Fluctuations and the QCD critical point

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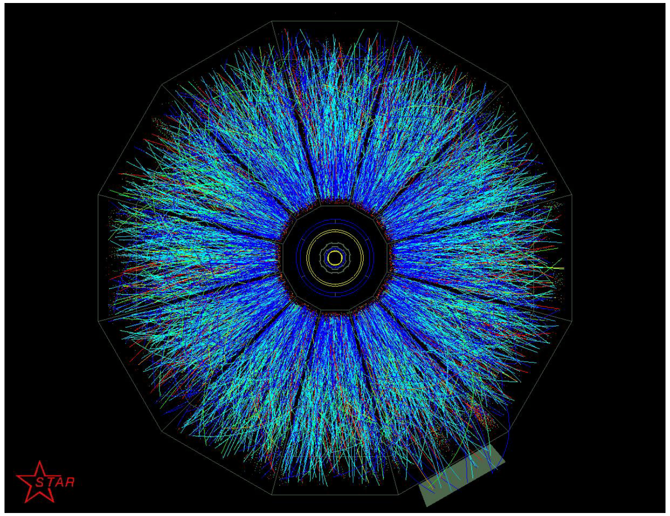
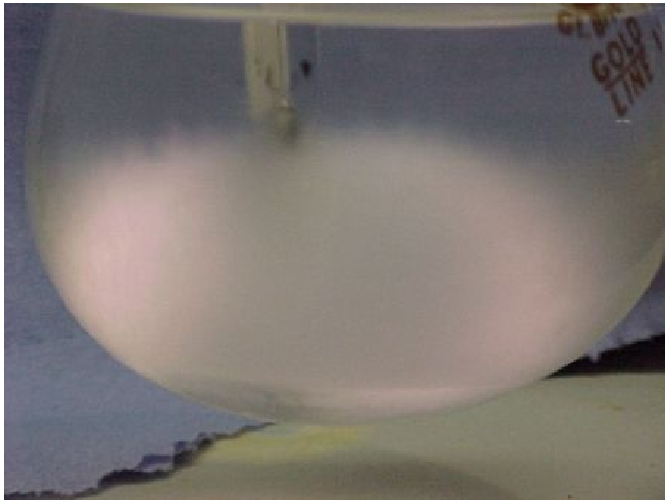
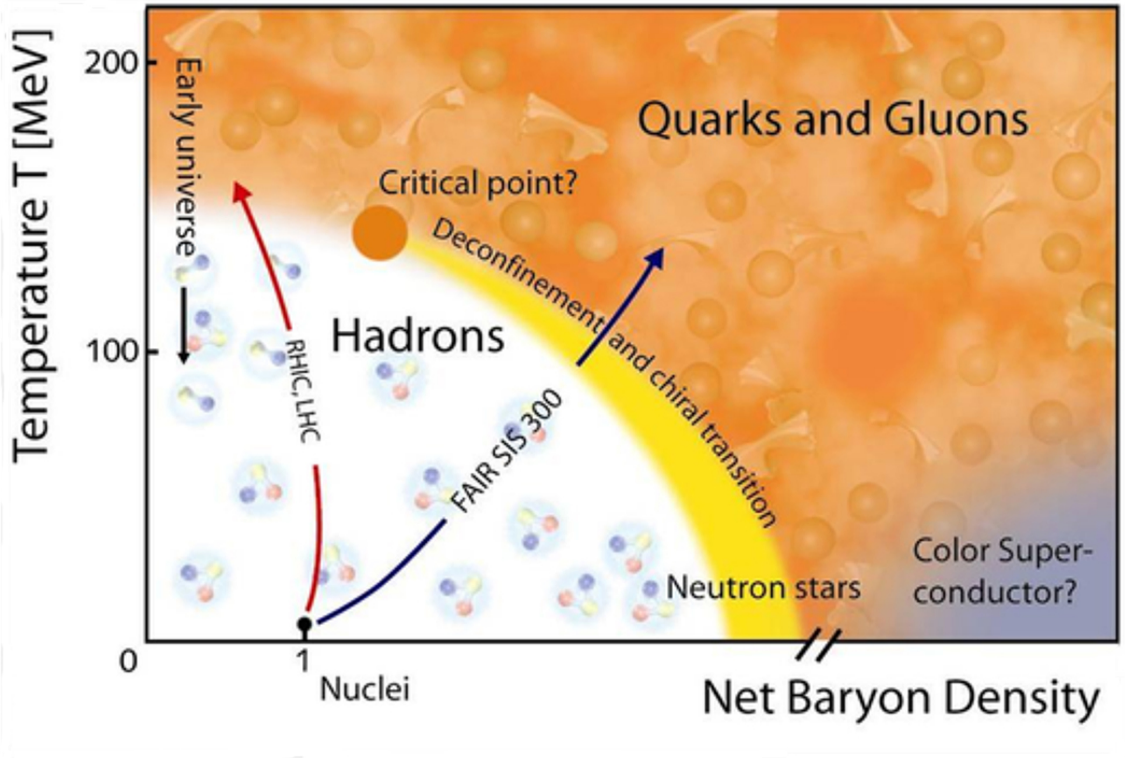
BEST COLLABORATION



