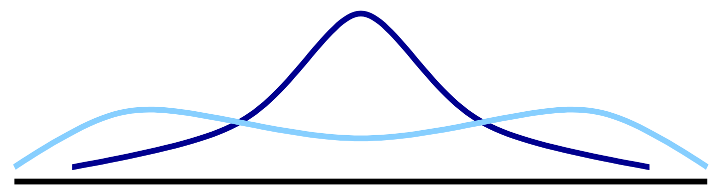
A fluid that saturates the bound is a "perfect fluid".

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



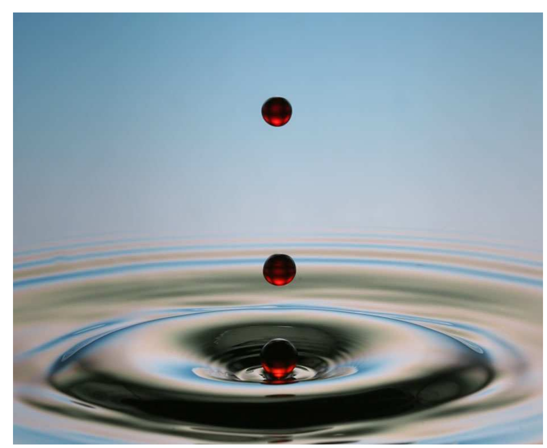
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

Conservation laws: mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

[Euler/Navier-Stokes equation]

Constitutive relations: Energy momentum tensor

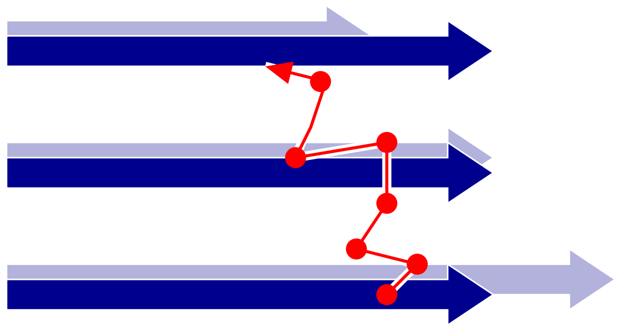
$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive dissipative 2nd order

Kinetic Theory

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$



$$\eta \sim \frac{1}{3} n \, \bar{p} \, l_{mfp}$$

Normalize to density. Uncertainty relation suggests

$$\frac{\eta}{n} \sim \bar{p} \, l_{mfp} \geq \hbar$$

Also: $s \sim k_B n$ and $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as $\bar{p} \, l_{mfp} \sim \hbar$?

Effective Theories for Fluids (Here: Weak Coupling QCD)

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

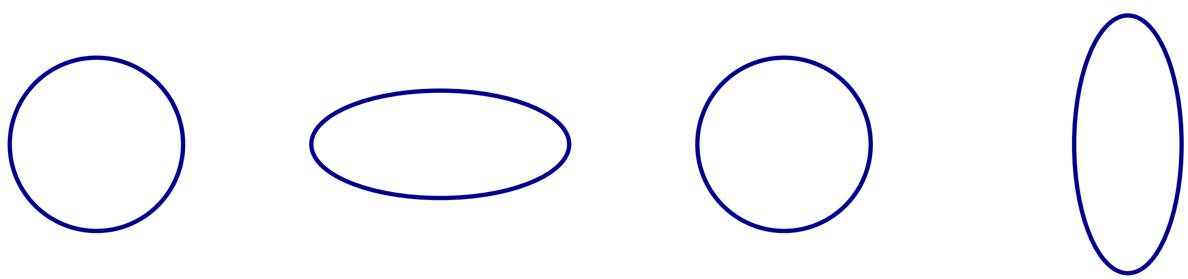
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{\delta S}{\delta g_{\mu\nu}} \qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic Duals: Transport Properties

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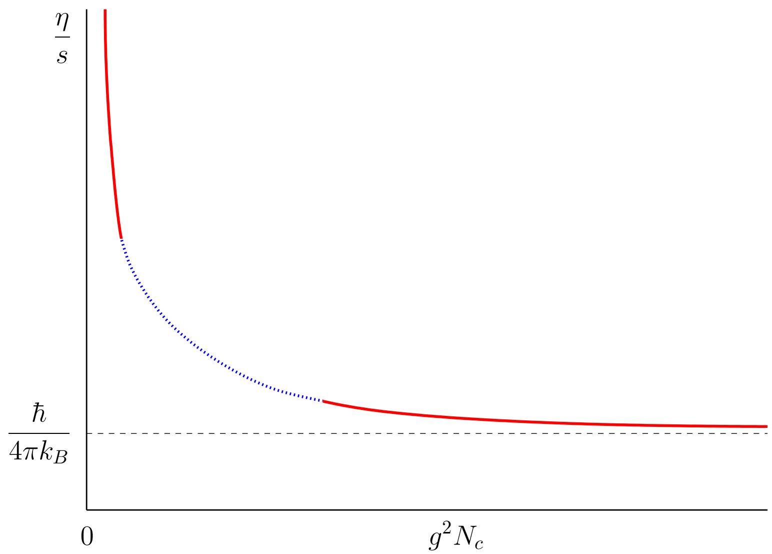
Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)



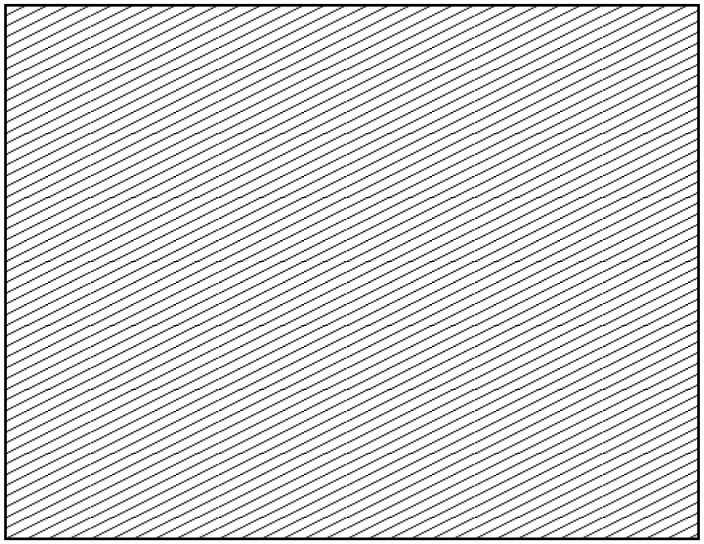
Strong coupling limit universal? Provides lower bound for all theories?

