QCD and Dense Matter: An Introduction

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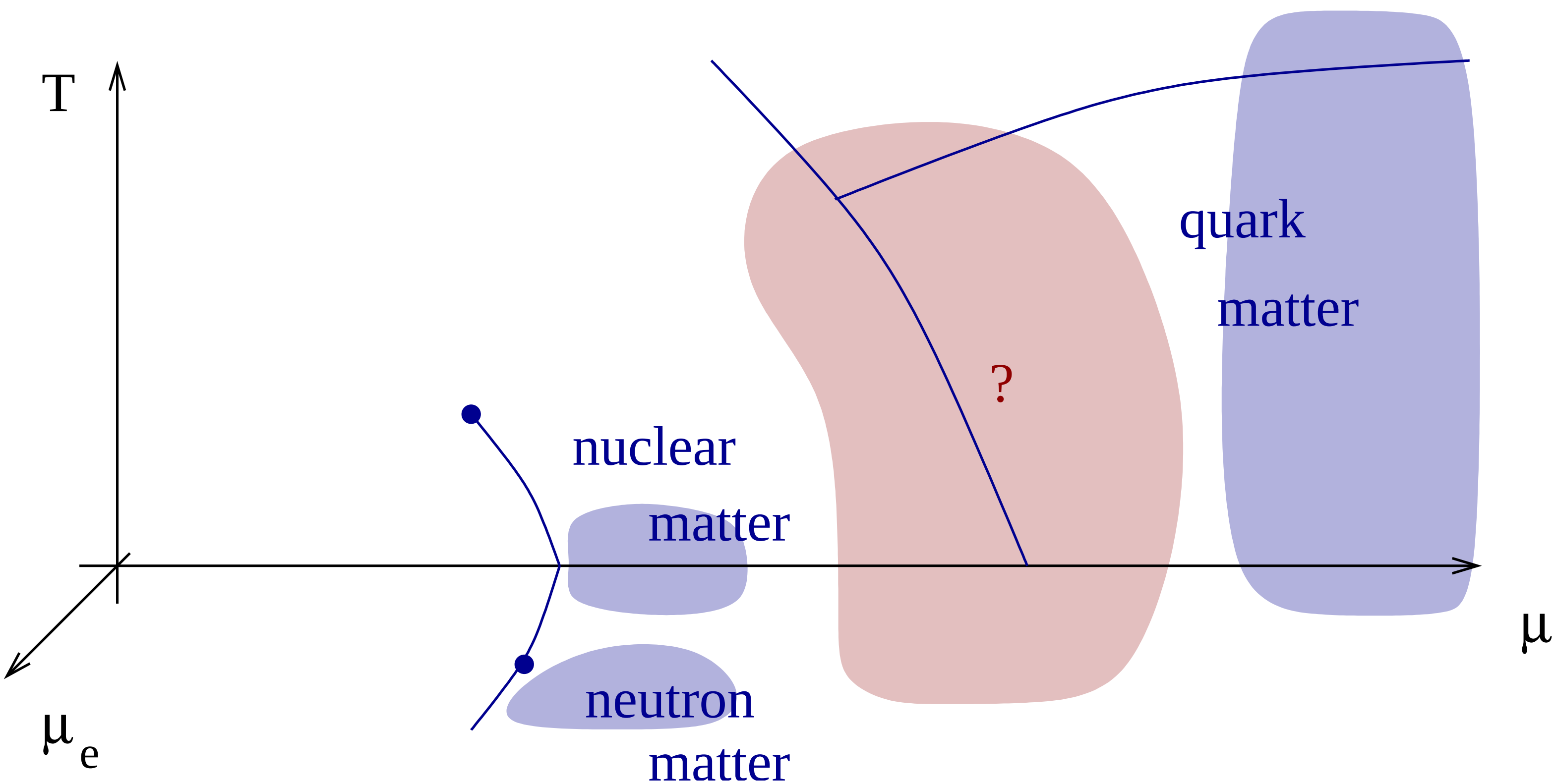
Schematic Phase Diagram



Dense Baryonic Matter

Low Density

- Equation of state of nuclear/neutron matter
- Neutron/proton superfluidity, pairing gaps

Moderate Density

- Pion/kaon condensation, hyperon matter
- Pairing, equation of state at high density

High Density

- Quark matter
- Color superconductivity, Color-flavor-locking

Dense Baryonic Matter

Low Density (constrained by NN interaction, phenomenology)

- Equation of state of nuclear/neutron matter
- Neutron/proton superfluidity, pairing gaps

Moderate Density (very poorly known)

- Pion/kaon condensation, hyperon matter
- Pairing, equation of state at high density