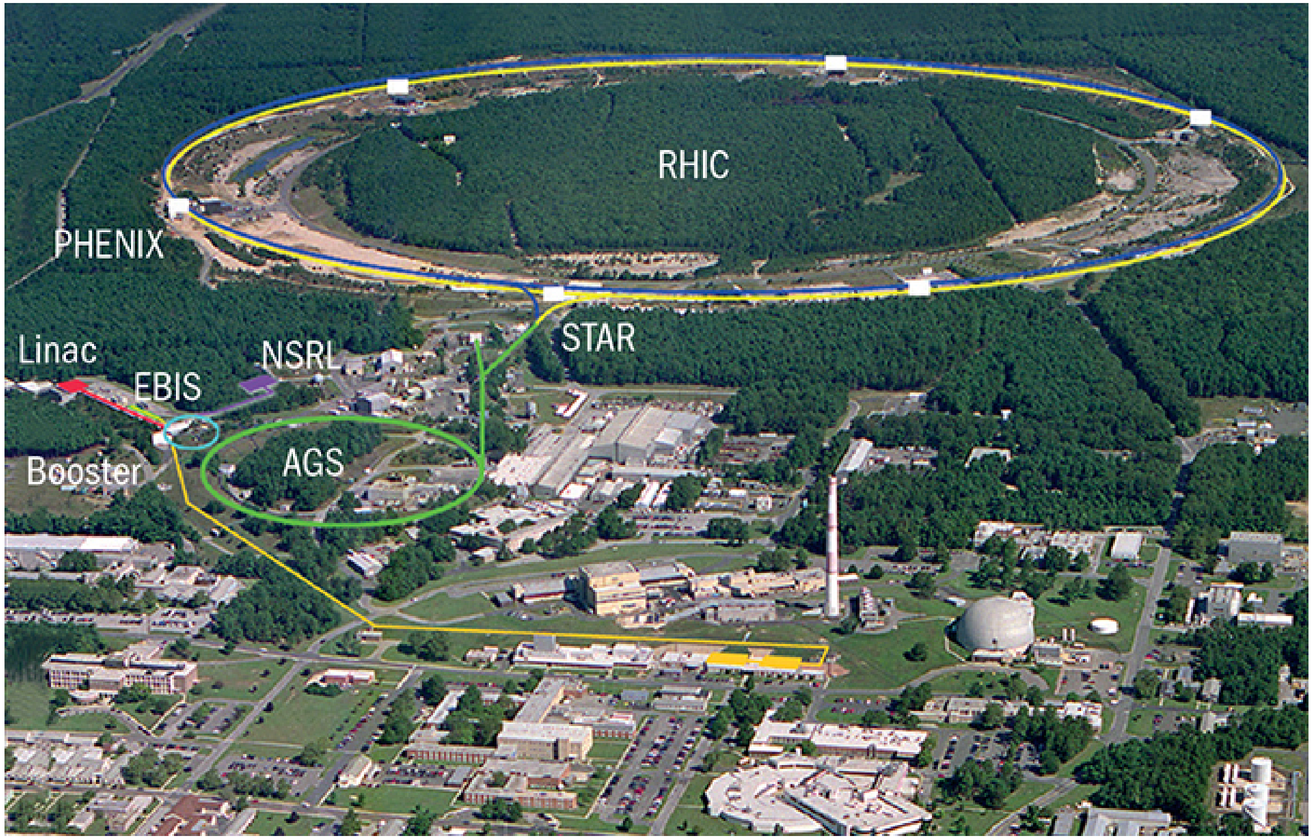


The phase diagram of QCD

$$\boxed{\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu}}$$



2000: Dawn of the collider era at RHIC

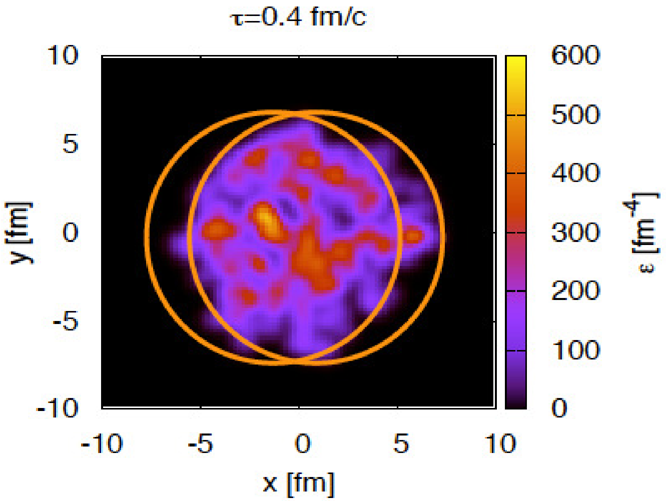


$Au + Au$ @200 $AGeV$

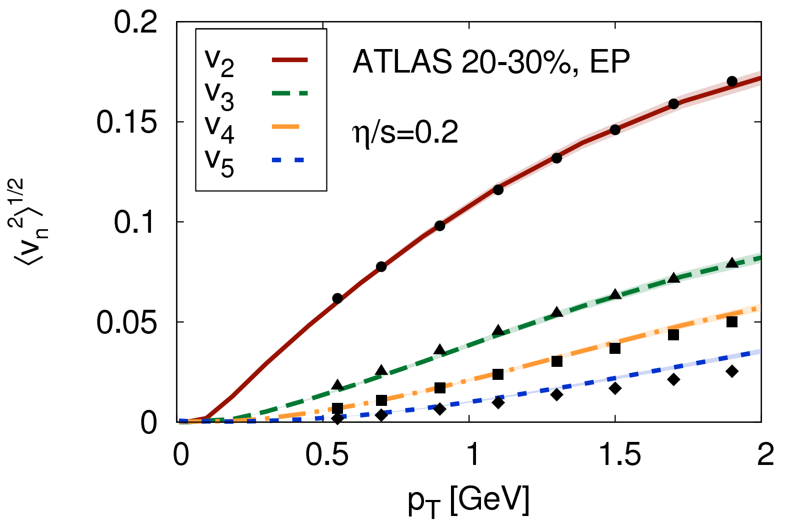
What did we find?

Heavy ion collisions at RHIC are described by a very simple theory:

$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota$ (*everything flows*)



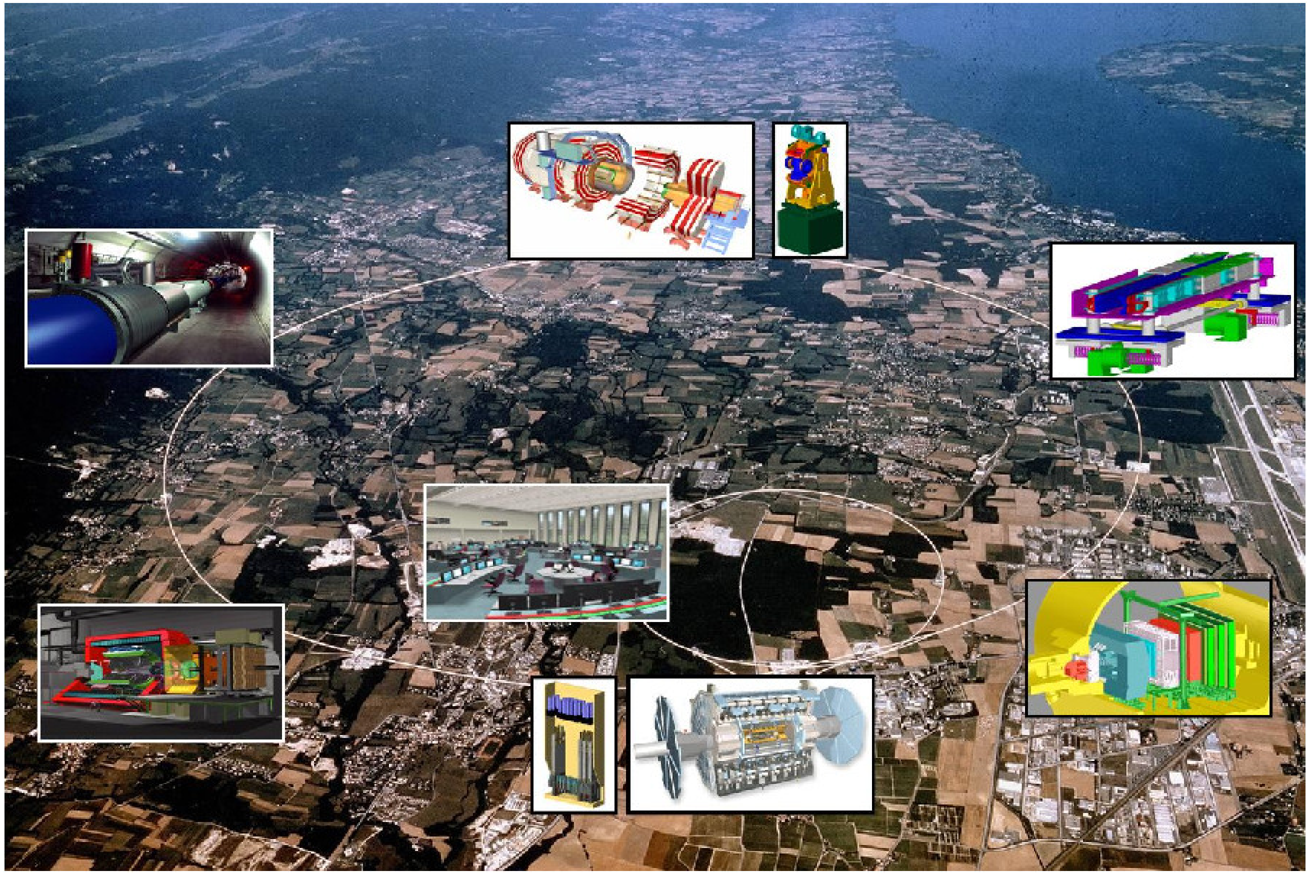
B. Schenke



C. Gale et al.

Hydro converts initial state geometry, including fluctuations, to flow. Attenuation coefficient is small, $\eta/s \simeq 0.08\hbar/k_B$, indicating that the plasma is strongly coupled.