Effective Theories (Strong coupling)

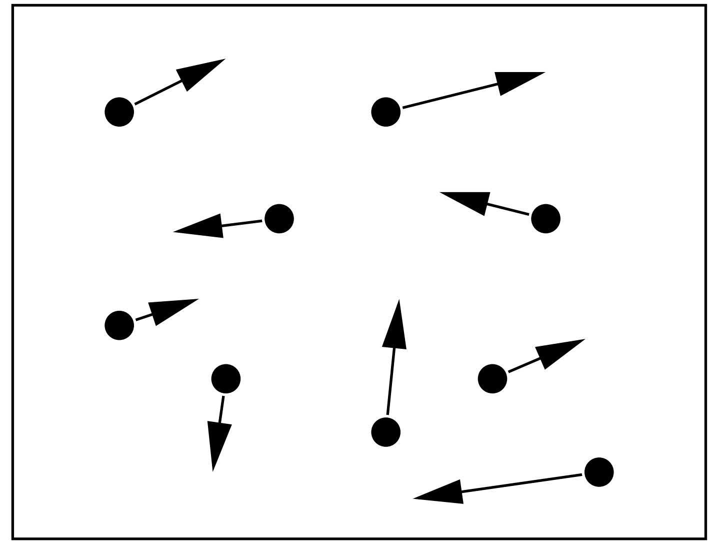
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics



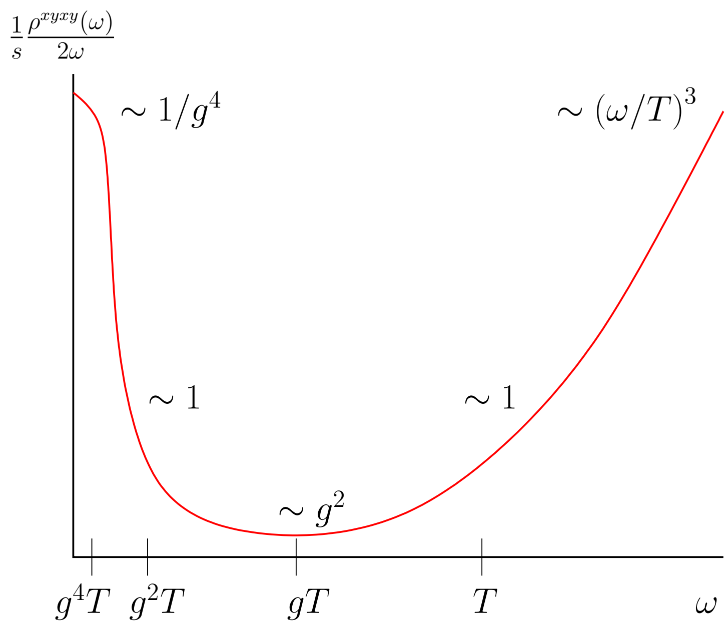
AdS/CFT low viscosity goo



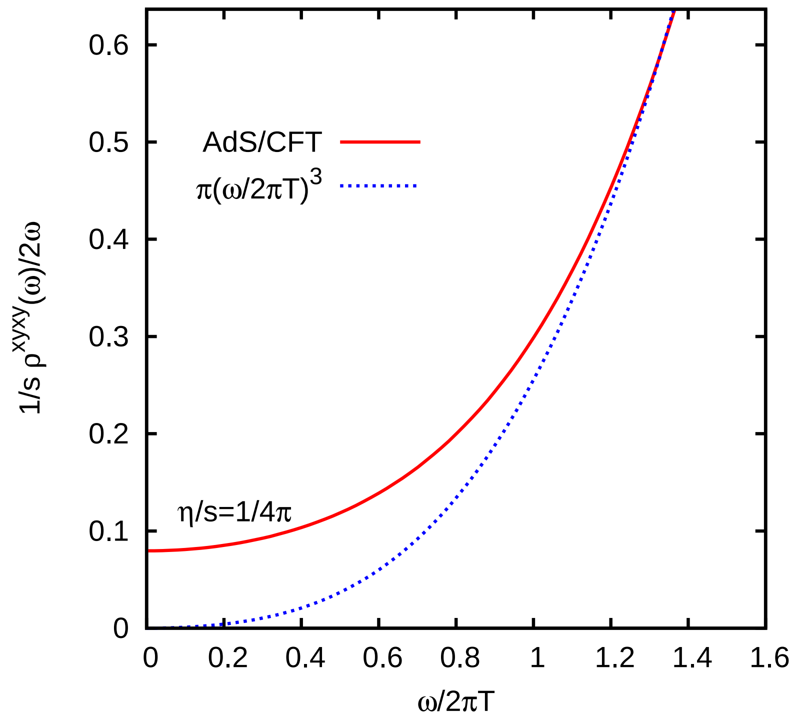
pQCD kinetic plasma

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \mathrm{Im} G_R(\omega, 0)$ associated with T_{xy}



weak coupling QCD



strong coupling AdS/CFT

transport peak vs no transport peak

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

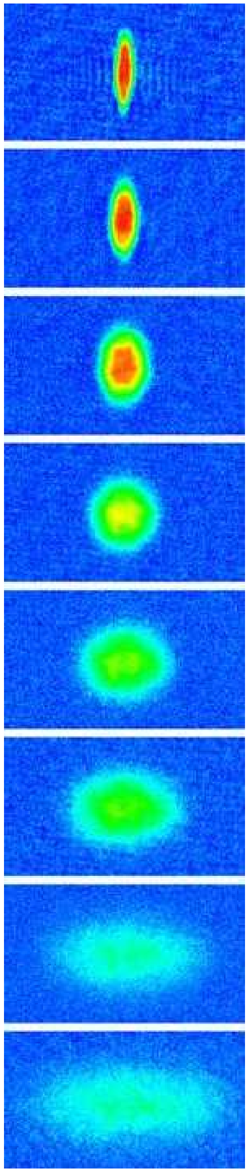
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

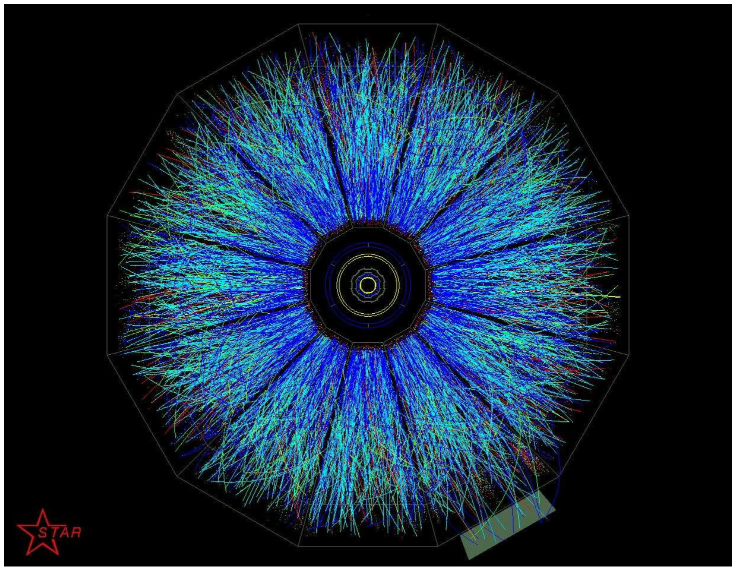
(Almost) scale invariant systems

Perfect Fluids: The contenders



Trapped Atoms

(T=0.1 neV)

