Fluid dynamics as an effective theory

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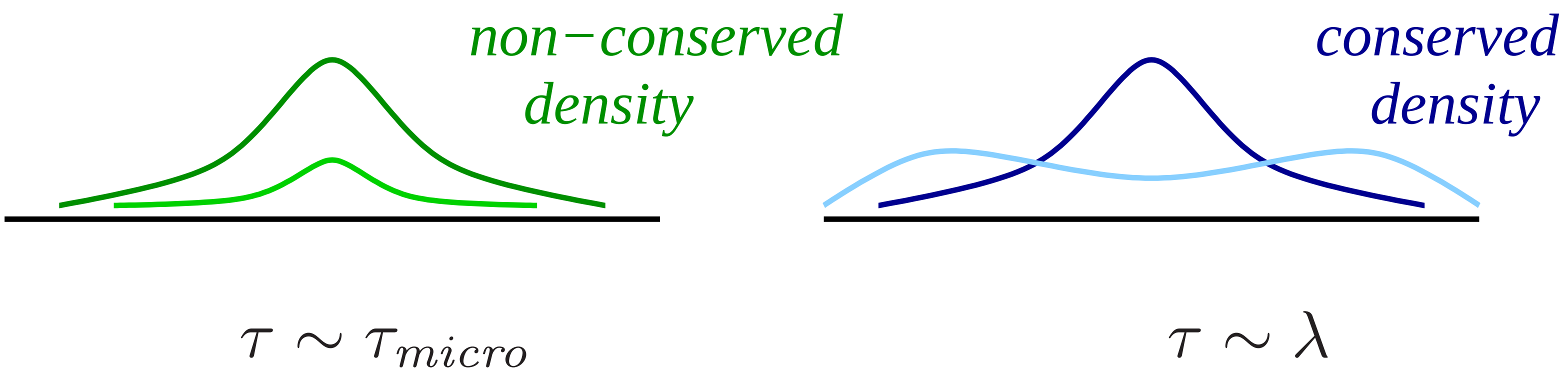
Hydroynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.

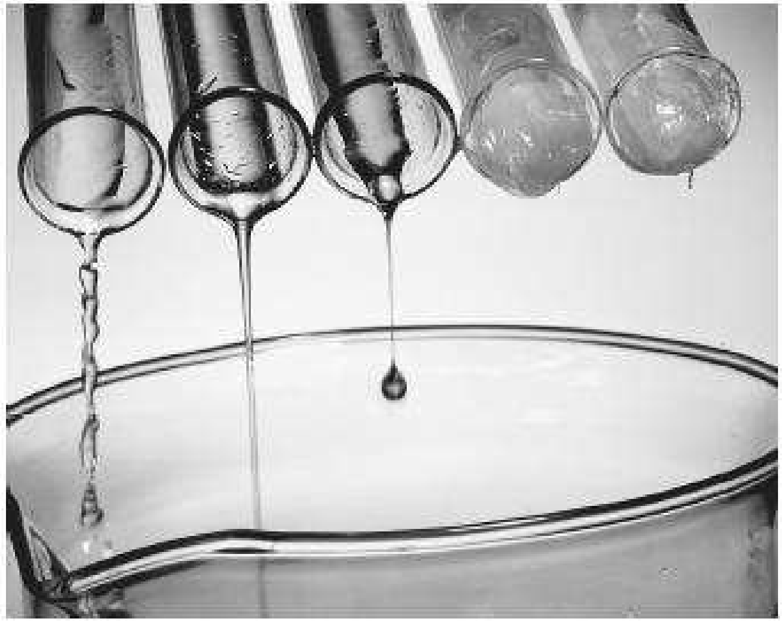
Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of any many-body system.



$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$



$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota$ (*everything flows*)

Heraclitus

The mountains flowed before the Lord.