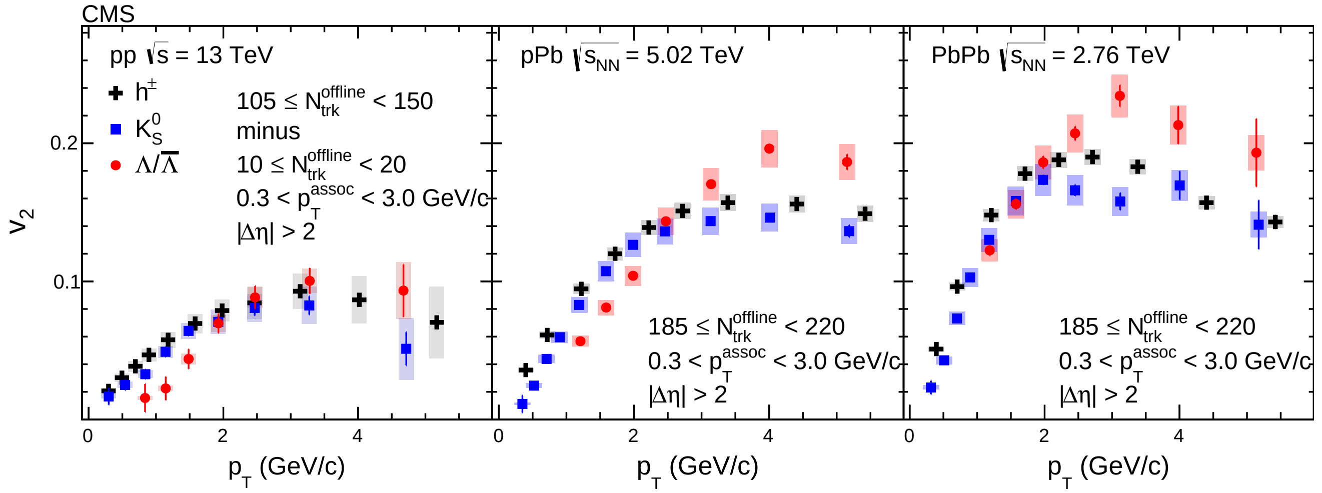
2010: The energy frontier at LHC



$Pb + Pb$ @$2.76\,ATeV$, now $5.5\,ATeV$

What did we find?

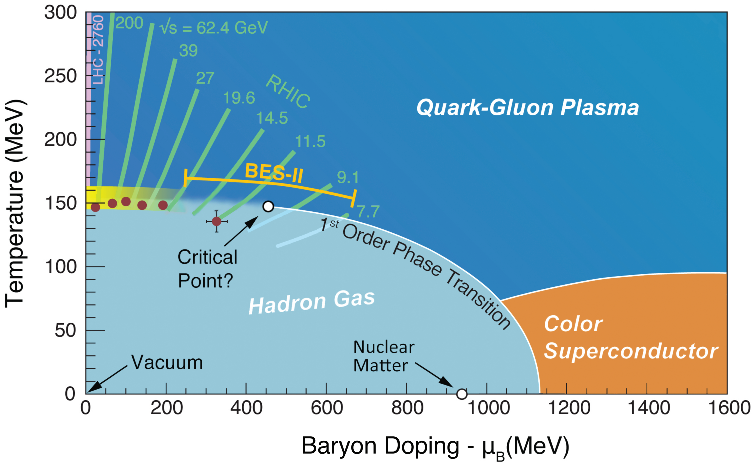
Even the smallest droplets of QGP fluid produced in (high multiplicty) pp and pA collisions exhibit collective flow.

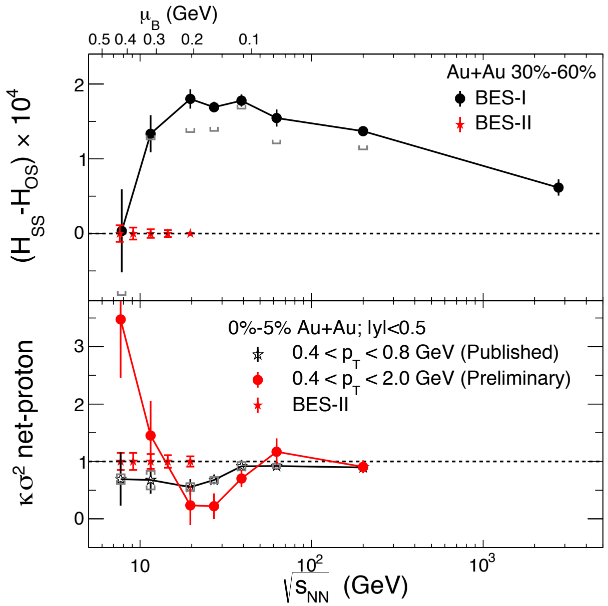


Small viscosity $\eta/s \simeq 0.08\hbar/k_B$ implies short mean free path and rapid hydrodynamization.

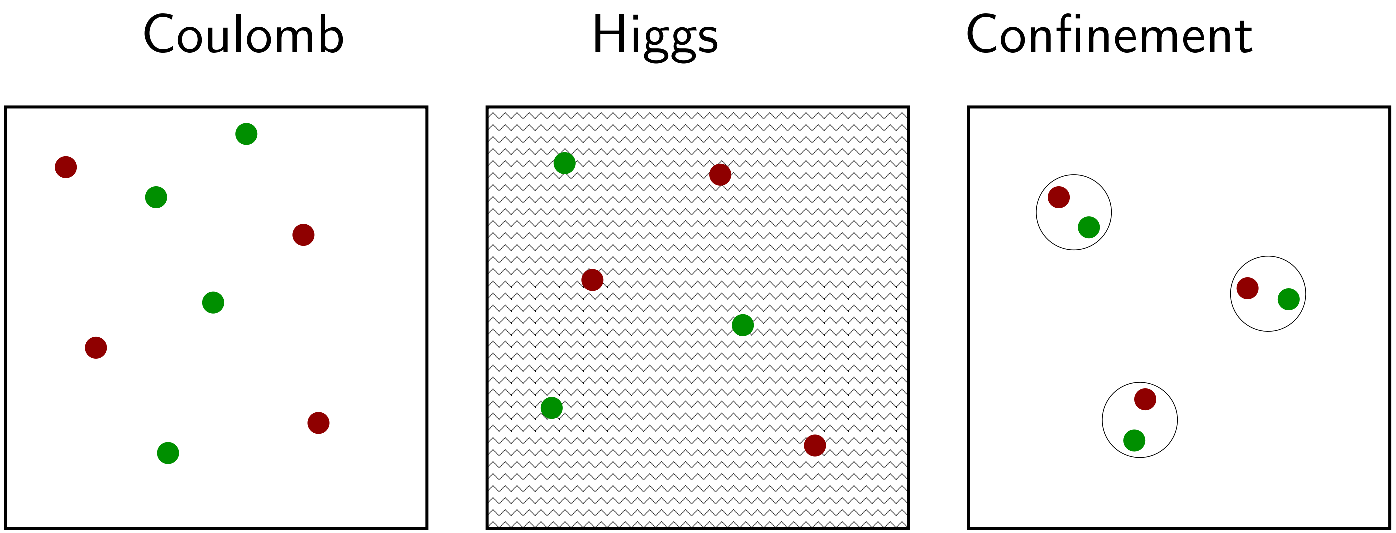
The next step (2010-21): RHIC beam energy scan (BES I/II)

Can we locate the phase transition itself, either by locating a critical point, or identifying a first order transition?