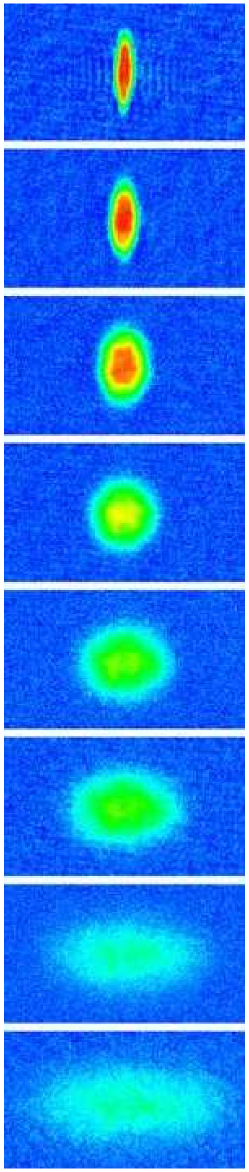


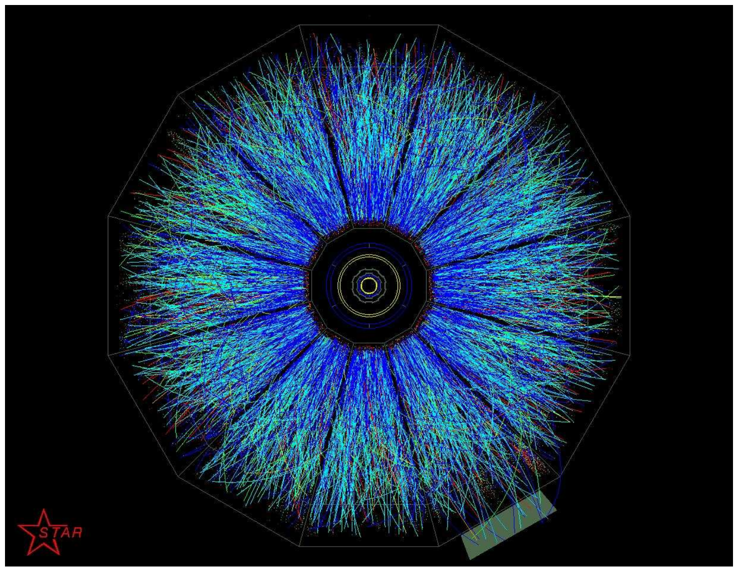
QGP (T=180 MeV)

Liquid Helium

(T=0.1 meV)

Perfect Fluids: The contenders

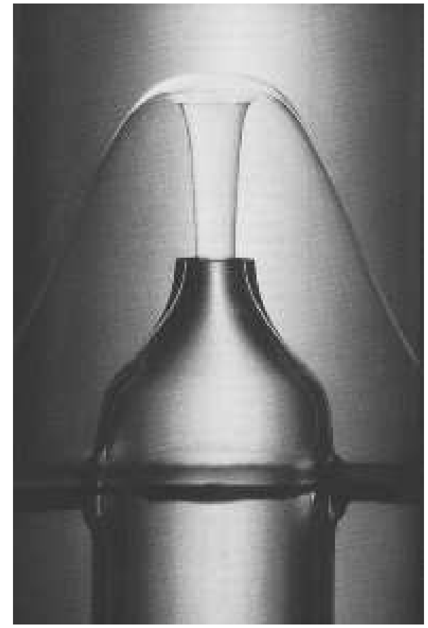




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



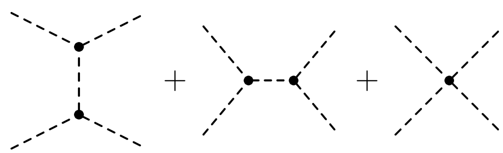
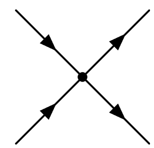
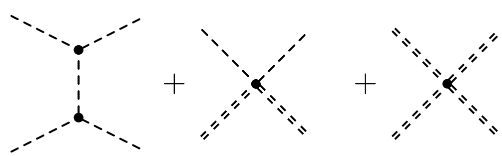
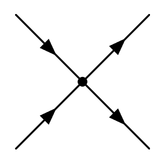
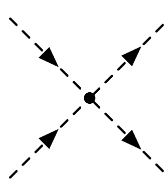
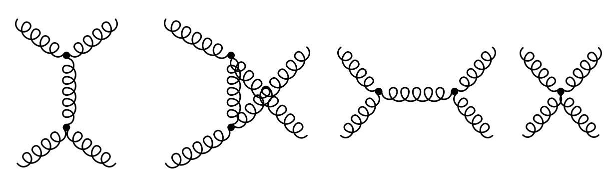
Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η/s

Kinetic Theory: Quasiparticles

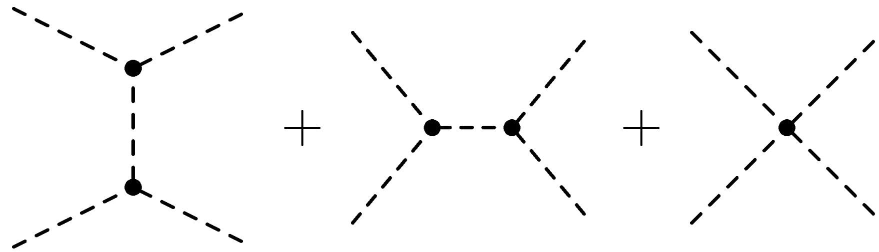
	low temperature	high temperature
unitary gas	phonons	atoms
		
helium	phonons, rotons	atoms
		
QCD	pions	quarks, gluons
		

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

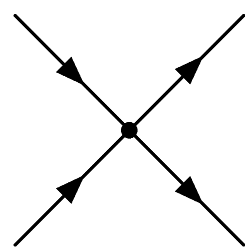
$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



T.S., G.R. (2007)

High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



Bruun (2005)

Low T: Phonons and Rotons Effective lagrangian

$$\mathcal{L} = \varphi^*(\partial_0^2 - v^2)\varphi + i\lambda\dot{\varphi}(\vec{\nabla}\varphi)^2 + \ldots$$

$$+ \varphi^*_{R,v}(i\partial_0 - \Delta)\varphi_{R,v} + c_0(\varphi^*_{R,v}\varphi_{R,v})^2 + \ldots$$

Shear viscosity

$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}}\, e^{\Delta/T} + \frac{c_{ph}}{T^5} + \ldots$$

Landau & Khalatnikov

High T: Atoms Viscosity governed by hard core ($V \sim 1/r^{12}$)

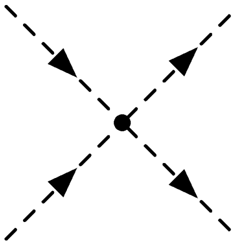
$$\eta = \eta_0 (T/T_0)^{2/3}$$

Low T: Pions Chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

Viscosity dominated by $\pi\pi$ scattering

$$\eta = A\frac{f_\pi^4}{T}$$



High T: Quasi-Particles HTL theory (screening, damping, ...)

$$\mathcal{L}_{HTL} = \int d\Omega\, G^a_{\mu\alpha} \frac{v^\alpha v_\beta}{(v\cdot D)^2} G^{a,\mu\beta}$$

