In Search of the Perfect Fluid

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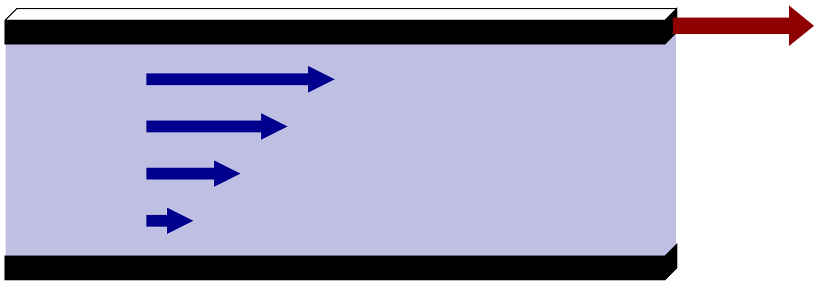
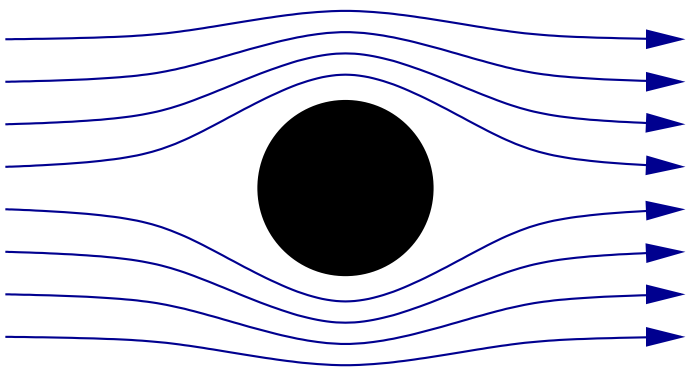
Measures of Perfection

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Dimensionless measure of shear stress: Reynolds number



$$Re \quad = \quad \frac{n}{\eta} \quad \times \quad mvr$$

fluid property — flow property

- $[\eta/n] = \hbar$
- Relativistic systems $Re = \dfrac{s}{\eta} \times \tau T$

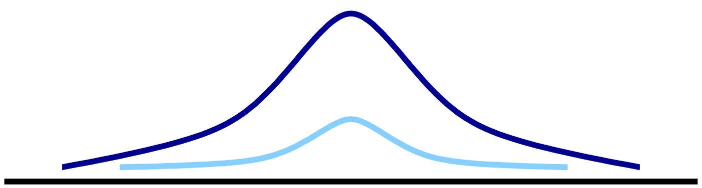
Other sources of dissipation (thermal conductivity, bulk viscosity, ...) vanish for certain fluids, but shear viscosity is always non-zero.

There are reasons to believe that η is bounded from below, possibly by some constant times $\hbar s/k_B$.

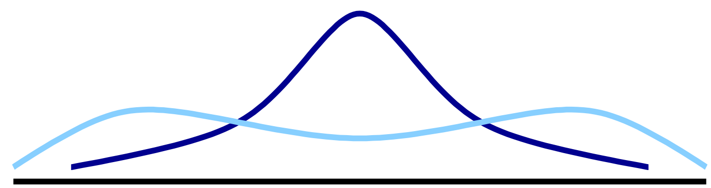
A fluid that saturates the bound is a "perfect fluid".

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



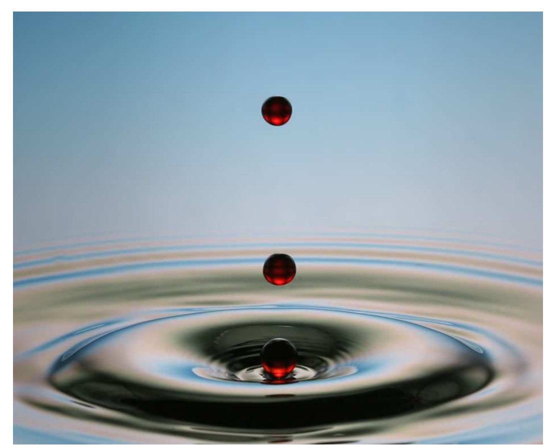
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

Conservation laws: mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\,\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

[Euler/Navier-Stokes equation]

Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + \ldots$$

reactive dissipative

Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

$$T_{ij} = \int d^3p \, \frac{p_i p_j}{E_p} f_p$$

Boltzmann equation

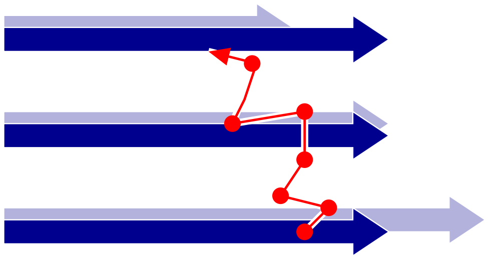
$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

Collision term $C[f_p]$

Linearized theory (Chapman-Enskog): $f_p = f_p^0 (1 + \chi_p / T)$

suitable for transport coefficients

shear viscosity $\chi_p = g_p p_x p_y \partial_x v_y$



Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

$$\eta \sim \frac{1}{3} n\, \bar{p}\, l_{mfp} \qquad\qquad (\mathrm{Note}: l_{mfp} \sim 1/(n\sigma))$$

Normalize to density. Uncertainty relation implies

$$\frac{\eta}{n} \sim \bar{p}\, l_{mfp} \geq \hbar$$

Also: $s \sim k_B n$ and $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as $\bar{p}\, l_{mfp} \sim \hbar$?

Effective Theories for Fluids (Here: Weak Coupling QCD)

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

What if (the coupling is strong)? Kubo Formula

Linear response theory provides relation between transport coefficients and Green functions

$$G_R(\omega, 0) = \int dt\, d^3x\, e^{i\omega t}\Theta(t)\langle[T_{xy}(t,x), T_{xy}(0,0)]\rangle$$

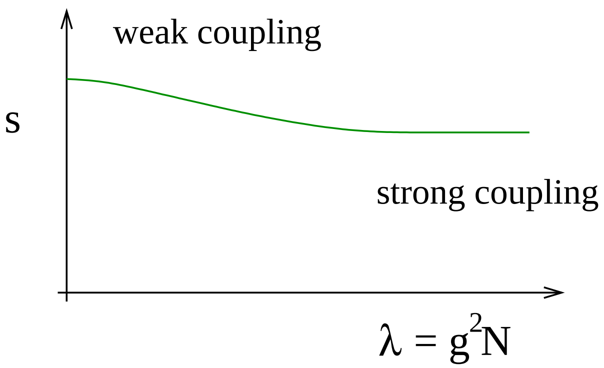
$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}G_R(\omega, 0)$$

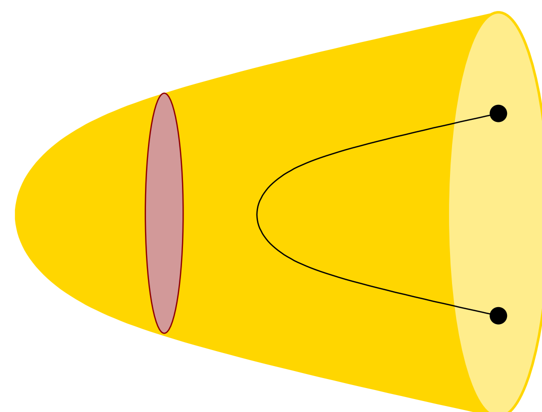
This result is hard to use for quantum fluids, but there are some heroic efforts by lattice QCD theorists, e.g. Meyer (2007).

Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon





$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov (1996)

Holographic Duals: Transport Properties

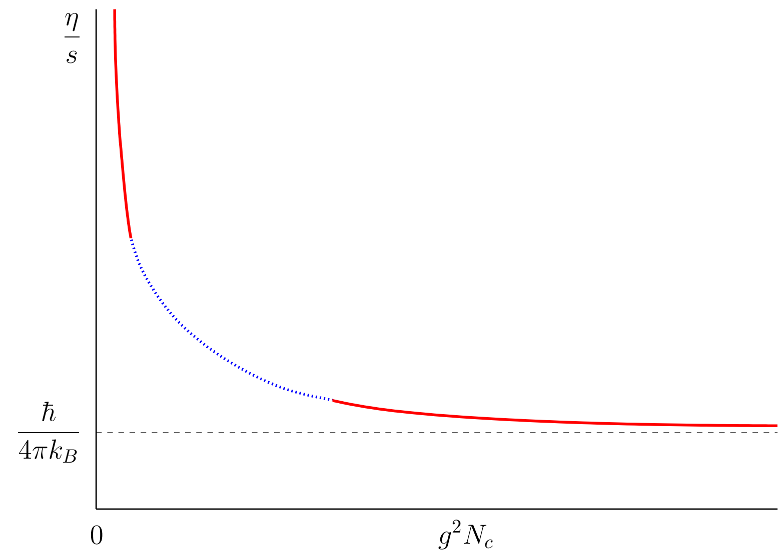
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

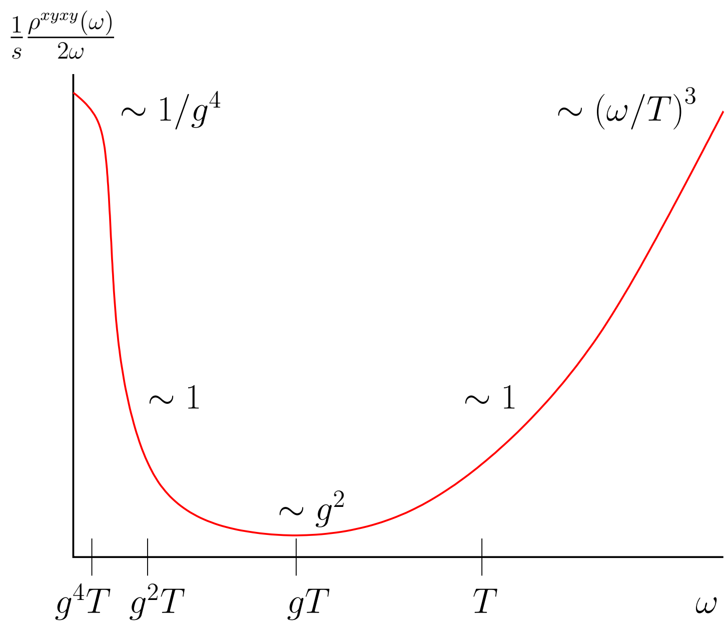
Effective Theories (Strong coupling)

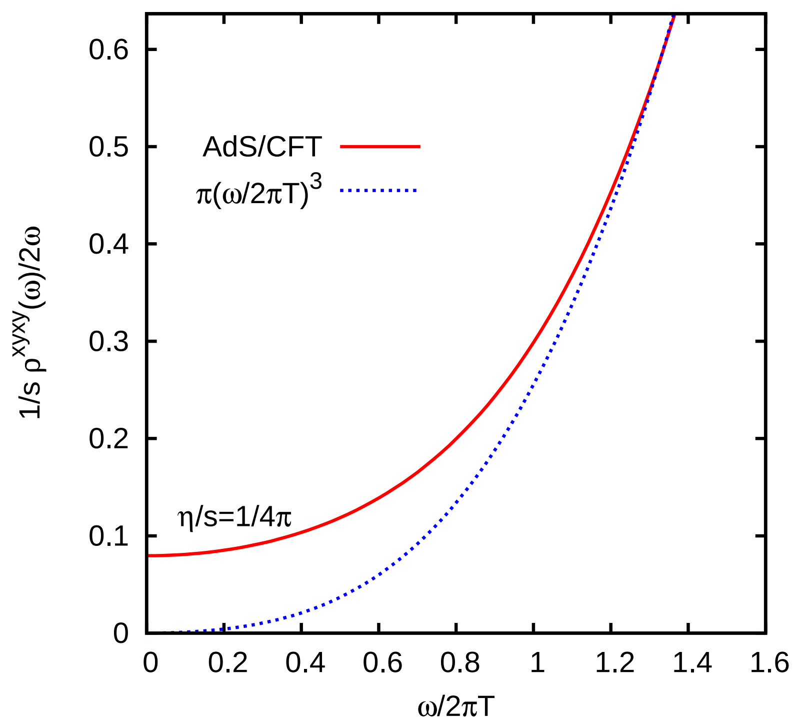
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \mathrm{Im}G_R(\omega, 0)$ associated with T_{xy}





weak coupling QCD — strong coupling AdS/CFT

transport peak vs no transport peak

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

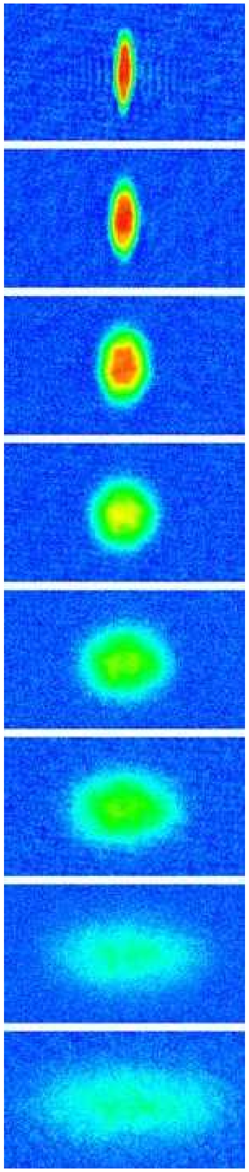
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