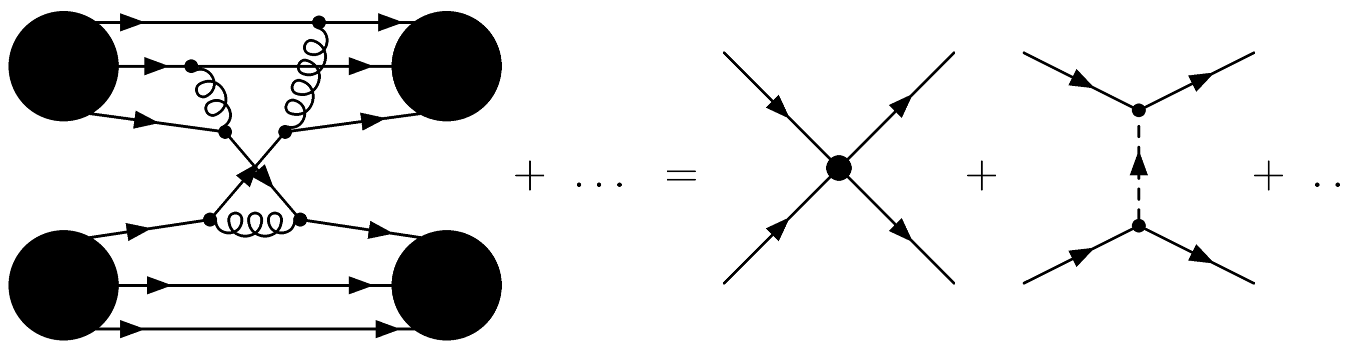
High Density (weak coupling methods apply)

- Quark matter
- Color superconductivity, Color-flavor-locking

Low Density: Nuclear Effective Field Theory

Low Energy Nucleons:

- Nucleons are point particles
- Interactions are local
- Long range part: pions



Advantages:

- Systematically improvable
- Symmetries manifest (Chiral, gauge, ...)
- Connection to lattice QCD

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \ldots$$

Simplifications: neutrons only, no pions (very low energy)

Effective range expansion

$$p \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} \sum_n r_n p^{2n}$$

Coupling constants

$$C_0 = \frac{4\pi a}{M}, \qquad C_2 = \frac{4\pi a^2}{M} \frac{r}{2}, \quad \ldots \qquad a = -18\,\mathrm{fm},\ r = 2.8\,\mathrm{fm}$$

Neutron Matter

Consider limiting case ("Bertsch" problem)

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

Scale invariant system; Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A} \right)_0 = \xi \frac{3}{5} \left(\frac{k_F^2}{2M} \right)$$

No Expansion Parameters!

How to find ξ?

Numerical Simulations

Experiments with trapped fermions

Analytic Approaches

Epsilon Expansion

Bound state wave function $\psi \sim 1/r^{d-2}$. Nussinov & Nussinov

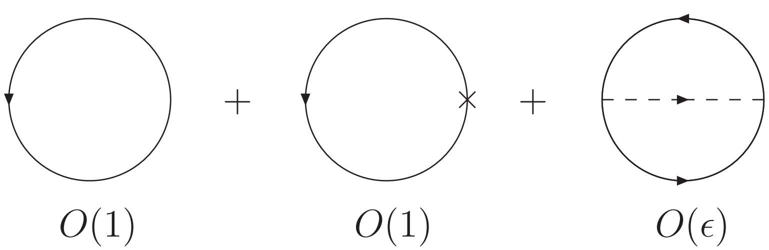
$d \geq 4$: Non-interacting bosons $\xi(d=4)=0$

$d \leq 4$: Effective lagrangian for atoms $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$ and dimers ϕ

$$\mathcal{L} = \Psi^\dagger \left(i\partial_0 + \frac{\sigma_3 \nabla^2}{2m} \right) \Psi + \mu \Psi^\dagger \sigma_3 \Psi + \Psi^\dagger \sigma_+ \Psi \phi + h.c.$$

Nishida & Son (2006)

Perturbative expansion: $\phi = \phi_0 + g\varphi$ $(g^2 \sim \epsilon)$



$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2} \ln \epsilon - 0.0246\, \epsilon^{5/2} + \ldots$$

$\xi = 0.475$ $\Delta = 0.62 E_F$

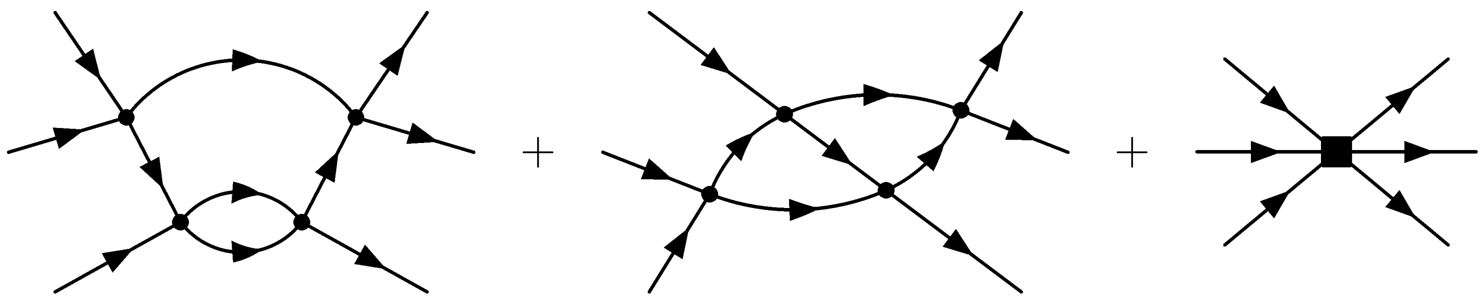
Nuclear Matter

isospin symmetric matter: first order onset transition

$$\rho_0 \simeq 0.14\,\mathrm{fm}^{-3} \qquad (k_F \simeq 250\,\mathrm{MeV}) \qquad B/A = 15\,\mathrm{MeV}$$

can be reproduced using accurate V_{NN} (V_{3N} crucial, $V_{4N} \approx 0$)