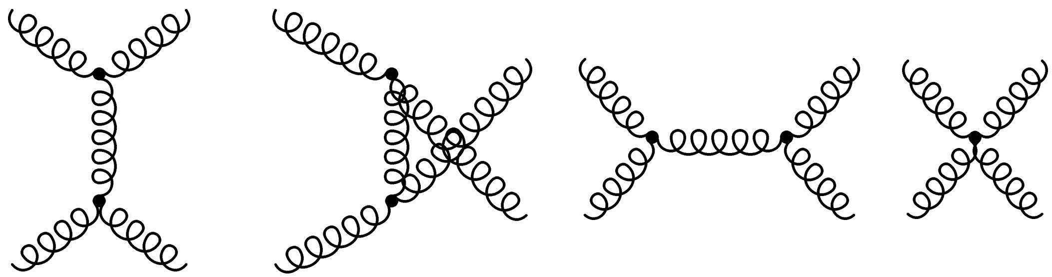
quasi-particle width

$$\gamma \sim g^2 T$$

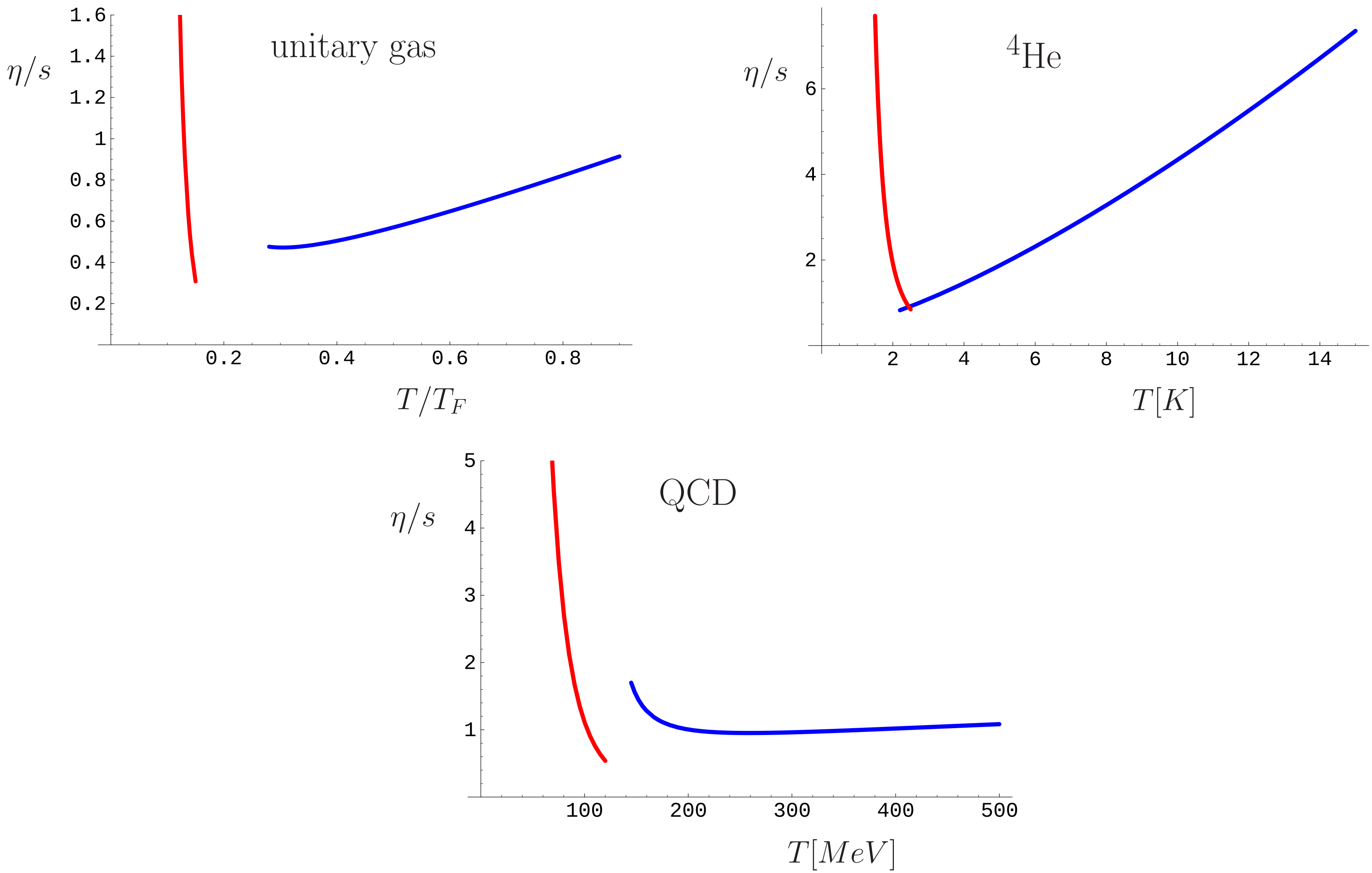
Viscosity dominated by t-channel gluon exchange

$$\eta = \frac{27.13 T^3}{g^4 \log(2.7/g)}$$

AMY (2003)



Theory Summary



I. Experiment (Liquid Helium)

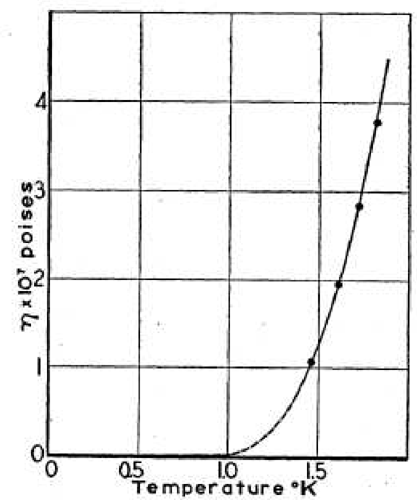
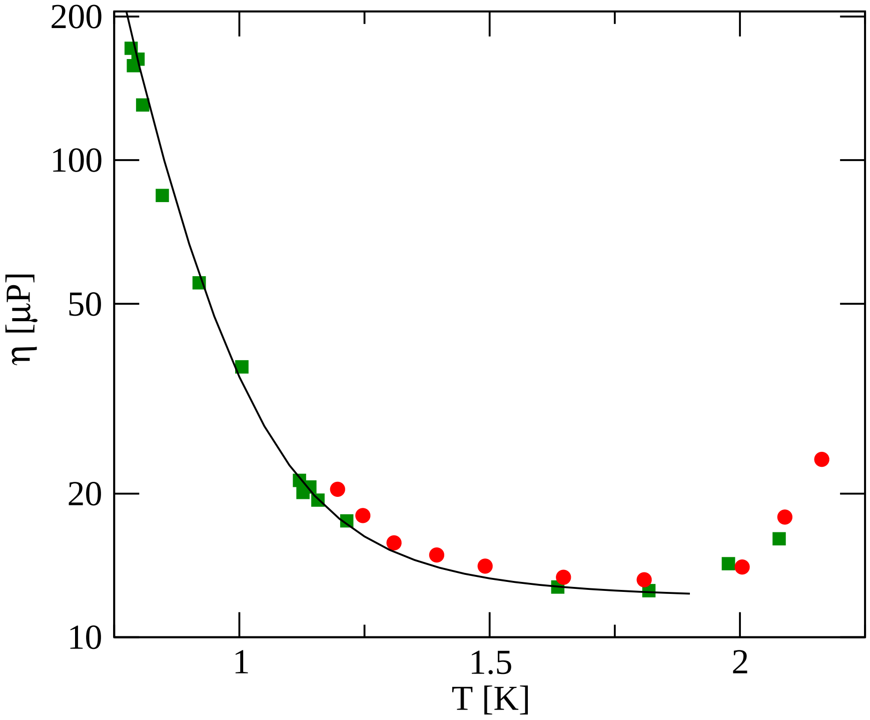