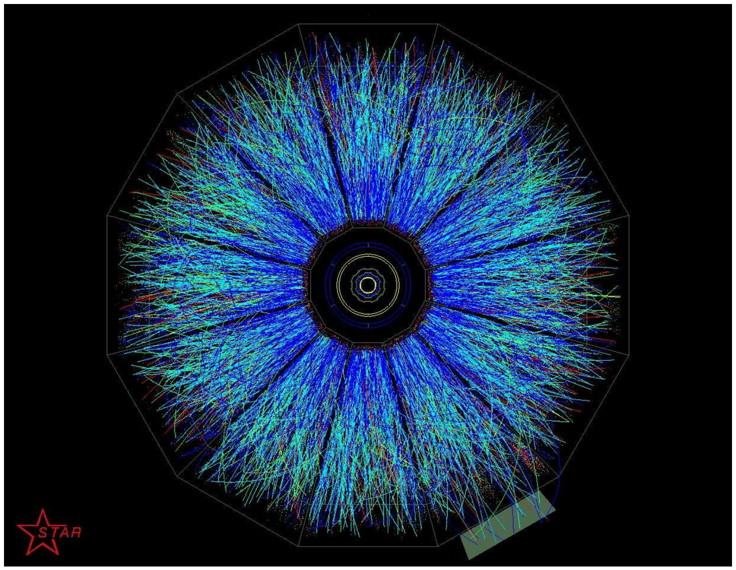
(Almost) scale invariant systems

Perfect Fluids: The contenders

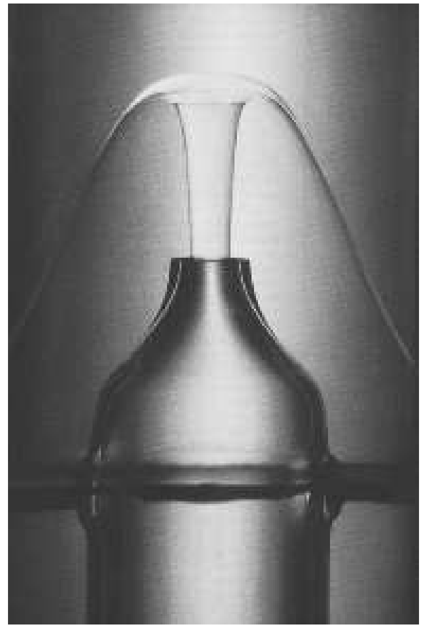


Trapped Atoms

(T=0.1 neV)



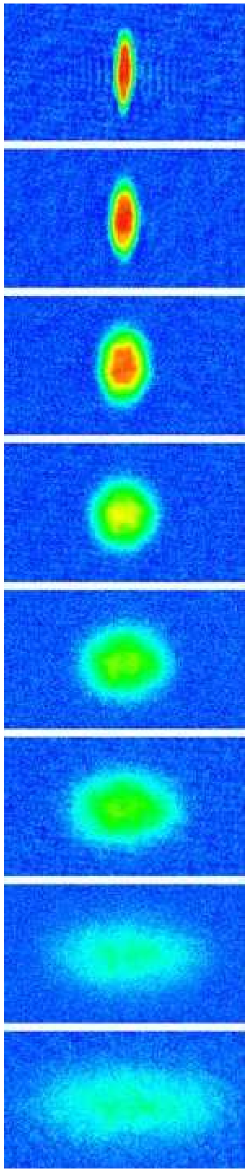
QGP (T=180 MeV)

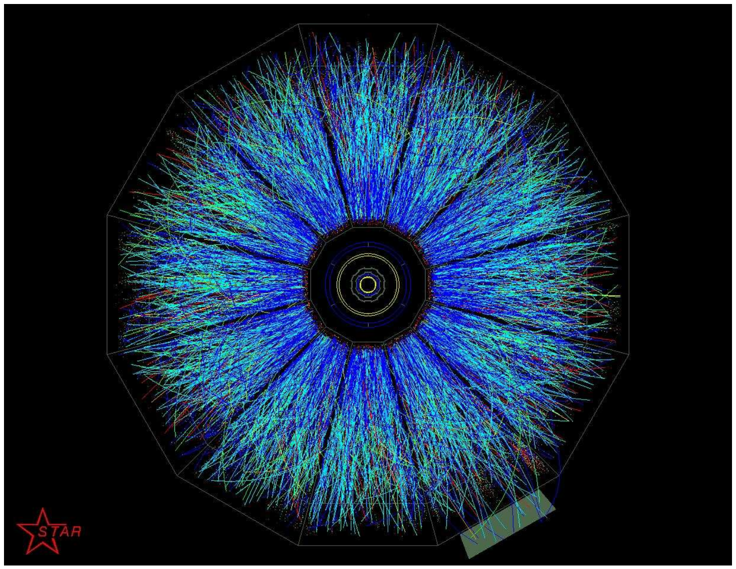


Liquid Helium

(T=0.1 meV)

Perfect Fluids: The contenders





QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

Liquid Helium

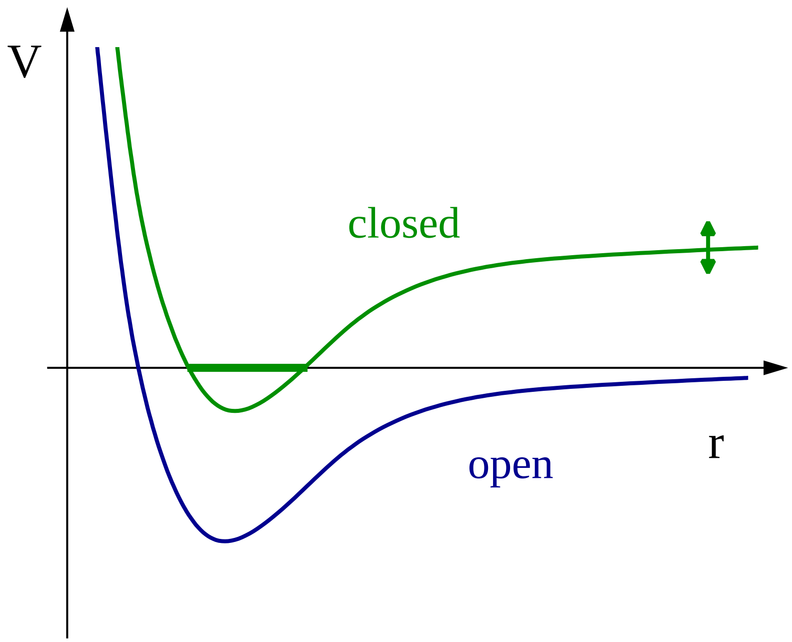
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

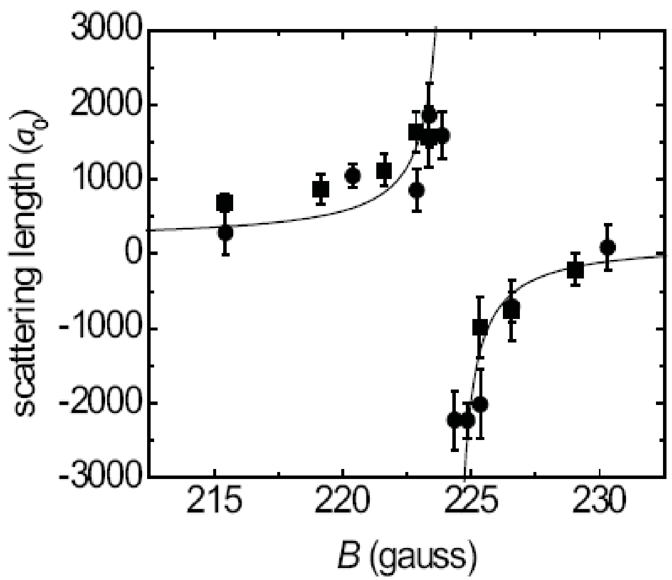
Consider ratios

η / s

I. Unitary Fermi Gas

Atomic gas with two spin states: "↑" and "↓"