EFT methods: explain need for V_{3N} if $N_f > 1$ (and $V_{4N} \ll V_{3N}$)

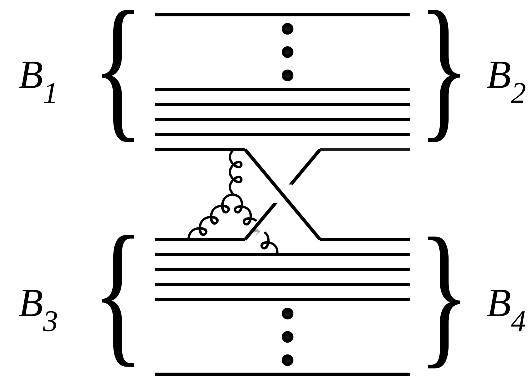


systematic calculations difficult since $k_F a \gg 1$, $k_F r \sim 1$

Nuclear Matter at large N_c

Nucleon nucleon interaction is $O(N_c)$

$$m_N = O(N_c) \qquad r_N = O(1)$$



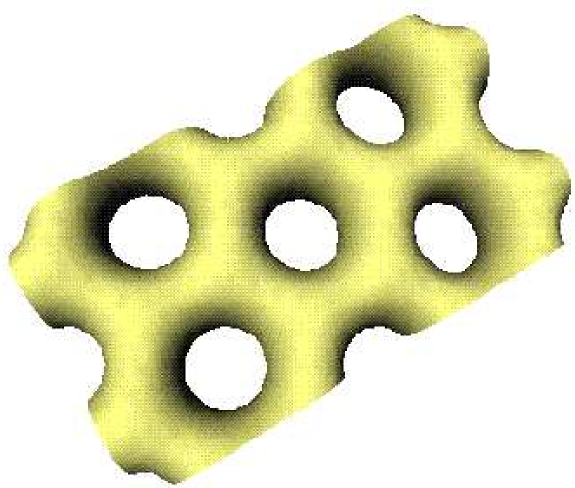
$$V_{NN} = O(N_c)$$

Get $SU(2N_f)$ (Wigner symmetry) relations

$$C_0(\psi^\dagger\psi)^2 \gg C_T(\psi^\dagger\vec{\sigma}\psi)^2$$

Dense matter: $k_F = O(1)$ $(E_F \sim 1/N_c)$

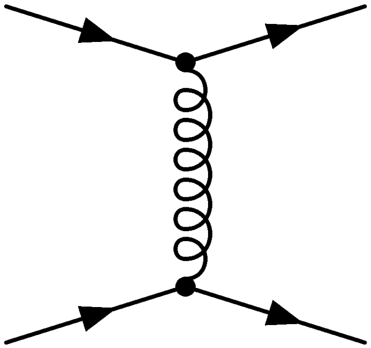
crystallization



Note: $E \sim N_c$ (no phase transition?)

High Density: Pairing in Quark Matter

QQ scattering in perturbative QCD

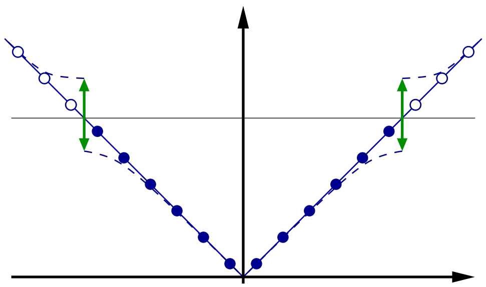


$$(\vec{T})_{ac}(\vec{T})_{bd} = -\frac{1}{3}(\delta_{ab}\delta_{cd}-\delta_{ad}\delta_{bc})+\frac{1}{6}(\delta_{ab}\delta_{cd}+\delta_{ad}\delta_{bc})$$

$$[3] \quad \times \quad [3] \quad = \quad [\bar{3}] \quad + \quad [6]$$

Fermi surface: pairing instability in weak coupling

$$\Phi_{ij}^{ab,\alpha\beta} = \langle \psi_i^{a,\alpha} C \psi_j^{b,\beta} \rangle$$