Prophet Deborah, Judges, 5:5

Effective theories for fluids (Unitary Fermi Gas, $T > T_F$)

$$\mathcal{L} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad \omega < T\frac{s}{\eta}$$

Effective theories (Strong coupling)

$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g}\mathcal{R} + \ldots$$

$$SO(d+2,2) \to Schr_d^2 \qquad AdS_{d+3} \to Schr_d^2$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \;\; (\omega < T)$$

Outline

I. Unitary Fermi gas

II. Gradient expansion

III. Symmetries: Galilean and conformal

IV. Fluctuations

V. Effective field theory?

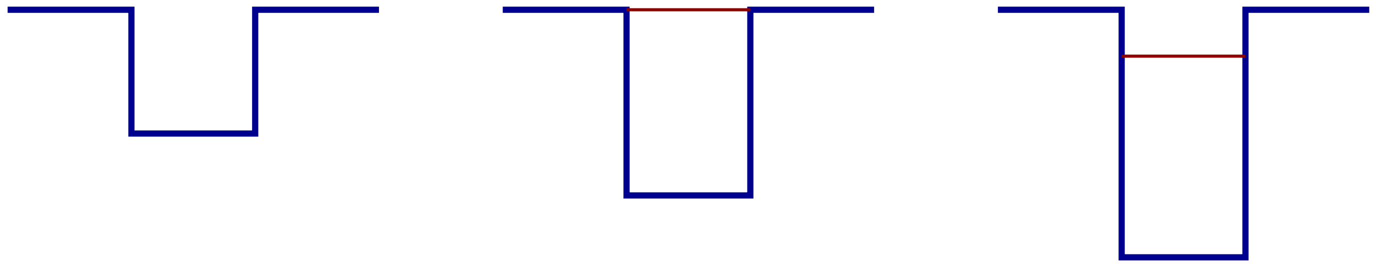
VI. Kinetic theory

VII. Quantum field theory

VIII. Holography

I. Non-relativistic fermions in unitarity limit

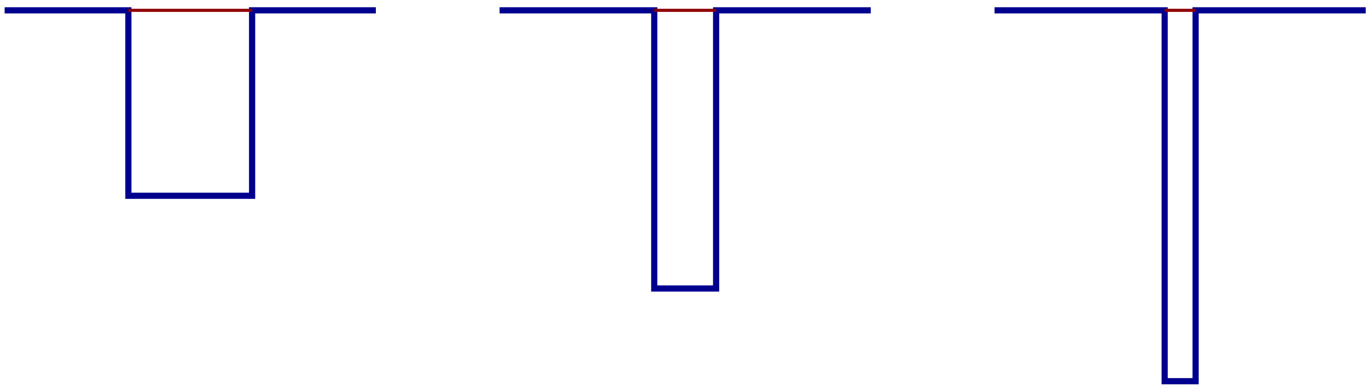
Consider simple square well potential



$a < 0$ $\quad$ $a = \infty,\ \epsilon_B = 0$ $\quad$ $a > 0,\ \epsilon_B > 0$

Non-relativistic fermions in unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r}\exp(-r/a)$$