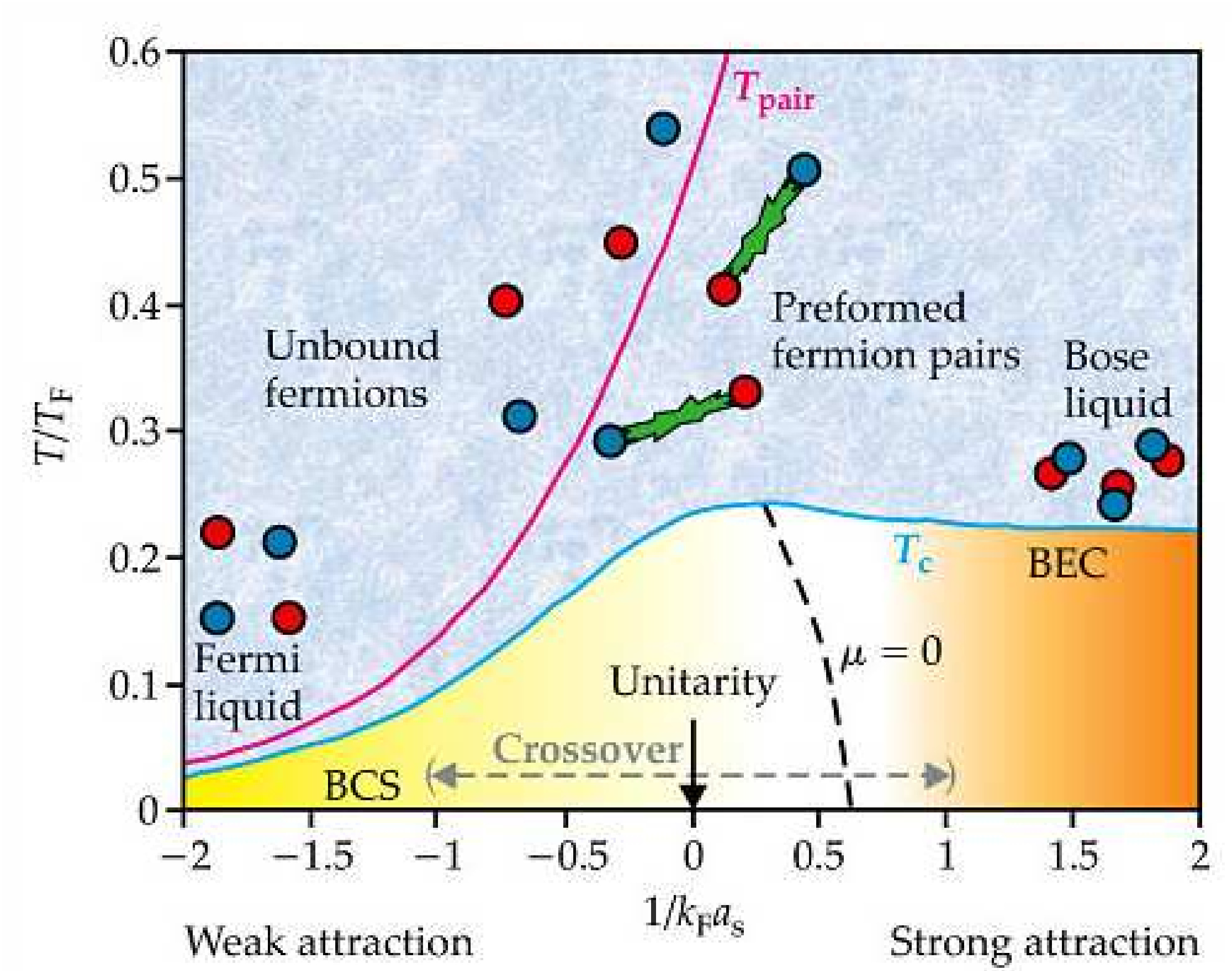




Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$ $$\sigma = \frac{4\pi}{k^2}$$

Fermi Gas at Unitarity: Phase Diagram



Fermi gas at unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

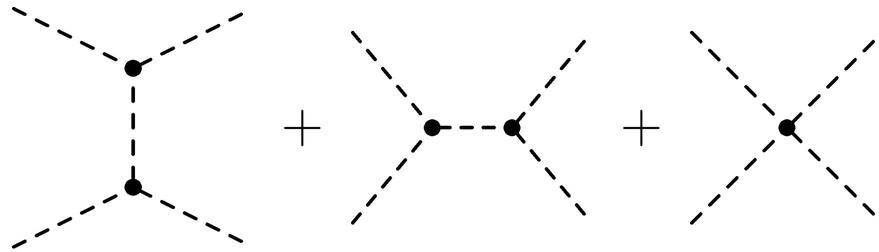
Low T ($T < T_c \sim \mu$): Pairing and superfluidity

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

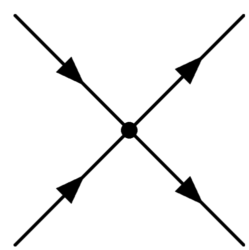
$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



T.S., G.R. (2007)

High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



Bruun (2005)