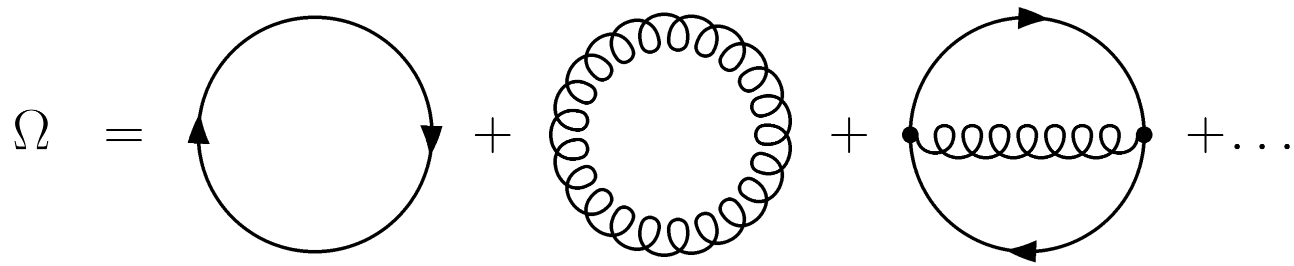
Phase structure in perturbation theory

$$\text{Minimize } \Omega(\Phi_{ij}^{ab,\alpha\beta})$$

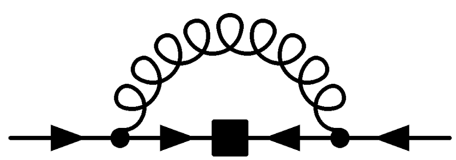
In practice: consider $\Phi_{ij}^{ab,\alpha\beta}$ with residual symmetries

Superconductivity

Thermodynamic potential



Variational principle $\delta\Omega/\delta\Phi$ gives gap equation



$$= \frac{g^2}{18\pi^2}\int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0 - q_0|}\right)\frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}$$

$\Lambda_{BCS} = c_i 256\pi^4 \mu g^{-5}$ determined by symmetries of order parameter

$$\Omega = \frac{\mu^2}{4\pi^2}\sum_i \Delta_i^2 \qquad \Delta_i = c_i \mu g^{-5}\exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

Remarks

Behavior of perturbative expansion

$$\Delta = \mu g^{-5} \exp\left(-\frac{c}{g}\right) \left(c_0 + c_1 g \log(g) + c_2 g + O(g^{n/3} \log^m(g))\right)$$

Also: non-perturbative effects (become large for $\mu \sim 1$ GeV)

$$\Delta \sim \mu \exp\left(-\frac{8\pi^2}{g^2}\right)$$

Magnitude of gap quite uncertain

But: Phase Structure not sensitive to interaction

$N_f = 1$: Color-Spin-Locking

$N_f = 1$, color-anti-symmetric: spin-1 condensate

$$\langle \psi^b C \gamma_i \psi^c \rangle = \Phi_i^a \epsilon^{abc}$$

Ground state: Color-Spin-Locking (CSL): $\Phi_i^a \sim \delta_i^a$

$SU(3)_c \times SO(3) \to SO(3)$: rotational symmetry

$U(1)_B$ broken: massless Goldstone boson

$(s=3/2) + (s=1/2)$ gapped fermions

$N_f = 2$: 2SC Phase

$N_f = 2$, color-anti-symmetric: spin-0 BCS condensate

$$\langle \psi_i^b C\gamma_5 \psi_j^c \rangle = \Phi^a \epsilon^{abc} \epsilon_{ij}$$

Order parameter $\phi^a \sim \delta^{a3}$ breaks $SU(3)_c \to SU(2)$

$SU(2)_L \times SU(2)_R$ unbroken

4 gapped, 2 (almost) gapless fermions

light $U(1)_A$ Goldstone boson

$SU(2)$ confined ($\Lambda_{conf} \ll \Delta$)

$N_f = 3$: CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

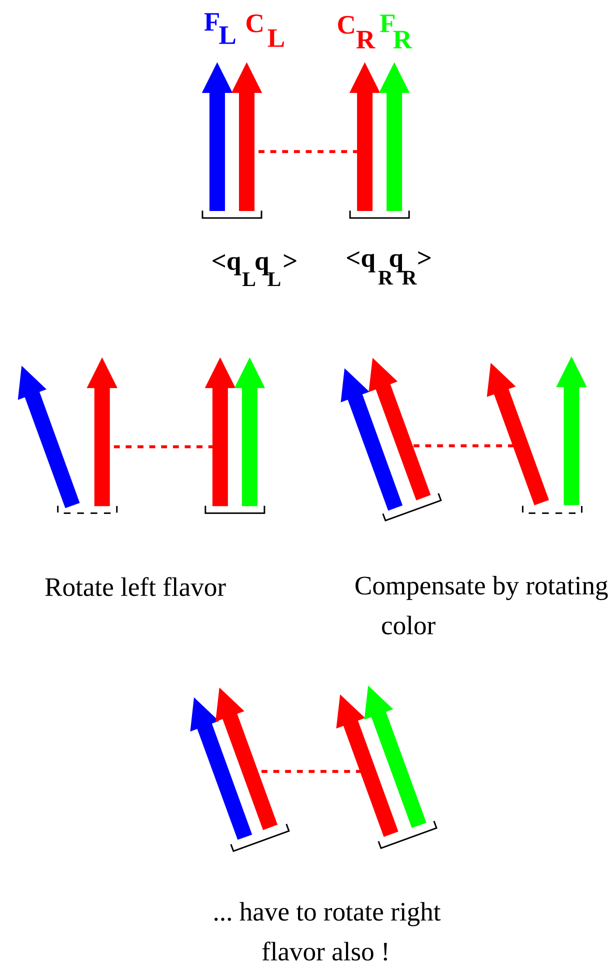
$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C$$
$$\times U(1) \to SU(3)_{C+F}$$