What is a Phase of QCD? Phases of Gauge Theories

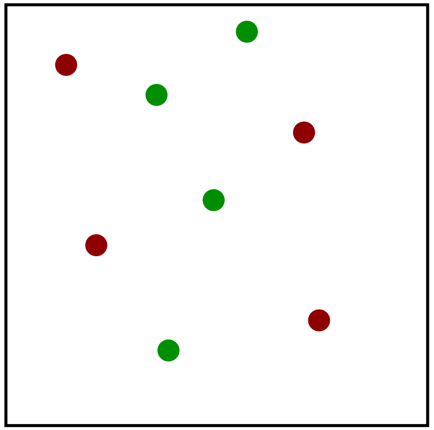


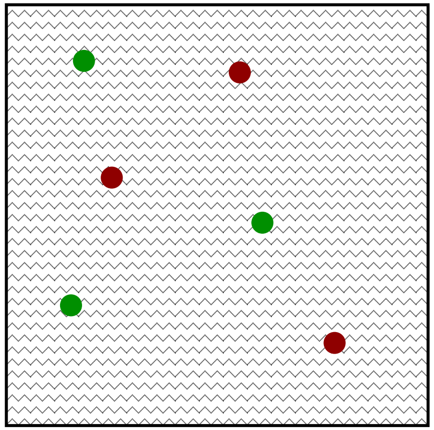
$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

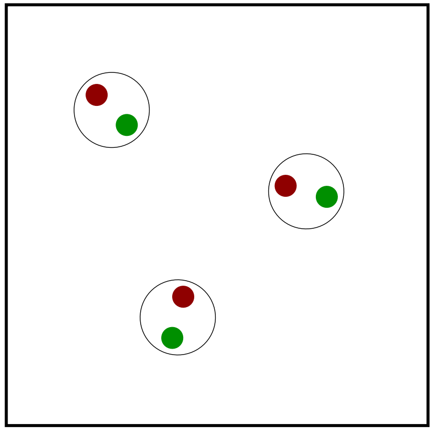
Standard Model: $U(1) \times SU(2) \times SU(3)$

What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement







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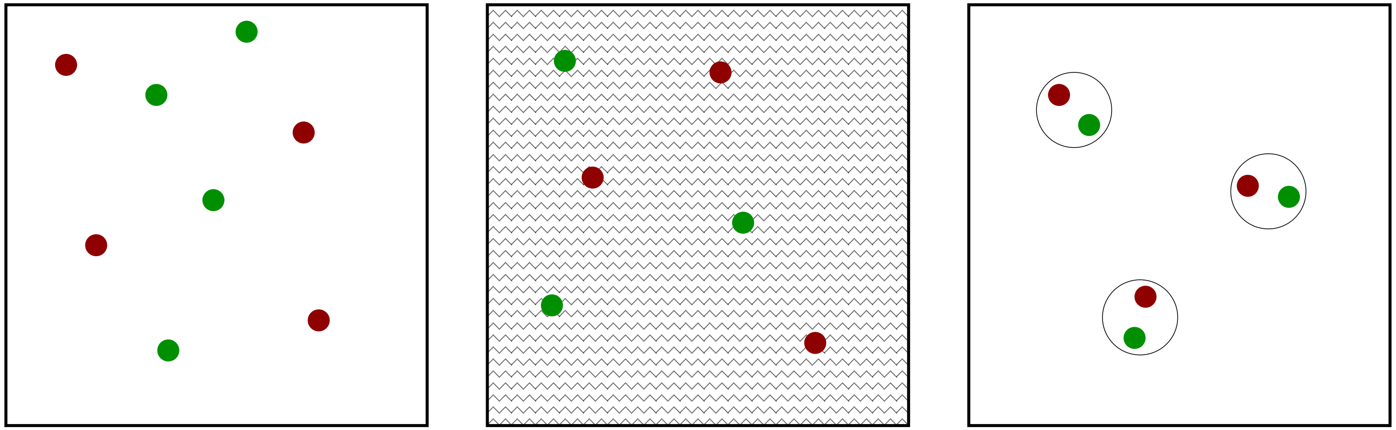
QCD: High T phase — High μ phase — Low T, μ phase

What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement



$$V(r) \sim -\frac{e^2}{r} \qquad\qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad\qquad V(r) \sim kr$$

No local order parameters: Phases can be continously connected.

Phases of QCD: Global symmetries

Local order parameters and change of symmetry: Sharp phase transitions.

$$\vec{M} \to \hat{R}\vec{M} \qquad \langle \vec{M} \rangle \neq 0 \quad \Longrightarrow \quad \textit{Broken Symmetry}$$

QCD: Baryon number (and charge) conservation

$$\psi_f^a \to e^{i\varphi}\psi_f^a$$

Approximate chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

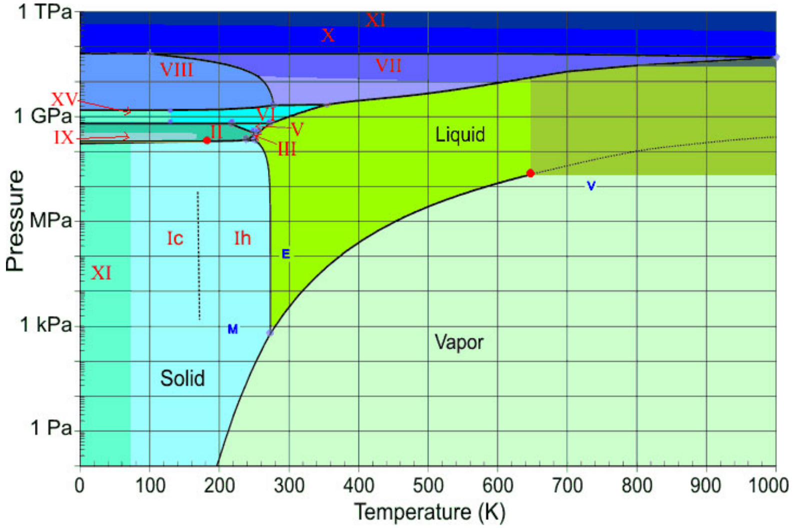
$$\psi_{L,f}^a \to L_{fg}\psi_{L,g}^a, \qquad \psi_{R,f}^a \to R_{fg}\psi_{R,g}^a$$

Broken explicitely by quark masses $m_f \ll \Lambda_{QCD}$, spontaneously by quark condensate

$$\langle \bar{\psi}_{f,L}\psi_{g,R} + \bar{\psi}_{f,R}\psi_{g,L} \rangle \simeq -\delta_{fg}\Sigma$$

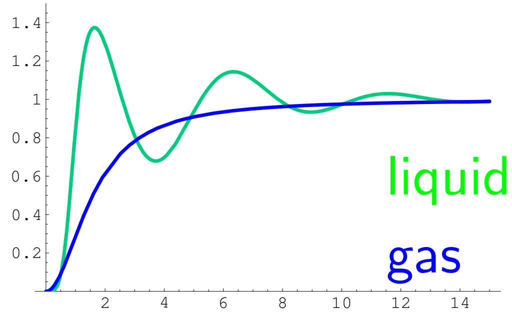
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