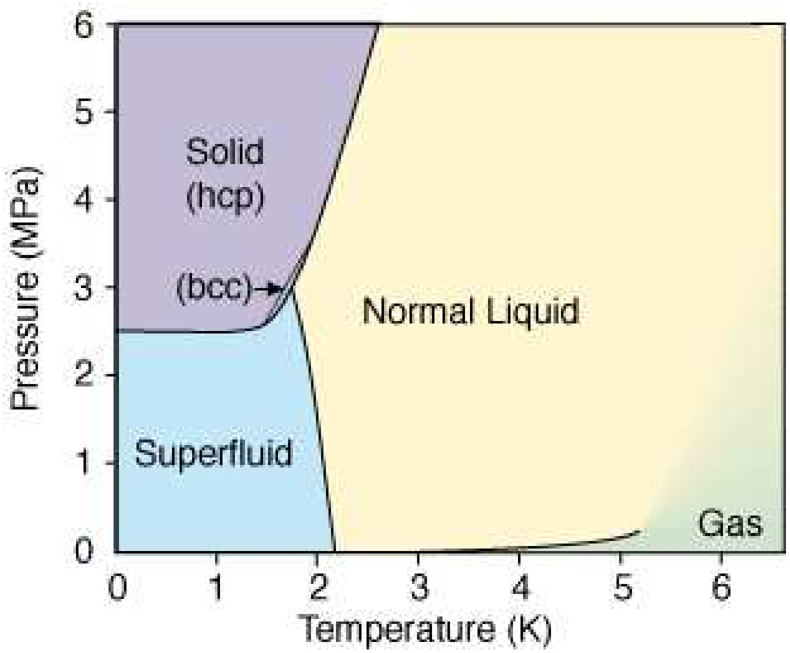
II. Liquid Helium

Bosons, van der Waals + short range repulsion

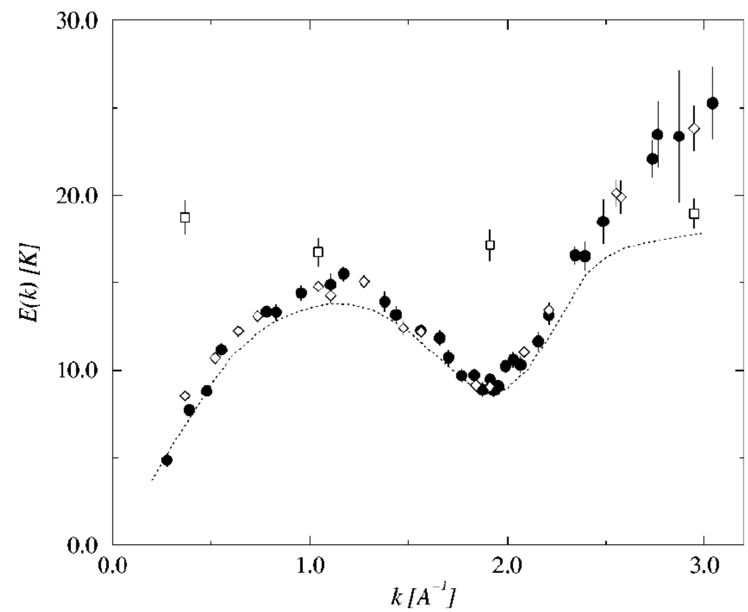
$$S = \int \Phi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \Phi + \int \int \left(\Phi^\dagger \Phi \right) V(x-y) \left(\Phi^\dagger \Phi \right)$$

with $V(x) = V_{sr}(x) - c_6/x^6$. Note: $a = 189 a_0 \gg a_0$

Phase Diagram



Excitations



Low T: Phonons and Rotons Effective lagrangian

$$\mathcal{L} = \varphi^*(\partial_0^2 - v^2)\varphi + i\lambda\dot{\varphi}(\vec{\nabla}\varphi)^2 + \ldots$$

$$+ \varphi^*_{R,v}(i\partial_0 - \Delta)\varphi_{R,v} + c_0(\varphi^*_{R,v}\varphi_{R,v})^2 + \ldots$$

Shear viscosity

$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}}\, e^{\Delta/T} + \frac{c_{ph}}{T^5} + \ldots$$

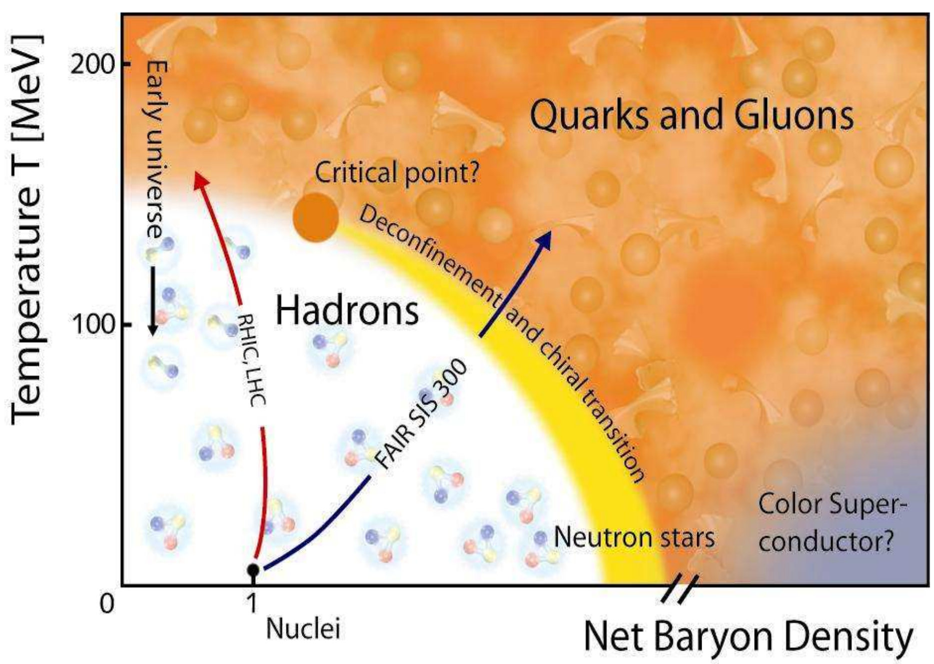
Landau & Khalatnikov

High T: Atoms Viscosity governed by hard core ($V \sim 1/r^{12}$)

$$\eta = \eta_0(T/T_0)^{2/3}$$

III. Quark Gluon Plasma

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

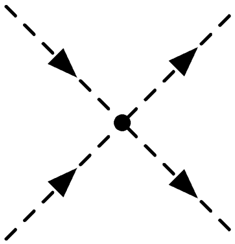


Low T: Pions Chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

Viscosity dominated by $\pi\pi$ scattering

$$\eta = A\frac{f_\pi^4}{T}$$



High T: Quasi-Particles HTL theory (screening, damping, ...)

$$\mathcal{L}_{HTL} = \int d\Omega\; G^a_{\mu\alpha}\frac{v^\alpha v_\beta}{(v\cdot D)^2}G^{a,\mu\beta}$$