Fermi gas at unitarity: Field Theory

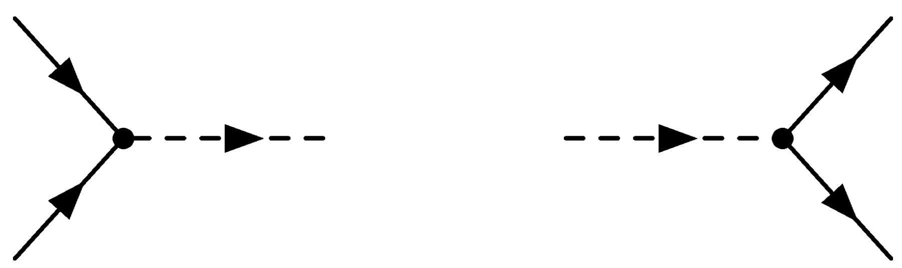
Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$ (DR: $C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$



II. Gradient expansion (simple non-relativistic fluid)

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \vec{j}^{\,\rho} = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\,\epsilon} = 0$$

$$\frac{\partial \pi_i}{\partial t} + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

Ward identity: mass current = momentum density

$$\vec{j}^{\,\rho} \equiv \rho \vec{v} = \vec{\pi}$$

Constitutive relations: Gradient expansion for currents

Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

Brief remark: Regimes of fluid dynamics

Ideal stress tensor $\Pi_{ij} = P\delta_{ij} + \rho v_i v_j$. Relative importance of the two terms governed by Mach number

$$Ma = \frac{v}{c_s} \qquad \text{sound speed } c_s^2 = \frac{\partial P}{\partial \rho}$$

$Ma \ll 1$: Incompressible flow, "gas dynamics"

$Ma \sim 1$: Compressible flow

Fluid dynamic expansion

Gradient expansion for currents, e.g. $\Pi_{ij} = \Pi_{ij}(\rho, v, T)$

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k \right) + O(\partial^2)$$

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$\frac{1}{Re} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{m v L}}_{\text{flow property}}$$

Consider $mvL \sim \hbar$: Hydrodynamics requires $\eta/(\hbar n) < 1$

"Nearly Perfect Fluid"

Additional remarks

1. Incompressible flows: Expansion parameter $Ma^2 Re^{-1}$.

   $Re \gg 1$: Turbulent flow $\qquad$ $Re \sim 1$: Highly viscous flow

2. Shocks: Breakdown of gradient expansion $\eta \nabla v \sim \rho v^2$

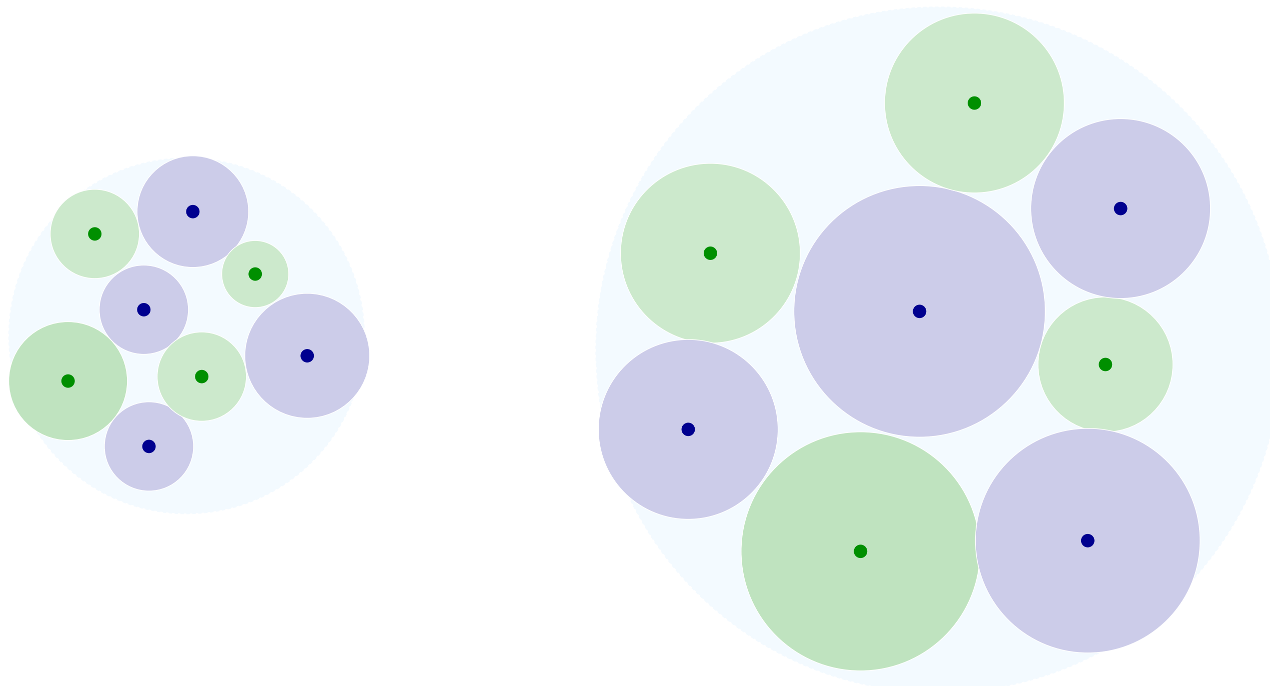
   Shock profile unreliable, but jump conditions insensitive to gradient corrections