All quarks and gluons acquire a gap

$[8] + [1]$ fermions, Q integer



$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

QCD with many flavors/colors

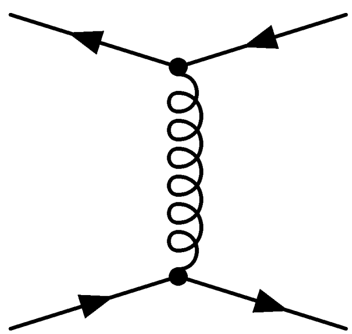
QCD with $N_f > 3$ flavors: CFL-like, fully gapped, phases

$N_f = 4 \quad SU(4)_L \times SU(4)_R \times U(1)_V \to [SU(2)_V]^2 \times SU(2)_A$

$N_f = 5 \quad SU(5)_L \times SU(5)_R \times U(1)_V \to SU(2)_V$

$N_f = 6 \quad SU(6)_L \times SU(6)_R \times U(1)_V \to SU(3)_V \times U(1)_V \times U(1)_A$

$N_c \to \infty$: qq pairing suppressed, qq^{-1} chiral density wave

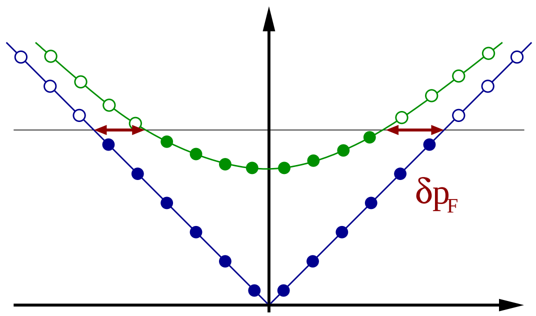


$$\langle \bar{q}(x) q(x) \rangle \sim \sum e^{2i\vec{q}_F\cdot\vec{x}}$$

Requires very large $N_c > 1000$

Towards the real world: Non-zero strange quark mass

Have $m_s > m_u, m_d$: Unequal Fermi surfaces



$$\delta p_F \simeq \frac{m_s^2}{2p_F}$$

Also: If $p_F^s < p_F^{u,d}$ have unequal densities

Charge neutrality not automatic

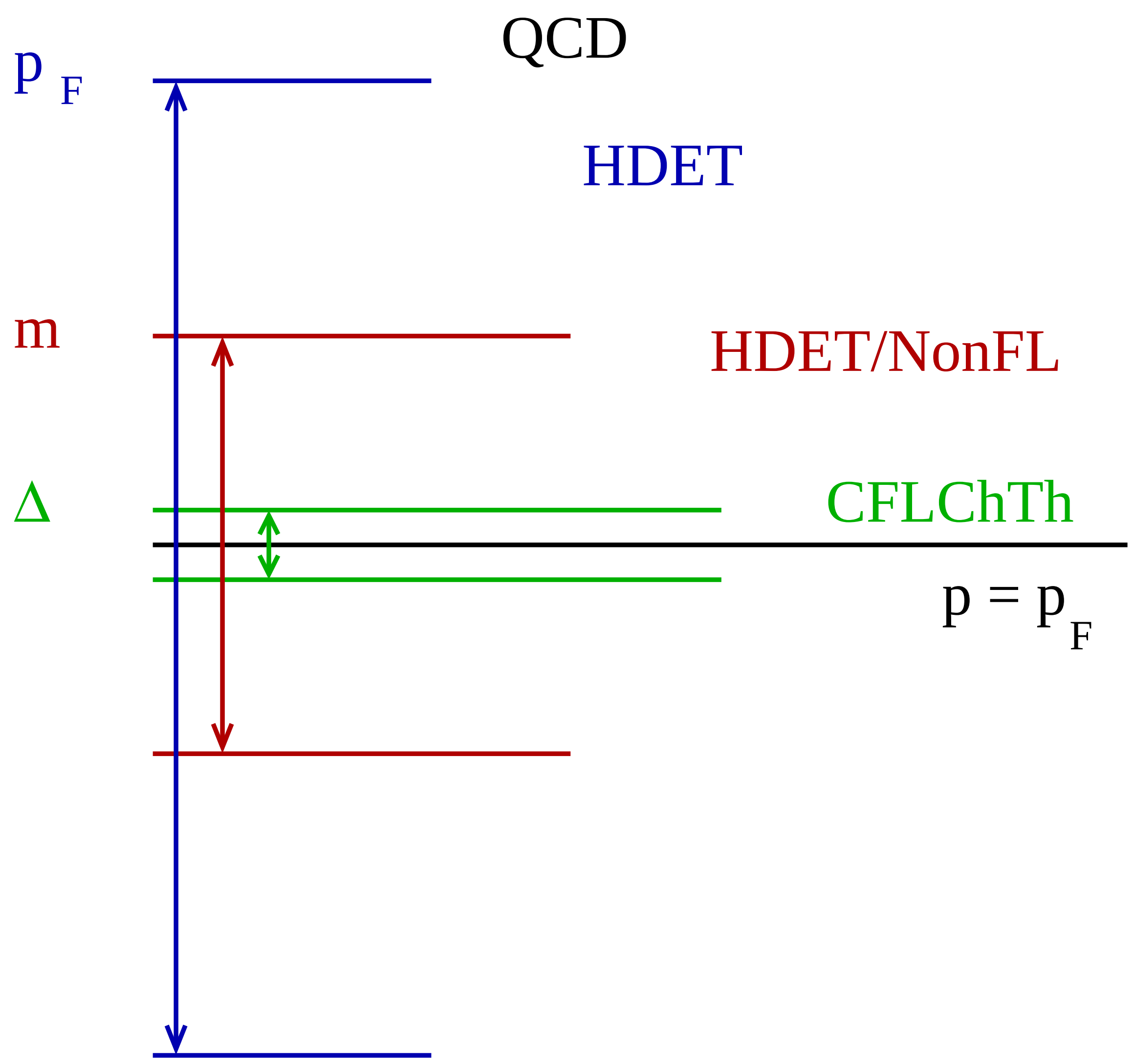
Strategy

Consider $N_f = 3$ at $\mu \gg \Lambda_{QCD}$ (CFL phase)

Study response to $m_s \neq 0$

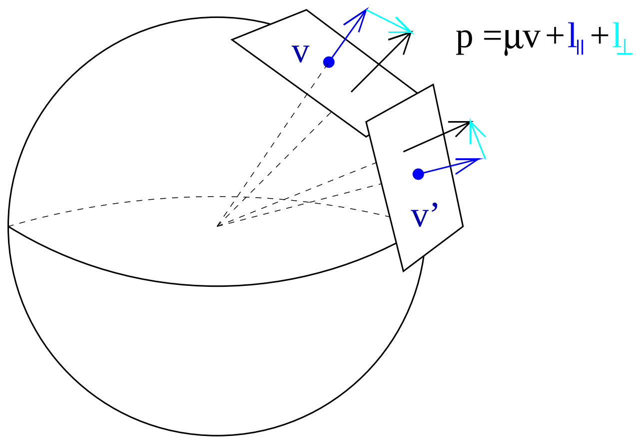
Constrained by chiral symmetry

Very Dense Matter: Effective Field Theories



High Density Effective Theory

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1\pm\vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

Effective lagrangian for $p_0 < m$

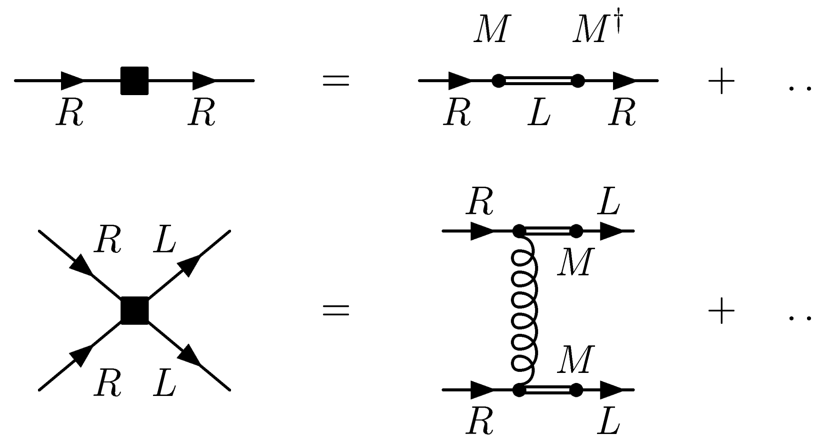
$$\mathcal{L} = \psi_v^\dagger\left(iv\cdot D - \frac{D_\perp^2}{2\mu}\right)\psi_v + \mathcal{L}_{4f} - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2}\sum_v G^a_{\mu\alpha}\frac{v^\alpha v^\beta}{(v\cdot D)^2}G^b_{\mu\beta}$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$