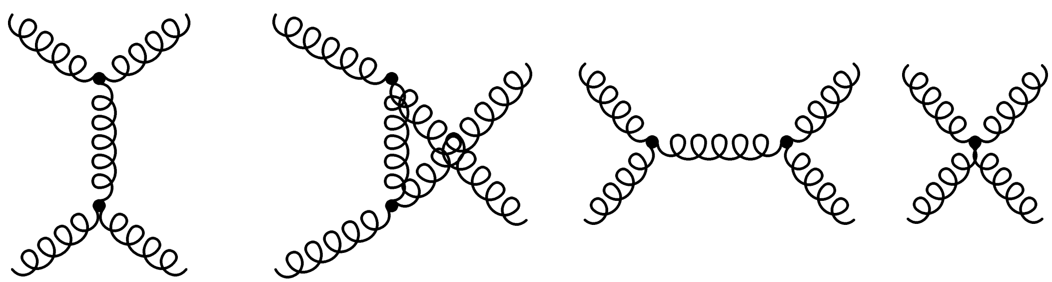
quasi-particle width

$$\gamma \sim g^2 T$$

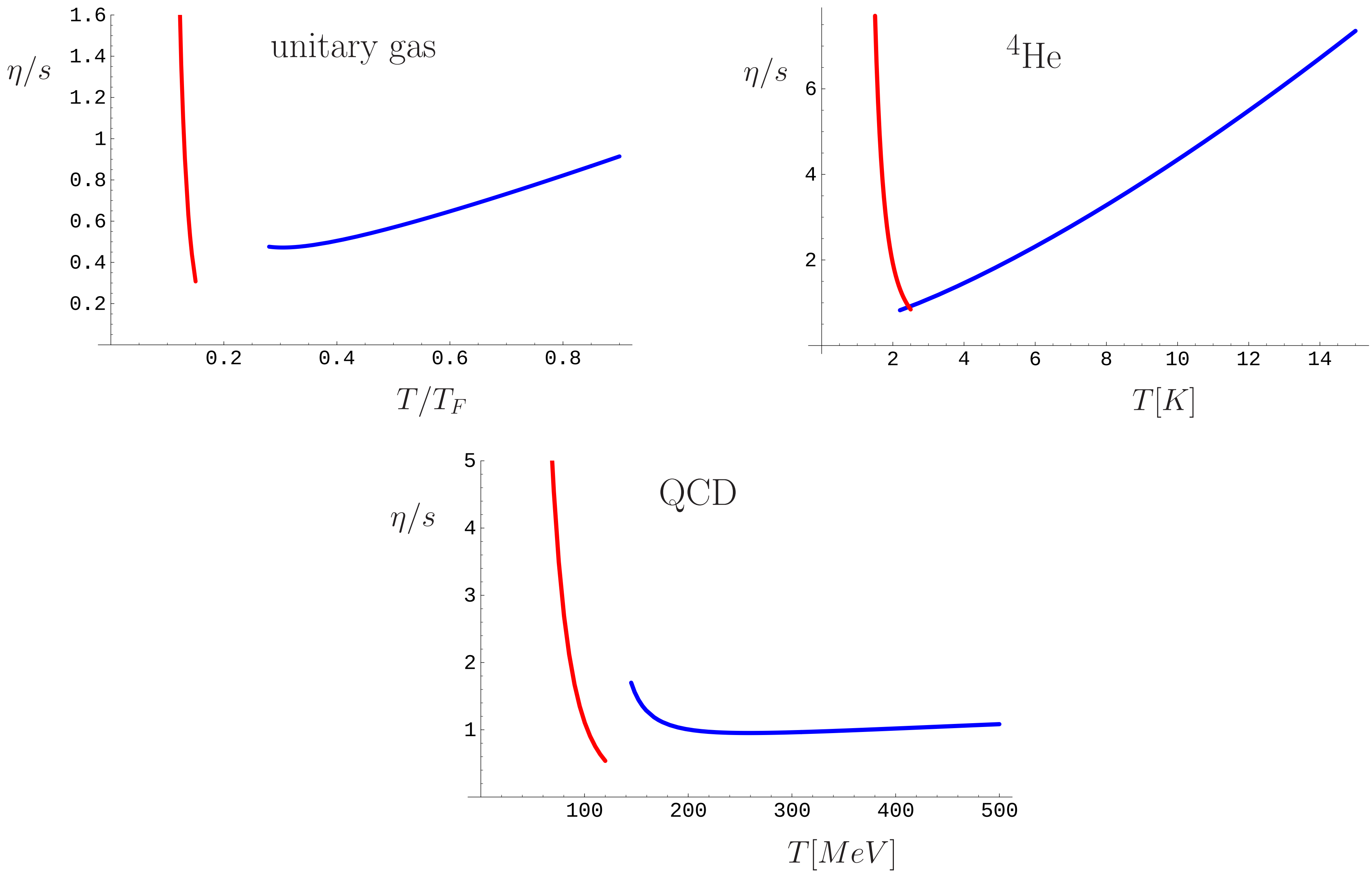
Viscosity dominated by t-channel gluon exchange

$$\eta = \frac{27.13T^3}{g^4\log(2.7/g)}$$

AMY (2003)



Theory Summary



I. Experiment (Liquid Helium)

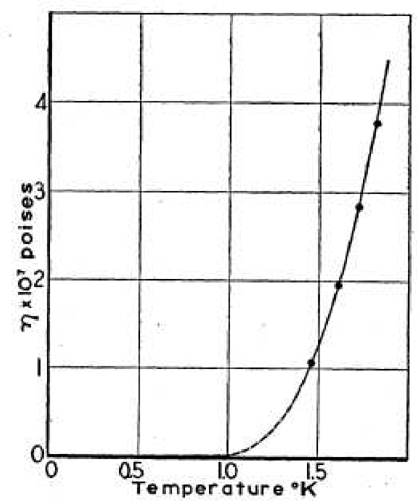


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.

2, § 3 LIQUID HELIUM II 23

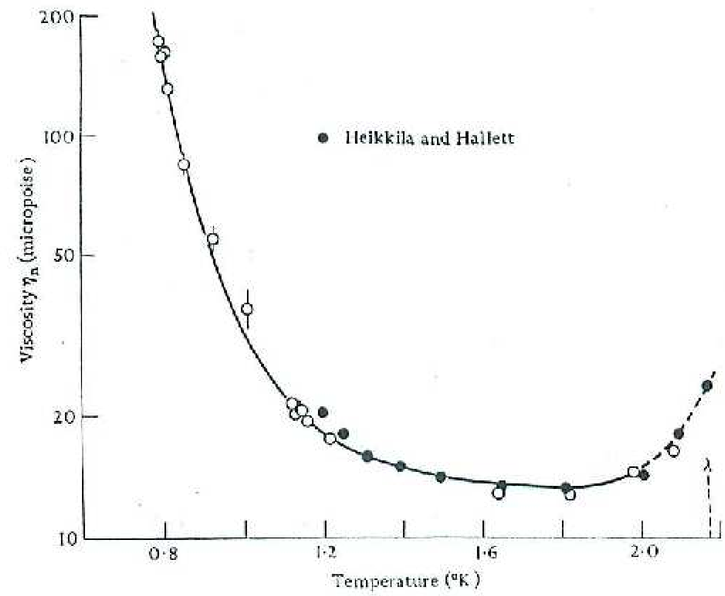


FIG. 11. The viscosity (η_n) of helium II as measured in a rotation viscometer (Woods and Hollis Hallett [50]). The full points show the earlier results of Heikkila and Hollis Hallett [51]).

Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

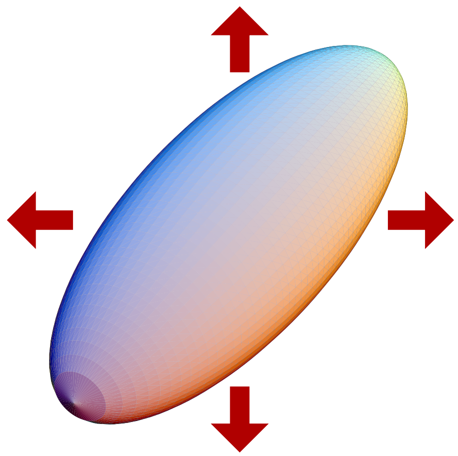
roton minimum, phonon rise

rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

II. Collective Modes (Fermions)

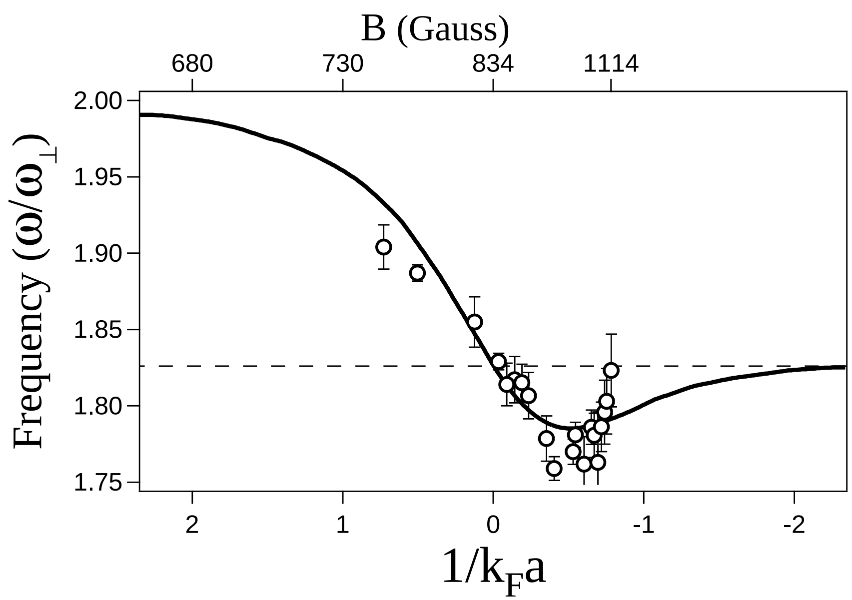
Radial breathing mode



Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{\vec{\nabla}P}{mn} - \frac{\vec{\nabla}V}{m}$$