3. Navier-Stokes problem (finite time blow-up). Relevant to physics?

   Not clear (to me). Blow-up could be irrelevant to coarse-grained observables.

III. Scale invariant fluid dynamics

Consider a many body system with $\sigma_{tr} \sim n^{-2/3}$



Systems remains hydrodynamic despite expansion

Conformal fluid dynamics: Symmetries

Symmetries of a conformal non-relativistic fluid

Galilean boost	$\vec{x}' = \vec{x} + \vec{v}t$	$t' = t$
Scale trafo	$\vec{x}' = e^s \vec{x}$	$t' = e^{2s} t$
Conformal trafo	$\vec{x}' = \vec{x}/(1+ct)$	$1/t' = 1/t + c$

This is known as the Schrödinger algebra (= the symmetries of the free Schrödinger equation)

Generators: Mass, momentum, angular momentum

$$M = \int dx\, \rho \quad P_i = \int dx\, \jmath_i \quad J_{ij} = \int dx\, \epsilon_{ijk} x_j \jmath_k$$

Boost, dilations, special conformal

$$K_i = \int dx\, x_i \rho \quad D = \int dx\, x \cdot \jmath \quad C = \int dx\, x^2 \rho/2$$

Spurion method: Local symmetries

Diffeomorphism invariance $\delta x_i = \xi_i(x,t)$

$$\delta g_{ij} = -\mathcal{L}_\xi g_{ij} = -\xi^k \partial_k g_{ij} + \ldots$$

Gauge invariance $\delta\psi = i\alpha(x,t)\psi$

$$\begin{aligned} \delta A_0 &= -\dot{\alpha} - \xi^k \partial_k A_0 - A_k \dot{\xi}^k \\ \delta A_i &= -\partial_i \alpha - \xi^k \partial_k A_i - A_k \partial_i \xi^k + m g_{ik} \dot{\xi}^k \end{aligned}$$

Conformal transformations $\delta t = \beta(t)$

$$\delta O = -\beta \dot{O} - \frac{1}{2} \Delta_O \dot{\beta} O$$

More recent work: Newton-Cartan geometry

Example: Stress tensor

Determine transformation properties of fluid dynamic variables

$$\delta\rho = -\mathcal{L}_\xi \rho \qquad \delta s = -\mathcal{L}_\xi s \qquad \delta v = -\mathcal{L}_\xi v + \dot{\xi}$$

Stress tensor: Ideal fluid dynamics

$$\Pi^0_{ij} = P g_{ij} + \rho v_i v_j , \qquad\qquad P = \frac{2}{3}\mathcal{E}$$