$$+ \; \frac{V_M^0}{\mu^2}(\psi_R^\dagger M\lambda^a\psi_L)(\psi_R^\dagger M\lambda^a\psi_L)$$



mass corrections to FL parameters $\hat{\mu}_{L,R}$ and $V^0(RR \to LL)$

EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger(iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\} + (L\leftrightarrow R, X\leftrightarrow Y)$$

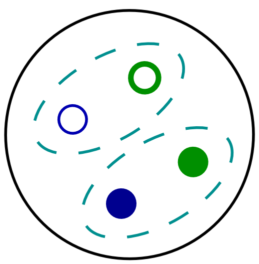
$$\psi_L \to L\psi_L C^T,\; X\to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = 1\!\!1$$

Quark loops generate a kinetic term for X, Y

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

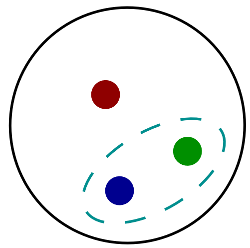
$$\Sigma = XY^\dagger$$

[8]+[1] GBs



$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons



Effective theory: (CFL) baryon chiral perturbation theory

$$\begin{aligned}
\mathcal{L} \; = \; & \frac{f_\pi^2}{4} \left\{ \mathrm{Tr}\left(\nabla_0 \Sigma \nabla_0 \Sigma^\dagger\right) - v_\pi^2 \mathrm{Tr}\left(\nabla_i \Sigma \nabla_i \Sigma^\dagger\right) \right\} \\
& + A \left\{ \left[\mathrm{Tr}\left(M\Sigma\right)\right]^2 - \mathrm{Tr}\left(M\Sigma M\Sigma\right) + h.c. \right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left\{\mathcal{A}_\mu, N\right\}\right) \\
& - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2} \left\{ \mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2 \right\}
\end{aligned}$$

$$\nabla_0 \Sigma = \partial_0 \Sigma + i\hat{\mu}_L \Sigma - i\Sigma\hat{\mu}_R$$

$$D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$$

$$f_\pi^2 = \frac{21 - 8\log 2}{18} \frac{\mu^2}{2\pi^2} \quad v_\pi^2 = \frac{1}{3} \quad A = \frac{3\Delta^2}{4\pi^2} \quad D = F = \frac{1}{2}$$