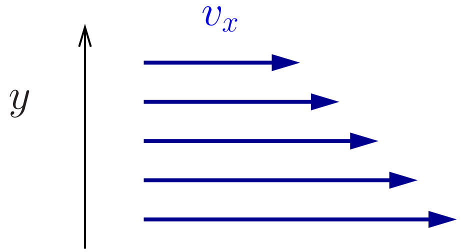


Characteristics of a liquid

Pair correlation function

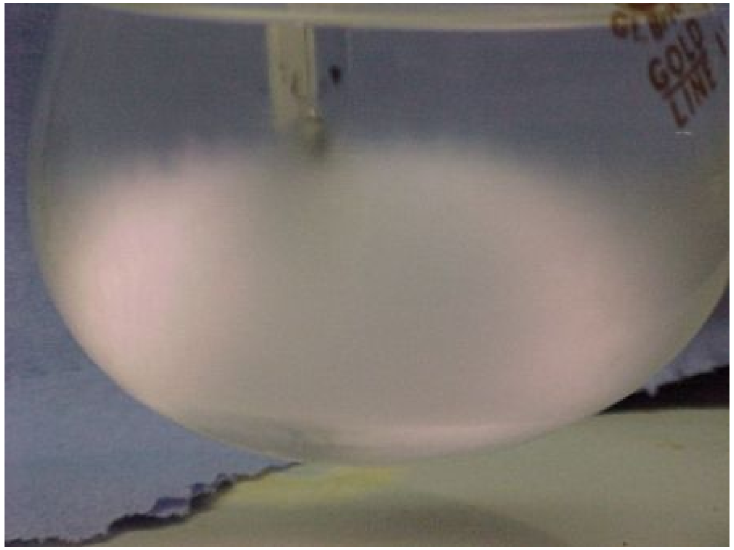


Good fluid: low viscosity



$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

Signatures of the critical endpoint



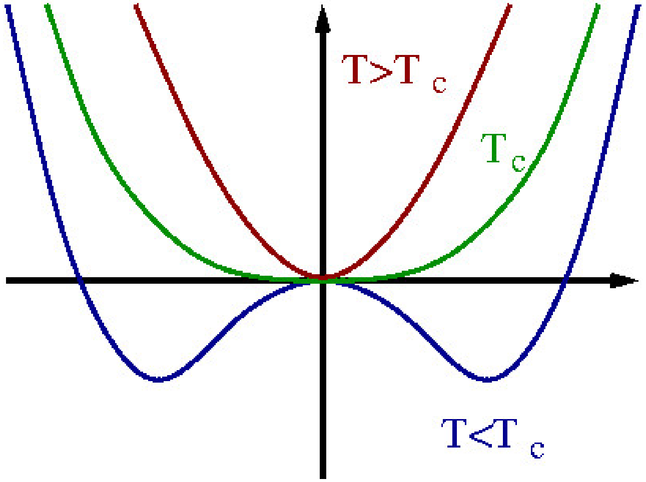
Correlation length diverges

Critical opalescence

Scalar order parameter $\phi = \rho - \rho_0$

$$F[\phi] = \int d^3x \left\{ \kappa (\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + h\phi \right\}$$

Free energy functional:



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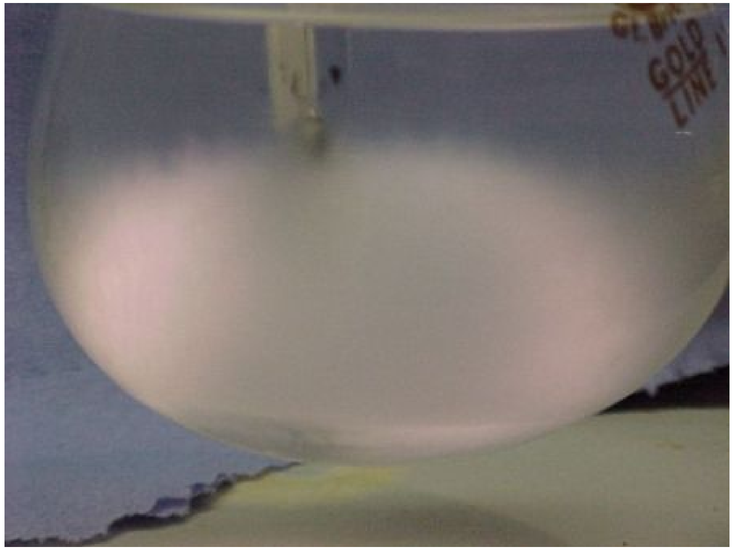

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Scalar order parameter $\phi = \rho - \rho_0$

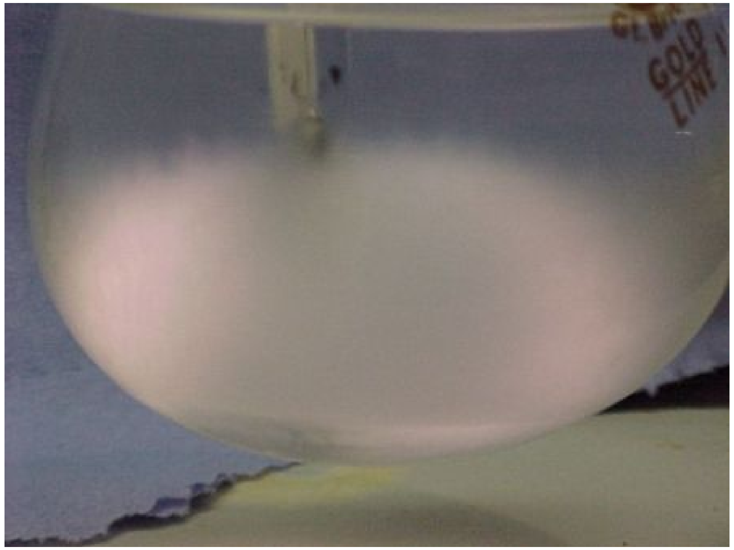
$$F[\phi] = \int d^3x \, \{\kappa(\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + h\phi\}$$

Predicts critical equation of state and correlation length ξ

$$\xi \sim t^{-\nu} \qquad t = \frac{T - T_c}{T_c}$$

$$\nu = \frac{3}{4}\epsilon + O(\epsilon^2) \simeq 0.58$$

Signatures of the critical endpoint



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Scalar order parameter $\phi = \rho - \rho_0$

$$F[\phi] = \int d^3x \left\{ \kappa(\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + h\phi \right\}$$

$F[\phi]$ universal, ϕ could be the magnetization of a spin system.

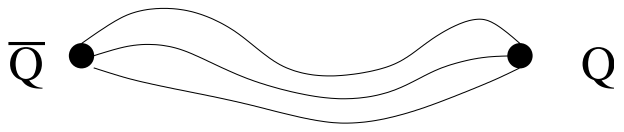
$$\xi \sim t^{-\nu} \qquad t = \frac{T - T_c}{T_c}$$

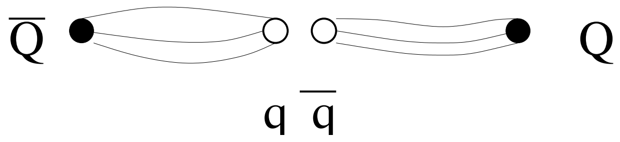
$$\nu = \frac{3}{4}\epsilon + O(\epsilon^2) \simeq 0.58$$

Classical fluids are in the universality class of the $3d$ Ising model.

Critical endpoint in QCD?

Light fermions: Confinement is not a sharp phase transition





Massless fermions: Chiral symmetry breaking is a sharp transition

$$\langle \bar{\psi}_{fL}\psi_{gR} + \bar{\psi}_{fR}\psi_{gR}\rangle \simeq -(230\,\mathrm{MeV})^3\ \delta_{fg}$$

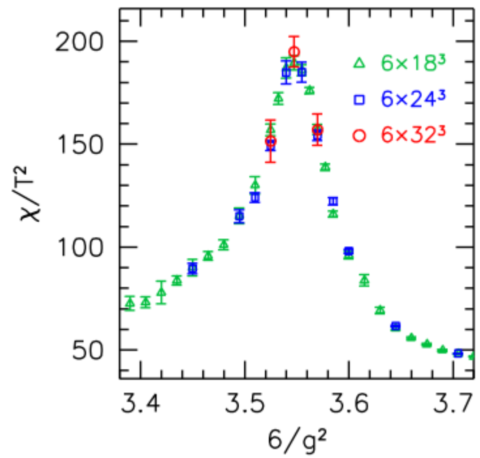
$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

$N_f = 2$: Second order.