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

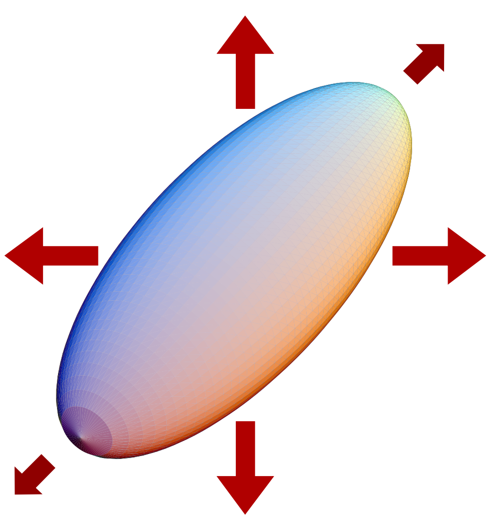
Hollis-Hallett (1955)

roton minimum, phonon rise

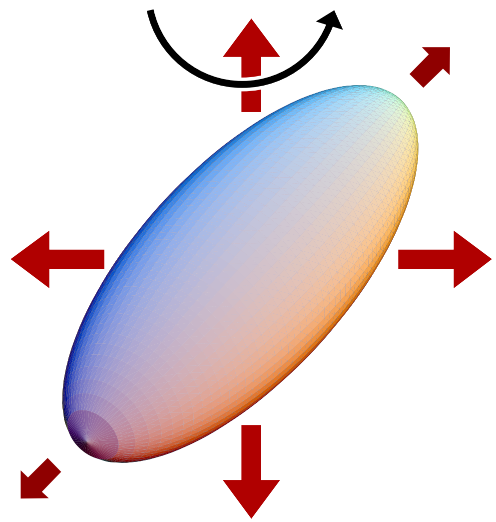
rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

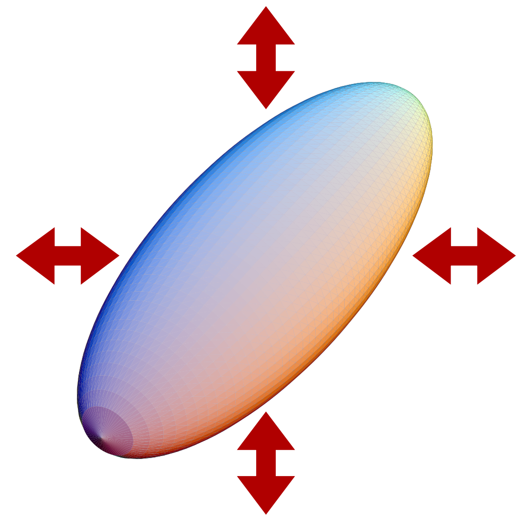
II. Scaling Flows (Cold Gases)



transverse expansion



expansion (rotating trap)



collective modes

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) \;=\; 0$$

$$mn\frac{\partial \vec{v}}{\partial t} + mn\left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} \;=\; -\vec{\nabla}P - n\vec{\nabla}V$$

Scaling Flows

Universal equation of state $P = \frac{n^{5/3}}{m} f\left(\frac{mT}{n^{2/3}}\right)$

Equilibrium density profile

$$n_0(x) = n(\mu(x), T) \qquad \mu(x) = \mu_0 \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

Scaling Flow: Stretch and rotate profile

$$\mu_0 \to \mu_0(t), \qquad T \to T_0(\mu_0(t)/\mu_0), \qquad R_x \to R_x(t), \;\; \ldots$$

Linear velocity profile

$$\vec{v}(x,t) = (\alpha_x x + (\alpha - \omega)y, \alpha_y y + (\alpha + \omega)y, \alpha_z z)$$

"Hubble flow"

Dissipation (Scaling Flows)

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\, \eta(x) \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\, \zeta(x)\, (\partial_i v_i)^2 - \frac{1}{T}\int d^3x\, \kappa(x)\, (\partial_i T)^2$$

Have $\zeta = 0$ and $T(x) = const$. Universality implies

$$\eta(x) = s(x)\, \alpha_s\left(\frac{T}{\mu(x)}\right)$$

$$\int d^3x\, \eta(x) = S\langle\alpha_s\rangle$$

Navier-Stokes equation

Option 1: Moment method

$$\int d^3x\, x_k \left(\rho \dot{v}_i + \ldots\right) = \int d^3x\, x_k \left(-\nabla_i P - \nabla_j \delta\Pi_{ij}\right)$$

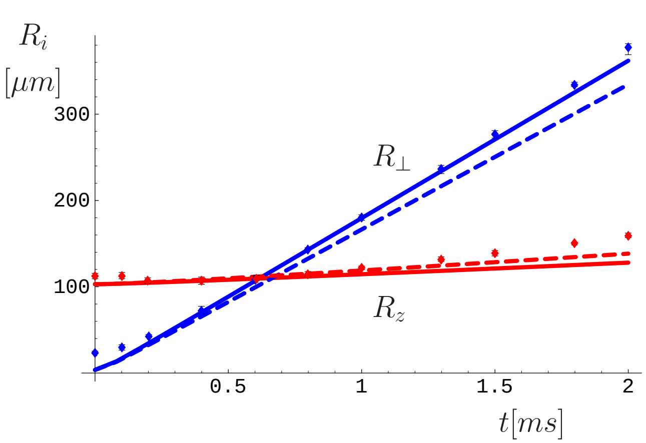
Only involves $\langle \eta \rangle / E_0$.

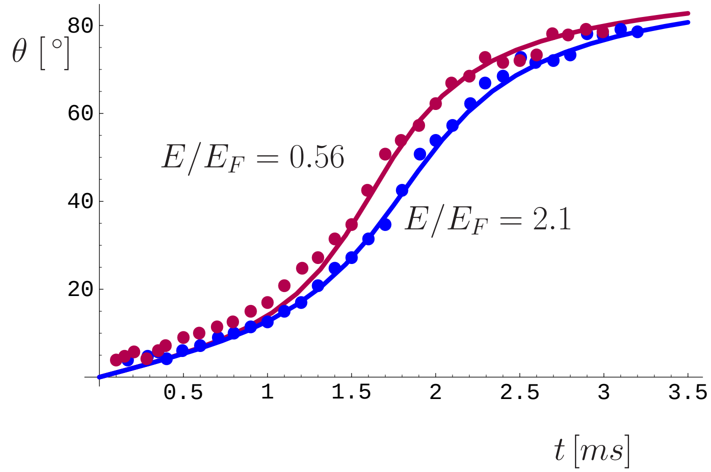
Option 2: Scaling ansatz for $\eta(\mu, T)$

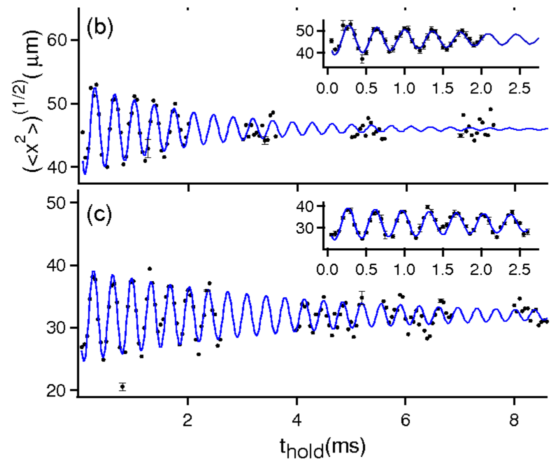
$$\eta(n,T) = \eta_0 (mT)^{3/2} + \eta_1 \frac{P(n,T)}{T}$$

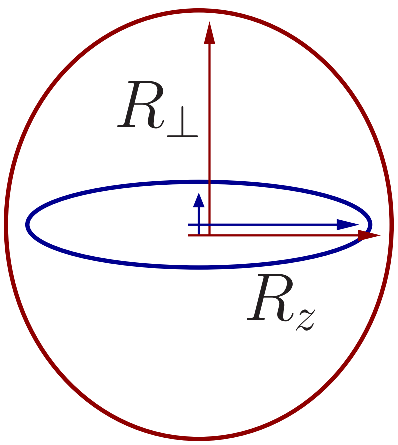
Option 3: Numerical solutions.