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(\hat{\mu}_L\Sigma\hat{\mu}_R\Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

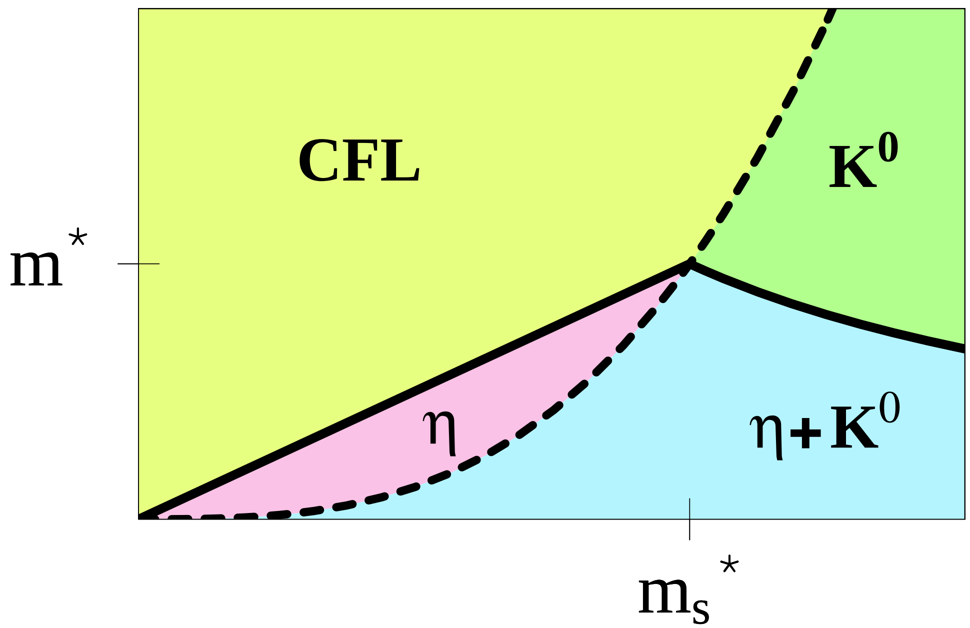
$$V(\Sigma_0) \equiv min$$

Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger\gamma_5\rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi\hat{\mu}_L\xi^\dagger \pm \xi^\dagger\hat{\mu}_R\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure of CFL Phase



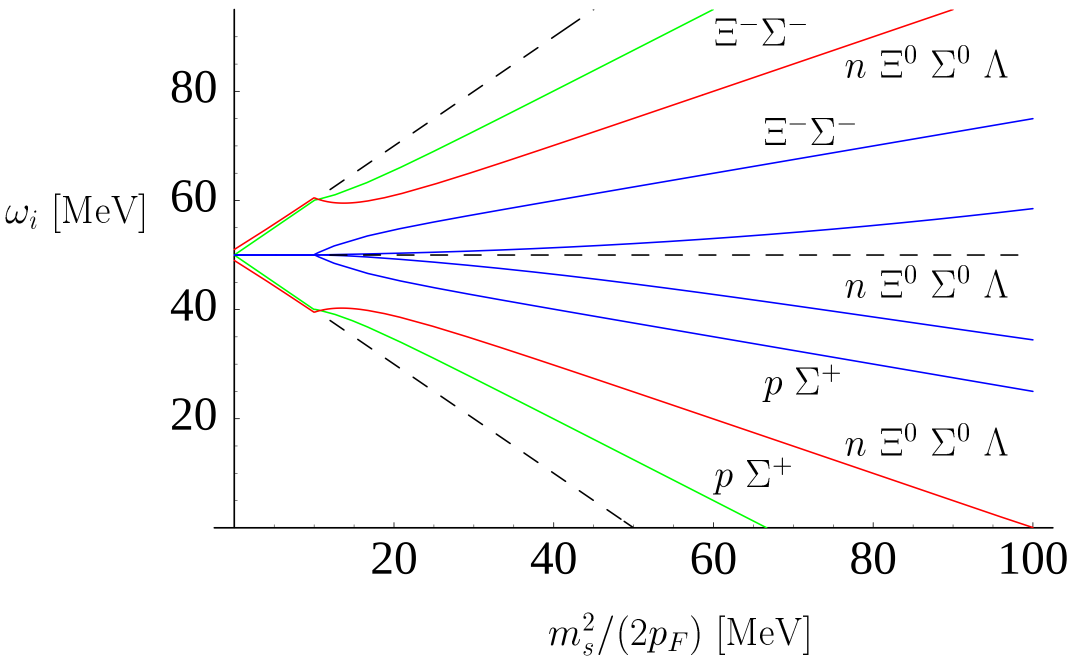
$$m_s^{crit} \sim 3.03\, m_d^{1/3} \Delta^{2/3}$$

$$m^* \sim 0.017\, \alpha_s^{4/3} \Delta$$

QCD realization of s-wave meson condensation

Driven by strangeness oversaturation of CFL state

Fermion Spectrum



$$m_s^{crit} \sim (8\mu\Delta/3)^{1/2}$$

gapless fermion modes (gCFLK)

(chromomagnetic) instabilities ?