$N_f = 3$: First order.

Real world, $m_s > m_{u,d} \neq 0$.

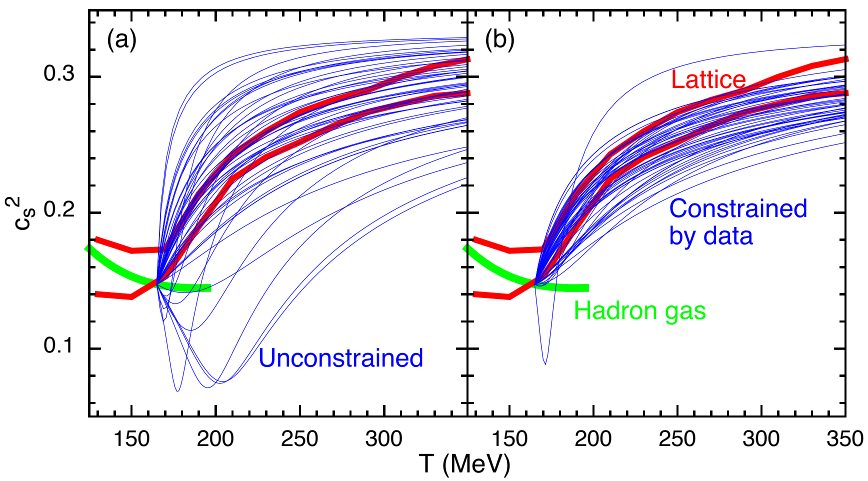
The $\mu = 0$ transition is a crossover.



Aoki et al., Nature (2006)

Crossover: Experimental indications

The speed of sound $c_s^2 = (\partial P)/(\partial \mathcal{E})$ determines the acceleration history of the fireball. Sharp phase transition: $c_s^2 = 0$. Crossover: Soft point $c_s^2(min) > 0$



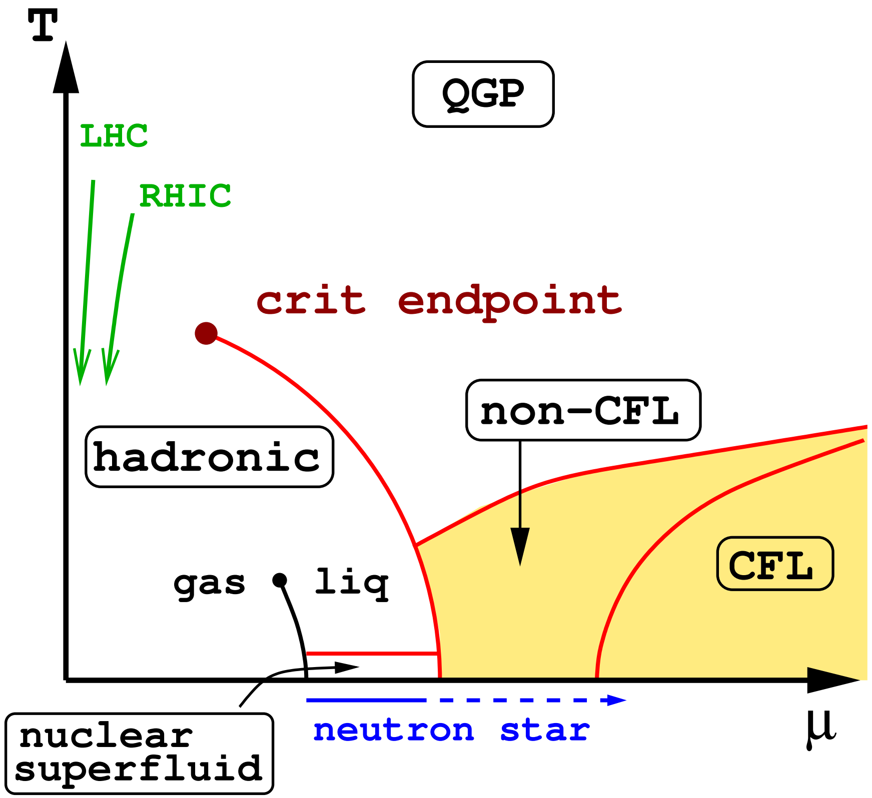
Pratt et al, PRL (2013)

Reconstruct sound speed from particle spectra, HBT source sizes and emission duration

Critical endpoint in QCD?

What happens for $\mu \neq 0$? Lattice calculations cannot tell (the QCD sign problem). Two options: The transition weakens, or it strengthens.

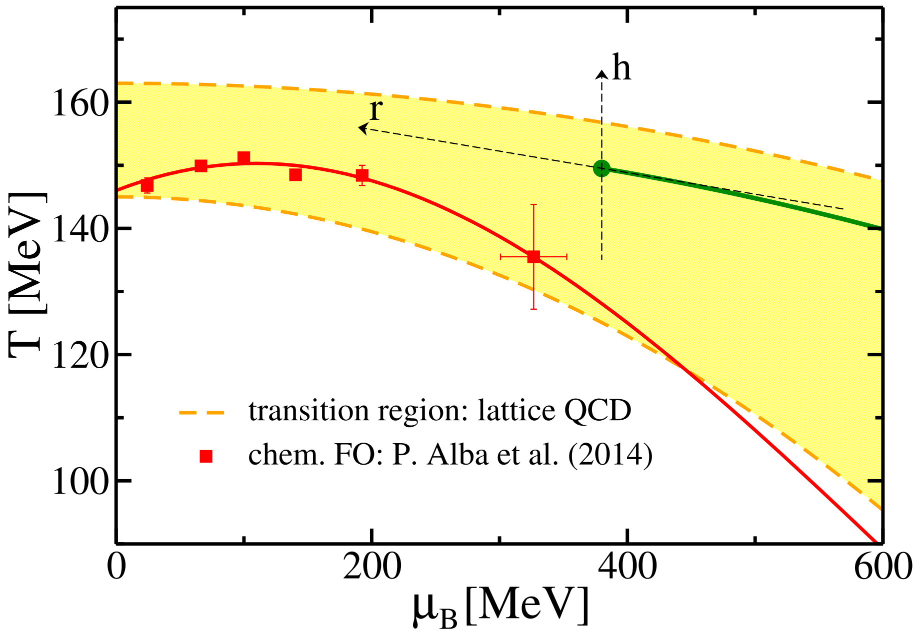
If the transition strengthens for $\mu > 0$ (as suggested by models) then there is a critical endpoint.



Critical endpoint in QCD?

Several possible order parameters: $\langle\bar{\psi}\psi\rangle - \Sigma_0$, $\rho - \rho_0$, $s - s_0$.

All of them mix, obtain one critical mode. Free energy in $d = 3$ Ising universality class.



$$\mathcal{F} = \kappa(\nabla\phi)^2 + r\phi^2 + \phi h$$
$$+ g\phi^3 + \lambda\phi^4$$

External field h.

Reduced temperature r.

$$\xi \sim r^{-\nu}$$

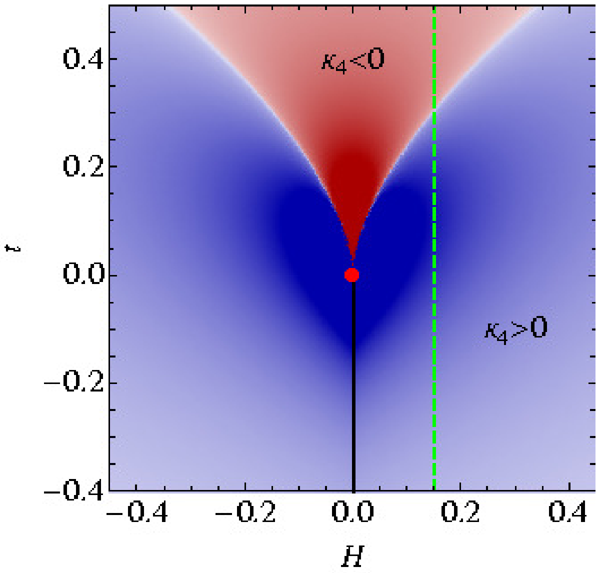
Freezout curve (exp). Transition regime (lattice). Critical line (model).