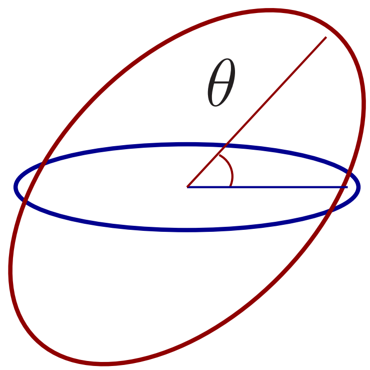
Dissipation

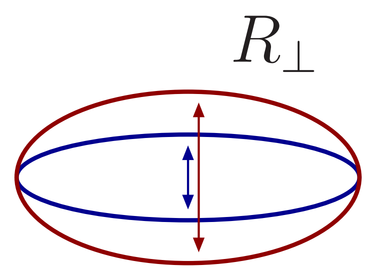






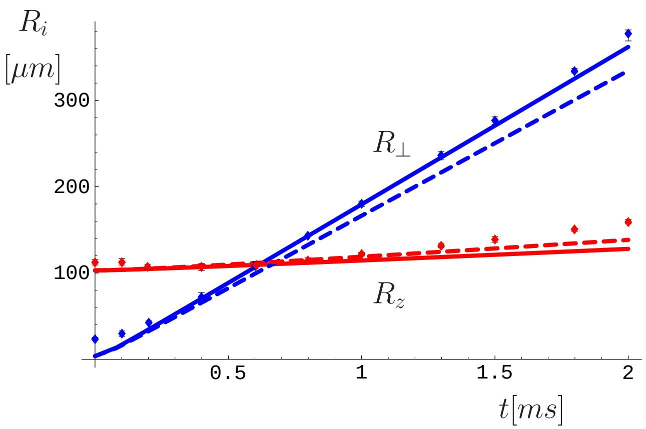


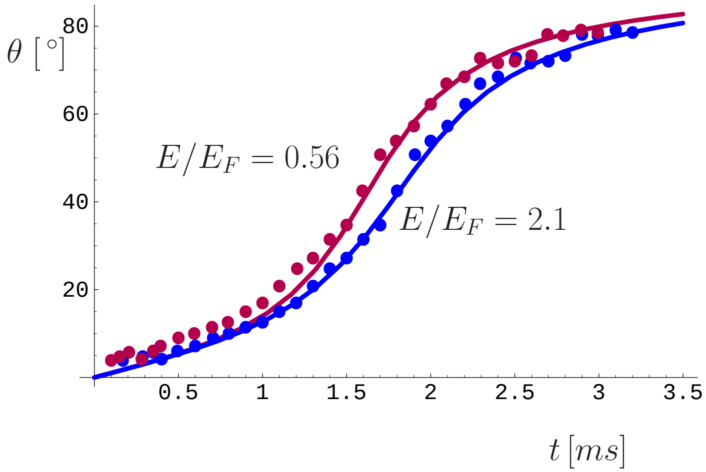


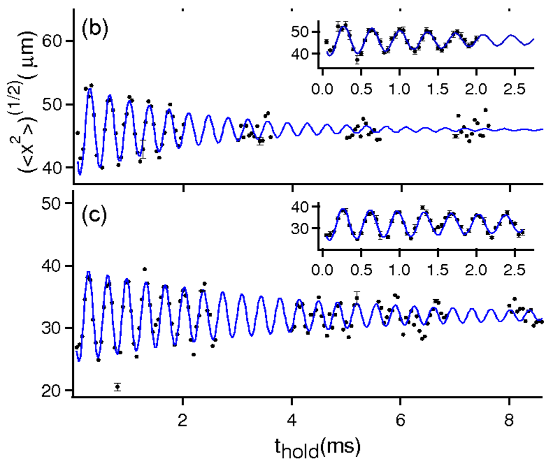


O'Hara et al (2002), Kinast et al (2005), Clancy et al (2007)

Dissipation







$$\left.\begin{matrix}(\delta t_0)/t_0 \\ (\delta a)/a\end{matrix}\right\} = \left\{\begin{matrix}0.008 \\ 0.024\end{matrix}\right\}\left(\frac{\langle\alpha_s\rangle}{1/(4\pi)}\right)\left(\frac{2\cdot 10^5}{N}\right)^{1/3}\left(\frac{S/N}{2.3}\right)\left(\frac{0.85}{E_0/E_F}\right)$$