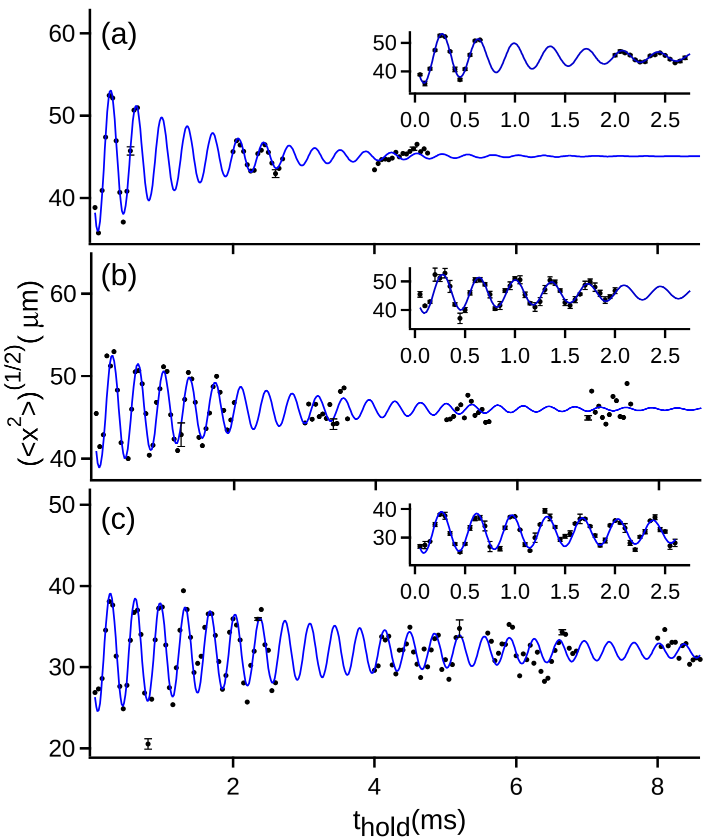


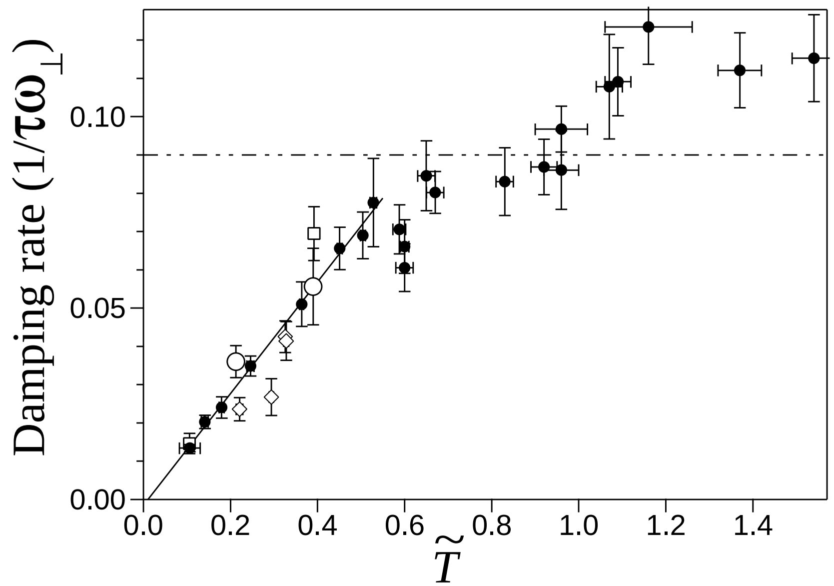
Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

experiment: Kinast et al. (2005)

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time × trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

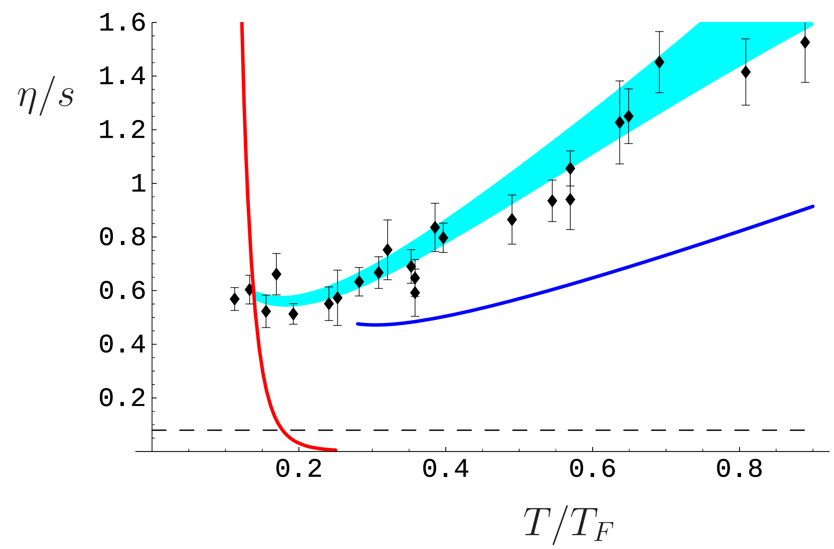
$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

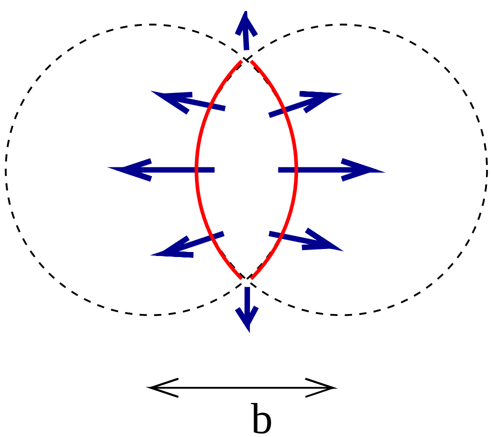
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{\bar{\omega}}{\omega_\perp}\frac{N}{S}$$