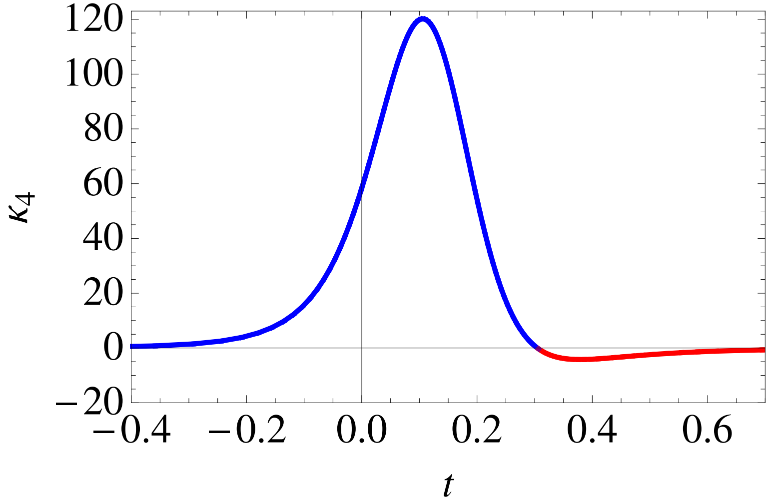
More sensitive observables: Higher order cumulants

Consider curtosis: $\kappa_4 = \langle \phi^4 \rangle - 3\langle \phi^2 \rangle^2$

Stronger divergence near critical point: $\kappa_4/\kappa_2^2 \sim \xi^3$

Non-trivial dependence on t ($\rightarrow$ beam energy)

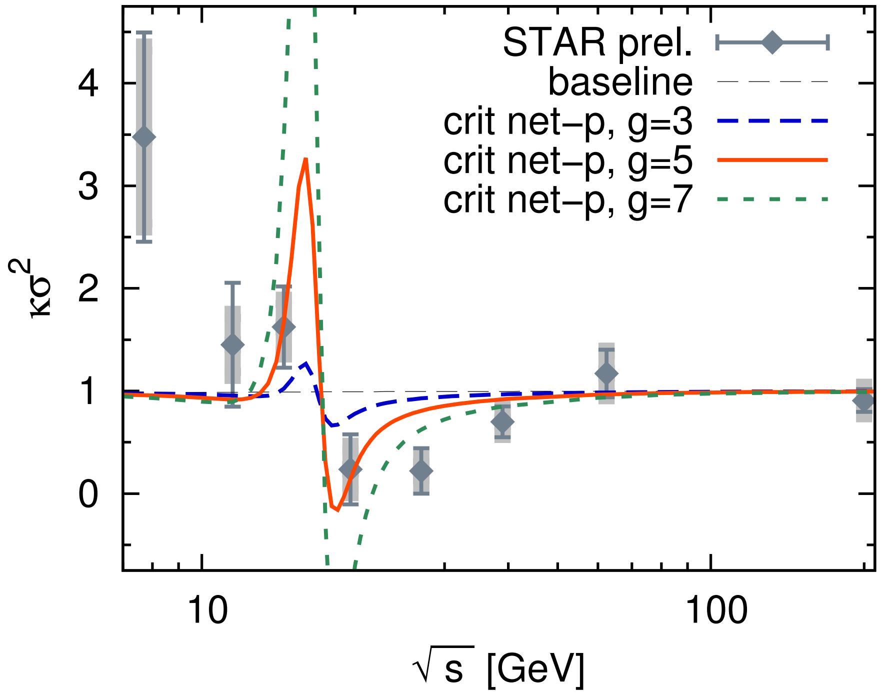




Stephanov, PRL (2011)

Compare to BES-I data

Many details: Couple fluctuations to particles $\delta N_p \sim \phi$, model freezeout curve, map Ising EOS to QCD phase diagram, include resonance decays.



M. Bluhm et al. (2016)

High energy baseline, fluctuations are Gaussian.

Some indication of non-Gaussian behavior at lower energy.

Dynamical Theory

What is the dynamical theory near the critical point?

The basic logic of fluid dynamics still aplies. Important modifications:

- Critical equation of state.
- Possible Goldstone modes (chiral field in QCD?)
- Stochastic fluxes, fluctuation-dissipation relations.

Digression: Diffusion

Consider a Brownian particle

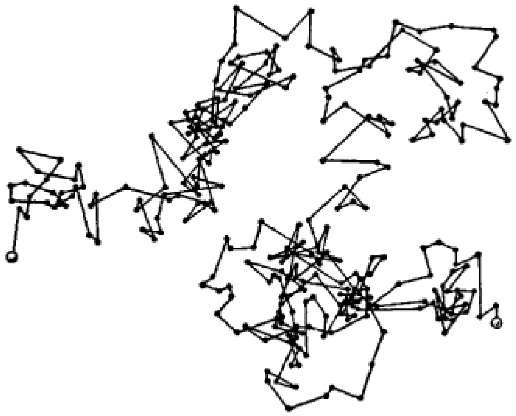
$$\dot{p}(t) = -\gamma_D p(t) + \zeta(t) \qquad \langle \zeta(t)\zeta(t')\rangle = \kappa \delta(t - t')$$

drag (dissipation) white noise (fluctuations)

For the particle to eventually thermalize

$$\langle p^2 \rangle = 2mT$$

drag and noise must be related



$$\kappa = \frac{mT}{\gamma_D}$$

Einstein (Fluctuation-Dissipation)

Hydrodynamic equation for critical mode

Equation of motion for critical mode ϕ ("model H")

$$\frac{\partial \phi}{\partial t} = \lambda \nabla^2 \frac{\delta \mathcal{F}}{\delta \phi} - g \vec{\nabla} \phi \cdot \frac{\delta \mathcal{F}}{\delta \vec{\pi}} + \zeta_\phi$$

Diffusion Advection Noise

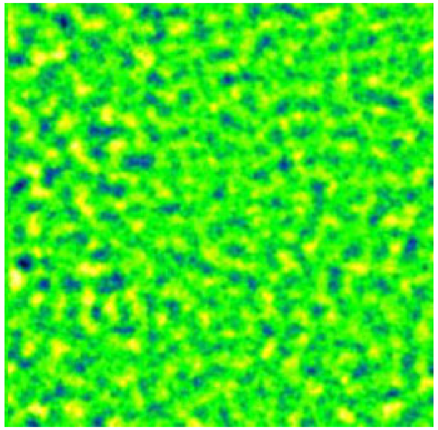
Free energy functional: Order parameter ϕ, momentum density $\vec{\pi} = \rho \vec{v}$

$$\mathcal{F} = \int d^d x \left[\frac{1}{2w} \vec{\pi}^2 + \frac{\kappa}{2} (\vec{\nabla} \phi)^2 + \frac{m^2}{2} \phi^2 + u \phi^4 \right] \qquad D = m^2 \lambda$$

Fluctuation-Dissipation relation

$$\langle \zeta_\phi(x,t) \zeta_\phi(x',t') \rangle = 2DT \delta(x - x') \delta(t - t')$$