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$

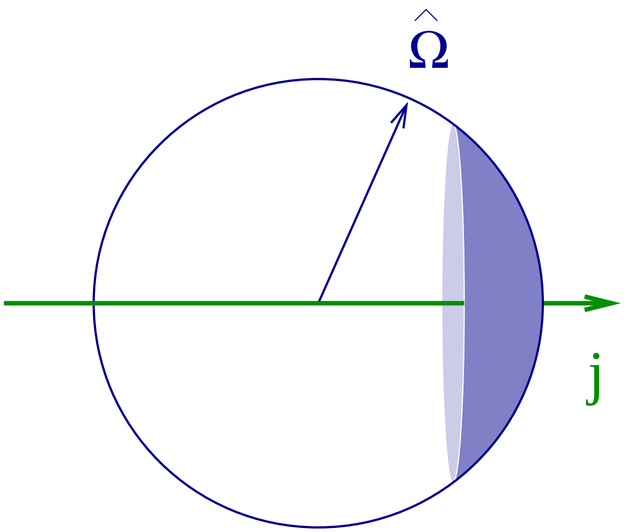
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2} v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$

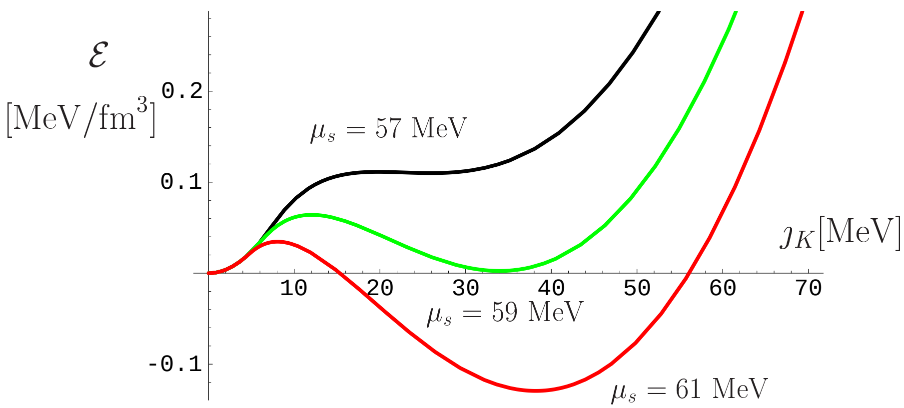
Fermion spectrum

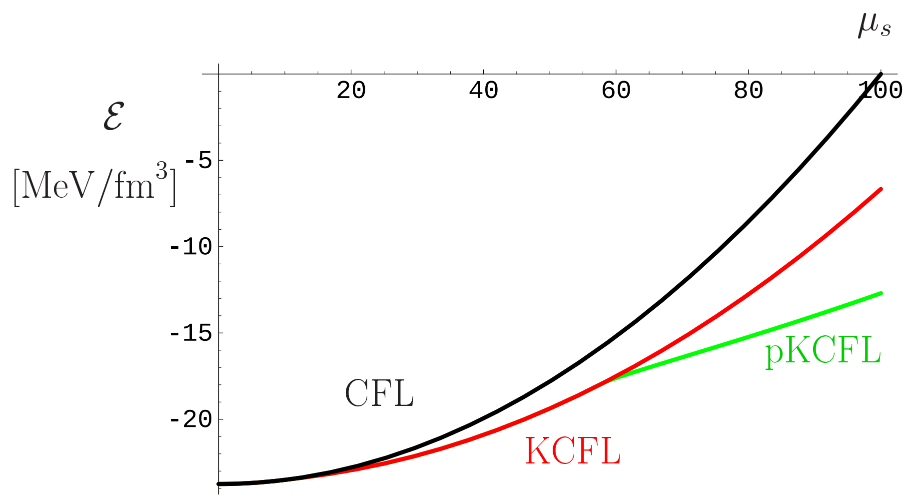
$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\; \omega_l \Theta(-\omega_l)$$



Energy Functional





$$\left.\frac{3\mu_s - 4\Delta}{\Delta}\right|_{crit} = a_{crit} \qquad \frac{\jmath_K}{\Delta} = c_{crit}$$

current strongly suppressed by electric charge neutrality

$m_s^2 \sim 2\mu\Delta$: multiple currents? crystalline state?

Summary

Rich, complicated phase diagram for both $\mu \sim \Lambda_{QCD}$ and $\mu \gg \Lambda_{QCD}$

Depends crucially on N_f, N_c and m_q

Color-flavor-locked (CFL) phase provides weak coupling realization of non-perturbative phenomena

Chiral symmetry breaking, s and p-wave meson condensation

Issues not covered in this talk: Transition to nuclear matter, nuclear exotics, etc.

Constraints from compact star phenomenology