t_0: "Crossing time" $(b_\perp = b_z,\ \theta = 45^\circ)$

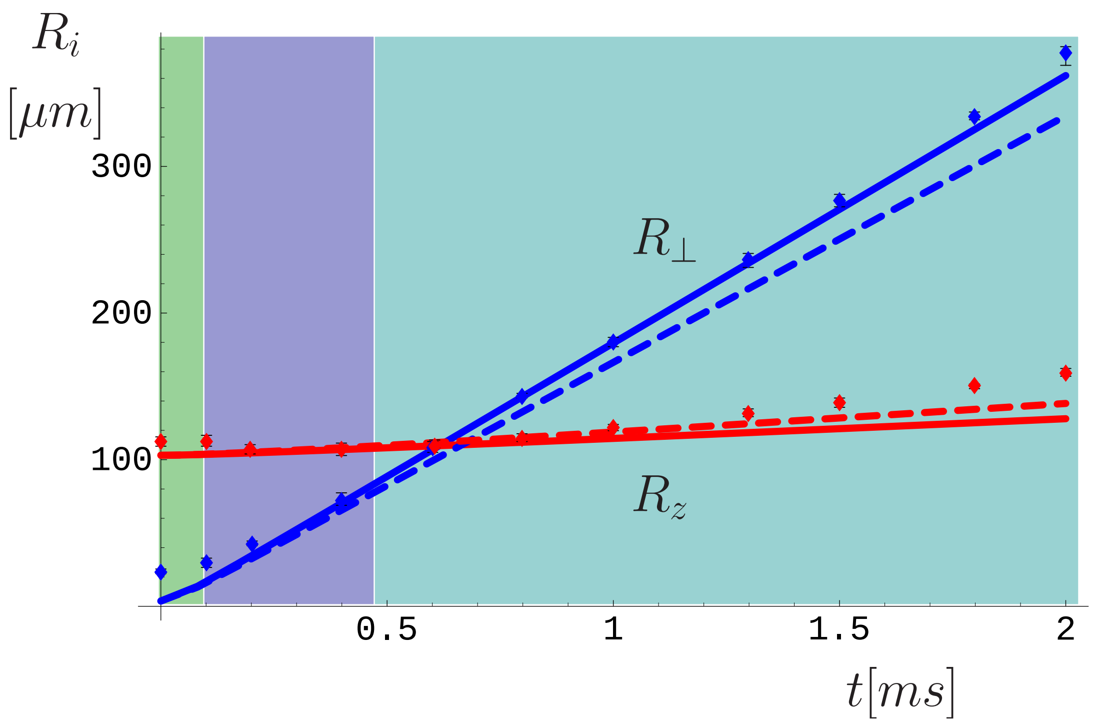
a: amplitude

Time Scales

dissipative

hydro/free streaming

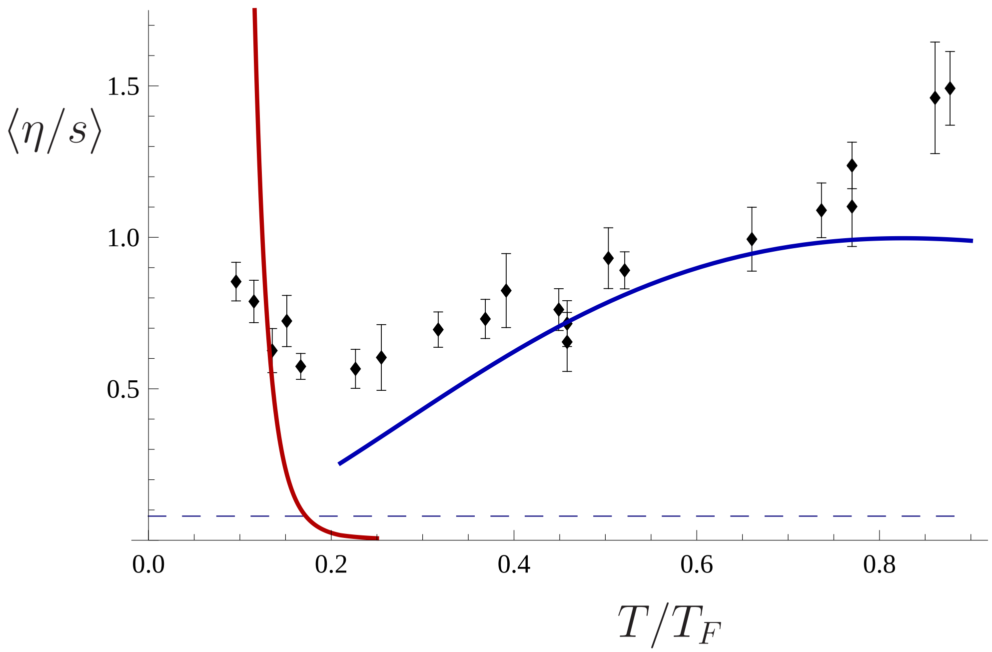
ballistic



Collective modes: Small viscous correction exponentiates

$$a(t) = a_0 \cos(\omega t) \exp(-\Gamma\, t)$$

$$\langle \eta/s \rangle = (3N\lambda)^{1/3} \left(\frac{\Gamma}{\omega_\perp}\right) \left(\frac{E_0}{E_F}\right) \left(\frac{N}{S}\right)$$



Kinast et al. (2006), Schaefer (2007)

Limitations of Scaling Flows

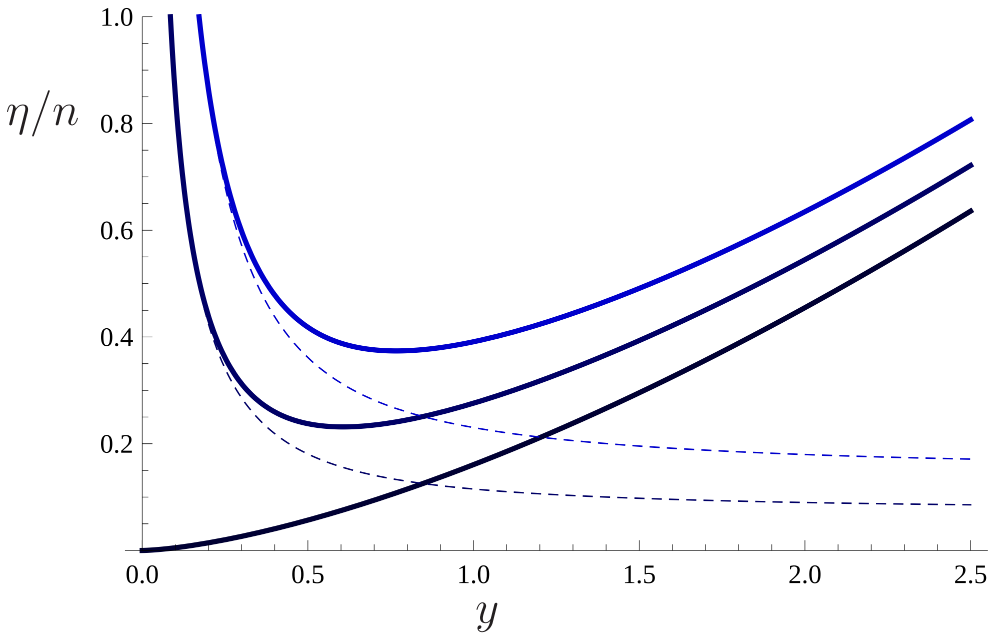
Simple model for $\eta(n,T)$

$$\eta(n,T) = \eta_0 (mT)^{3/2} + \eta_1 \frac{P(n,T)}{T} f$$

Find exact scaling solutions of the Navier Stokes equation

But: η_0 completely unconstrained by data

$$\nabla_j [\eta_0 (mT)^{3/2} (\nabla_i v_j + \ldots)] = 0$$



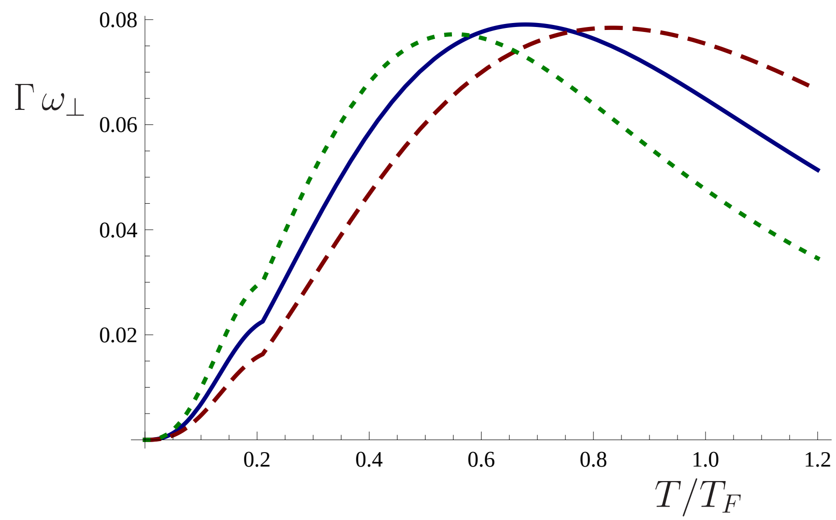
Relaxation Time Model

In real systems stress tensor does not relax to Navier-Stokes form instantaneously. Consider

$$\tau_R \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \delta\Pi_{ij} = \delta\Pi_{ij} - \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial \cdot v \right)$$