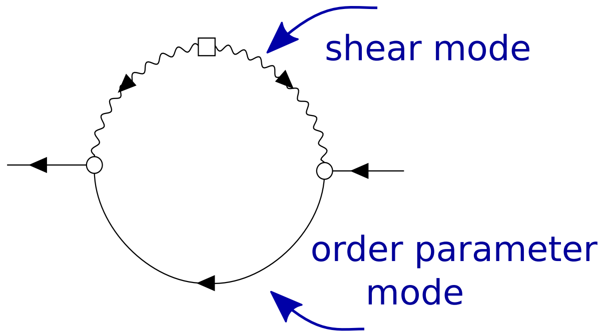
ensures $P[\phi] \sim \exp(-\mathcal{F}[\phi]/T)$



Critical Dynamics: Infinite Volume

Study relaxation of order parameter near equilibrium. "Mode Coupling" approximation: Use bare shear viscosity, and static susceptibility χ_k

$$G^{-1}(\omega,k) = i\omega - Dk^2 - \delta\Gamma_k$$



Order parameter relaxation rate ("Kawasaki function").

$$\Gamma_k = \frac{T\xi^{-3}}{6\pi\eta_0} K(k\xi) \qquad K(x) = \frac{3}{4}\left[1 + x^2 + \left(x^3 + x^{-1}\right)\arctan(x)\right] .$$

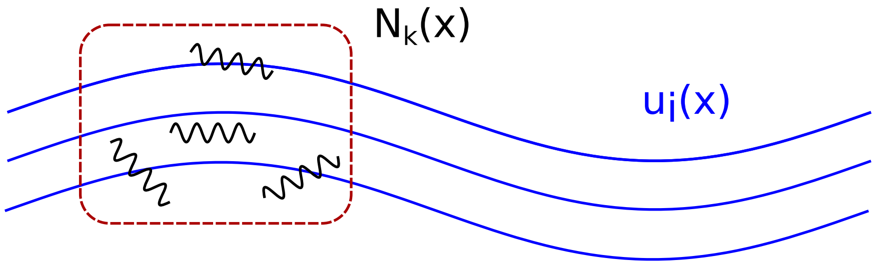
Dynamic critical exponent: $\Gamma_{\xi^{-1}} \sim \xi^{-z}$ with $z = 3$

Fluctuations in an expanding fluid

Consider linearized stochastic dynamics about a fluid background.

Determine eigenmodes: two sound $\phi_\pm$, three diffusive modes $\phi_\psi, \phi_{\vec{\pi}_T}$.

Noise average: Consider equal time 2-point fct $W_{ab} = \langle \phi_a(\tau, x)\phi_b(\tau, x')\rangle$.



Wigner function representation: $W_{ab}(\tau, x, k)$. Diagonal component $N_{a,k}(\tau, x)$ is a phase space density of hydro fluctuations.

Akamatsu et al. (2016), Martinez, T.S. (2017).

Critical mode in expanding system

Study transit of critical point: Consider $\hat{s} = s/n$ and follow "mode coupling" philosophy. Use static susceptibilty and critical relaxation rate $\Gamma_{\hat{s}}$.

$$\partial_t N_{\hat{s}}(t,k) = -2\Gamma_{\hat{s}}(t,k)\left[N_{\hat{s}}(t,k) - N^0_{\hat{s}}(t,k)\right] + \ldots,$$

$$\Gamma_{\hat{s}}(t,k) = \frac{\lambda_T}{C_p\xi^2}(k\xi)^2(1+(k\xi)^{2-\eta}), \qquad N^0_{\hat{s}}(t,k) = \frac{C_p(t)}{(1+(k\xi)^{2-\eta})},$$

Correlation length $\xi(t) = \xi(n(t),e(t)) = \xi_0 f_\xi(r(t),h(t))$

$$\text{hydro}: \ \frac{\partial_t n}{n} \sim \frac{\partial_t e}{e} \sim \frac{1}{\tau_{exp}} \qquad \text{Ising map}: \ (e,n) \to (r,h)$$

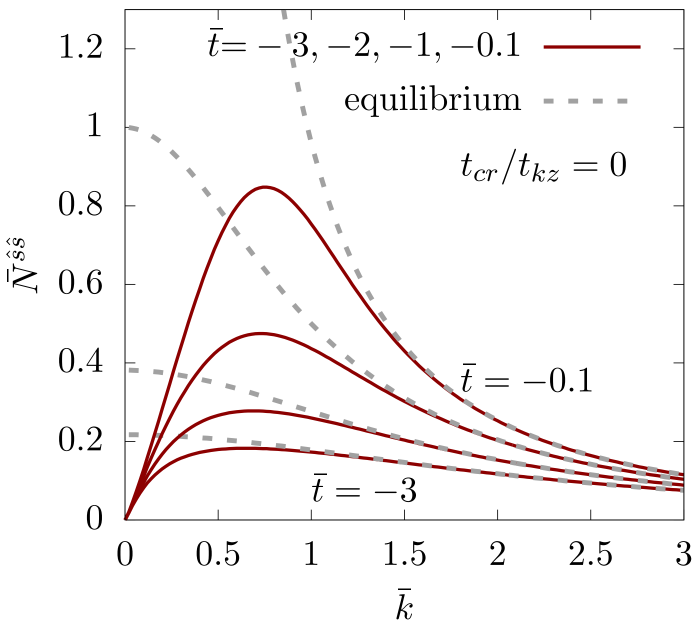
Emergent time scale t_{KZ}: Expansion rate matches relaxation time for modes with $k^* \sim \xi^{-1}$ (modes fall out of equilibrium).

Emergent length scale l_{KZ}: $l_{KZ} = \xi(t_{KZ})$. $l_{KZ} \sim 1.6$ fm

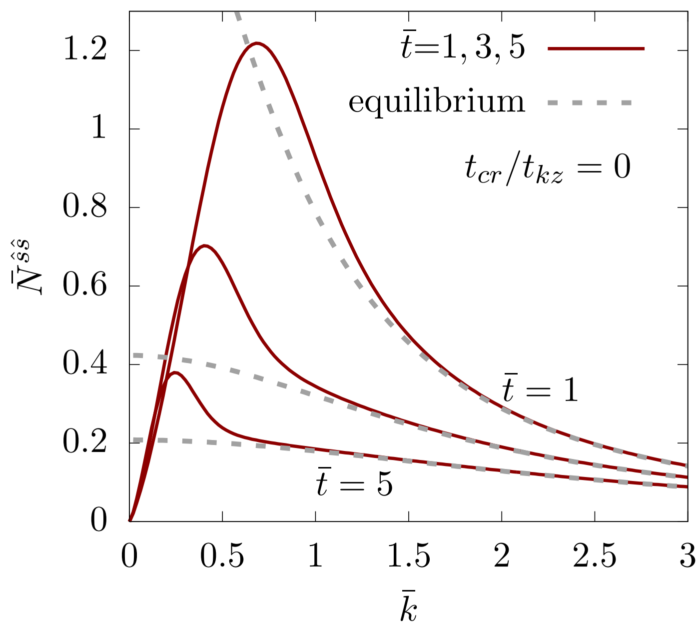
Akamatsu, Teaney, Yan, Yi [1811.05081], Martinez, Schaefer, Skokov [1906.11306], see also Berdnikov, Rajagopal [hep-ph/9912274]

Expanding System: Numerical Results

$$\bar{k} = kl_{KZ},\, \bar{t} = t/t_{KZ}$$



before CP



after CP

Akamatsu, Teaney, Yan, Yi [1811.05081]

Outlook

Opportunity: Discover QCD critical point by observing critical fluctuations in heavy ion collisions. Intriguing hints present in BES-I data.

Challenge: Propagate fluctuations of conserved charges in relativistic fluid dynamics. Describe initial state fluctuations and final state freezeout.

Experiment: BES-II is being analyzed.

Learned many things abot fluid dynamics along the way.