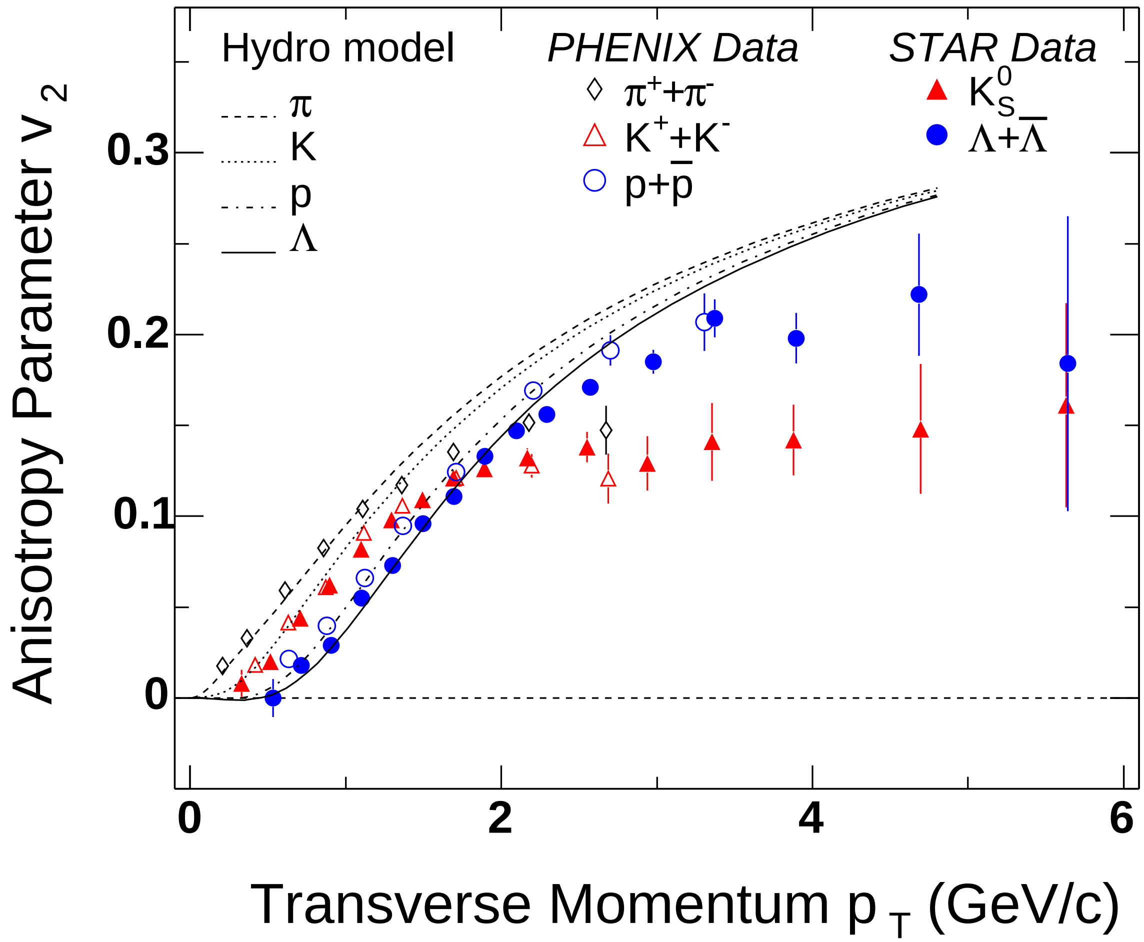
Schaefer (2007), see also Bruun, Smith



III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





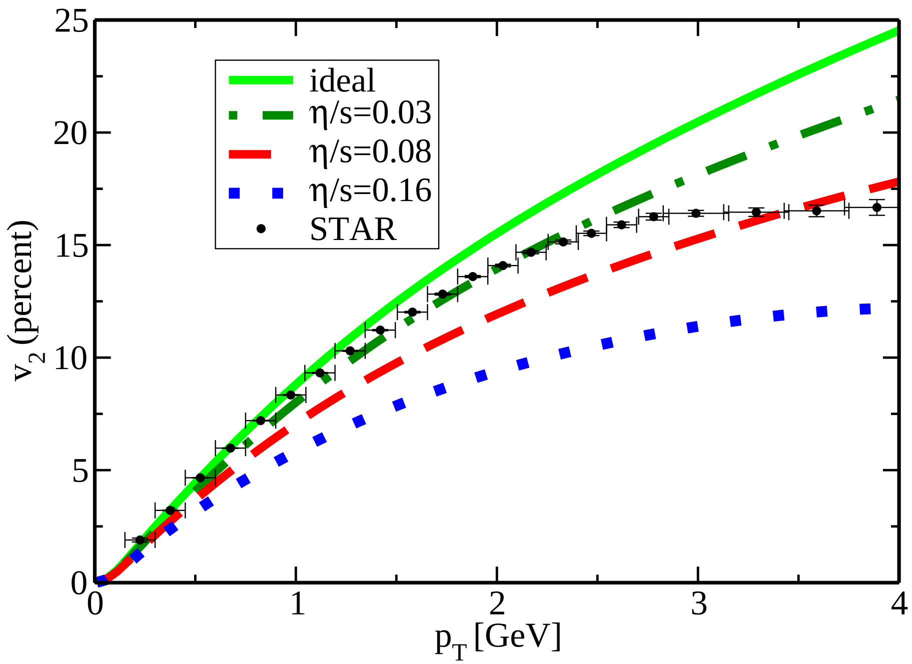
source: U. Heinz (2005)

Viscosity and Elliptic Flow

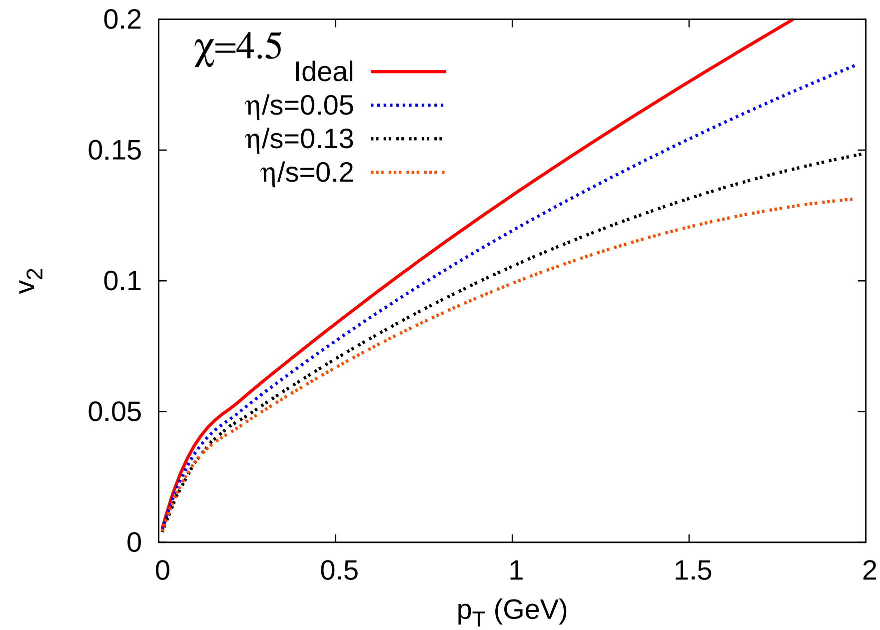
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$ (applicability of Navier-Stokes) very restrictive for $\tau < 1$ fm

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)



Romatschke (2007)



Dusling, Teaney (2008), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

Outlook

Too early to declare a winner.

$\eta/s \simeq 0.8$ (He), $\eta/s \leq 0.5$ (CA), $\eta/s \leq 0.5$ (QGP)

Other experimental constraints (irrot flow ..), more analysis needed.

Kinetic theory: o.k. in He (all T), o.k. close to T_c in CA, QGP?

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, . . .)