In kinetic theory $\tau_R \simeq (\eta/n)\, T^{-1}$

- disspiation from $\eta \sim (mT)^{3/2}$: corona excerts drag force.
- modified T dependence
- modifed N scaling

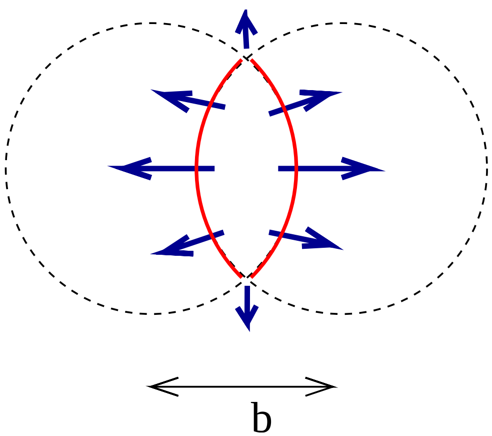


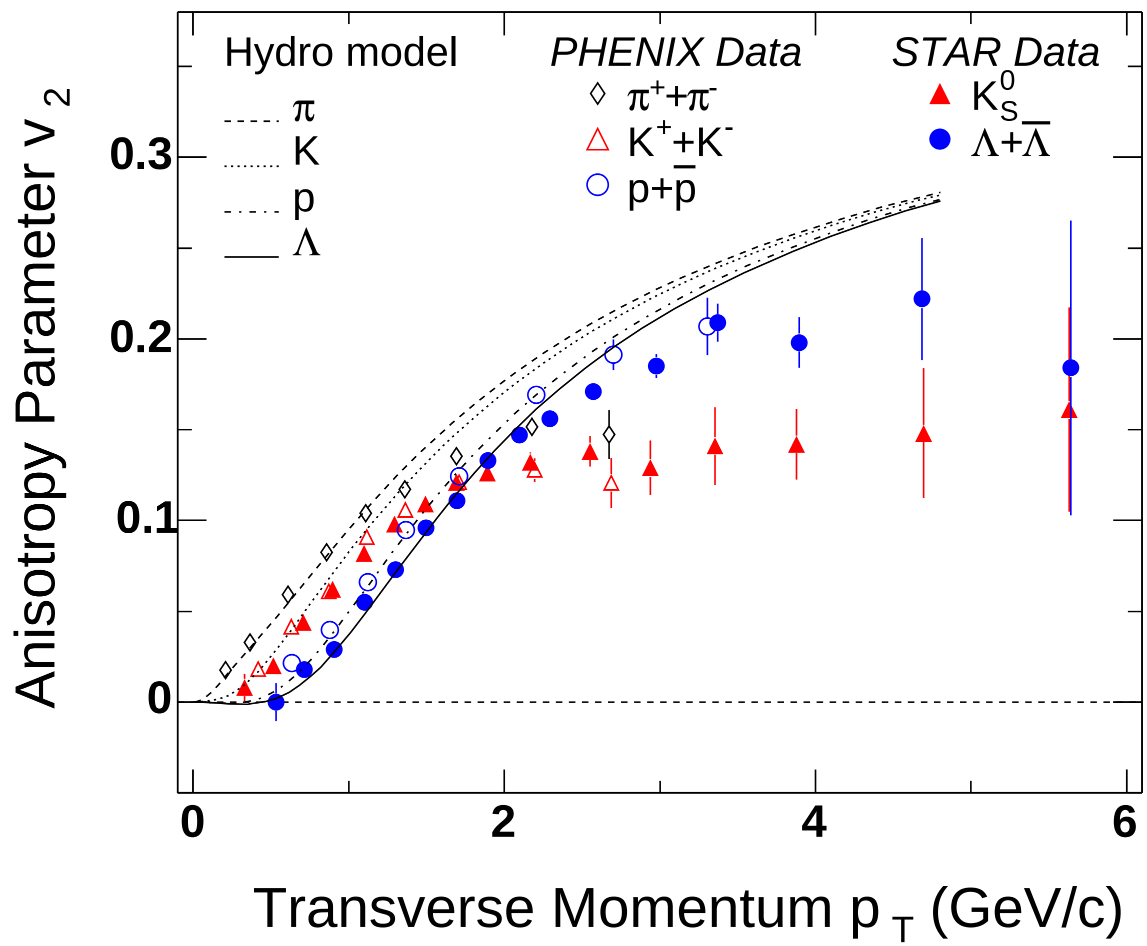
Where are we?

- high temperature ($T > 2.5T_c$) dominated by corona
- low temperature ($T \sim T_c$): evidence for low viscosity ($\eta/s < 0.4$) core
- also seen in "irrotational flow" data
- full (2nd order hydro or hydro+kin) analysis needed

III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

Viscosity and Elliptic Flow

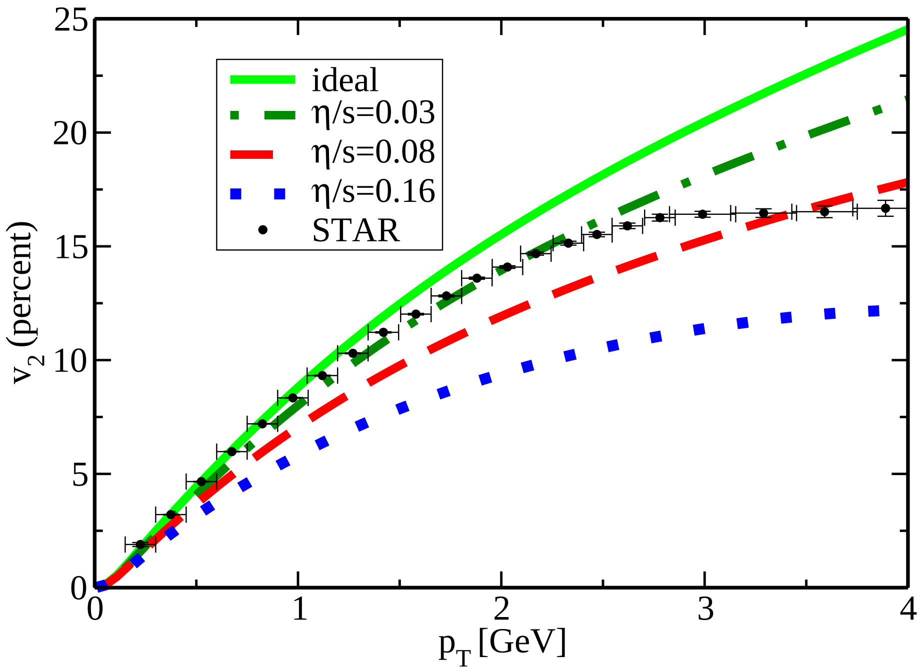
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

Outlook

Too early to declare a winner.

$$\eta/s \simeq 0.8 \text{ (He)}, \qquad \eta/s \leq 0.5 \text{ (CA)}, \qquad \eta/s \leq 0.5 \text{ (QGP)}$$

Other experimental constraints, more analysis needed.

Kinetic theory: o.k. in He (all T), o.k. close to T_c in CA, QGP?

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, ...)