First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij} - \zeta g_{ij}\langle\sigma\rangle \qquad \zeta = 0$$

$$\sigma_{ij} = \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3} g_{ij}\langle\sigma\rangle\right) \qquad\qquad \langle\sigma\rangle = \nabla\cdot v + \frac{\dot{g}}{2g}$$

Son (2007)

Simple application: Kubo formula

Consider background metric $g_{ij}(t,x) = \delta_{ij} + h_{ij}(t,x)$. Linear response

$$\delta\Pi^{xy} = -\frac{1}{2} G_R^{xyxy} h_{xy}$$

Harmonic perturbation $h_{xy} = h_0 e^{-i\omega t}$

$$G_R^{xyxy} = P - i\eta\omega + \ldots$$

Kubo relation: $$\eta = -\lim_{\omega \to 0} \left[\frac{1}{\omega} \mathrm{Im} G_R^{xyxy}(\omega, 0) \right]$$

Second order conformal hydrodynamics

Second order gradient corrections to stress tensor

$$\delta^{(2)}\Pi^{ij} = \eta\tau_\pi \left[{}^{\langle}D\sigma^{ij\rangle} + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] + \lambda_1 \sigma^{\langle i}{}_k \sigma^{j\rangle k} + \lambda_2 \sigma^{\langle i}{}_k \Omega^{j\rangle k} + \lambda_3 \Omega^{\langle i}{}_k \Omega^{j\rangle k} + O(\nabla^2 T)$$

$D = \partial_0 + v\cdot\nabla \quad A^{\langle ij\rangle} = \frac{1}{2}\left(A_{ij} + A_{ji} - \frac{2}{3} g_{ij} A^k{}_k\right) \quad \Omega^{ij} = \left(\nabla_i v_j - \nabla_j v_i\right)$

New transport coefficients $\tau_\pi, \lambda_i, \gamma_i$

Can be written as a relaxation equation for $\pi^{ij} \equiv \delta\Pi^{ij}$

$$\pi^{ij} = -\eta\sigma^{ij} - \tau_\pi \left[{}^{\langle}D\pi^{ij\rangle} + \frac{5}{3}(\nabla\cdot v)\pi^{ij}\right] + \ldots$$

Chao, Schaefer (2011)

Second order fluid dynamics: Causality

"Speed" of diffusive wave in Navier-Stokes theory

$$v_D = \frac{\partial |\omega|}{\partial k} = \frac{2\eta}{\rho}\, k$$

May encounter $v_D \gg c_s$

Not a fundamental problem (should impose $k < \Lambda$), but a nuisance in simulations.

Second order fluid dynamics, relaxation type

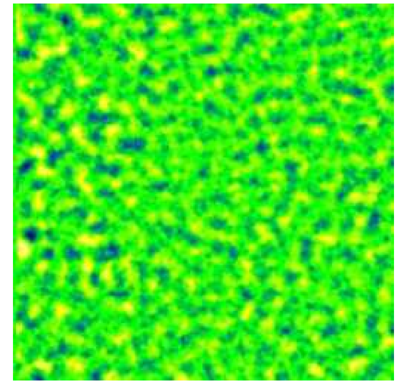
$$i\omega = \frac{\nu k^2}{1 - i\omega\tau_\pi} \qquad \text{("resummed hydro")}$$

Limiting speed $v_D^\infty \sim \sqrt{\eta/(\rho\tau_\pi)}$

Find $v_D^\infty \sim c_s$ for $\tau_\pi = \eta/P$.

IV. Beyond gradients: Hydrodynamic fluctuations

Hydrodynamic variables fluctuate



$$\langle \delta v_i(x,t)\delta v_j(x',t)\rangle = \frac{T}{\rho}\delta_{ij}\delta(x-x')$$

Linearized hydrodynamics propagates fluctuations as shear or sound

$$\langle \delta v_i^T \delta v_j^T\rangle_{\omega,k} = \frac{2T}{\rho}(\delta_{ij}-\hat{k}_i\hat{k}_j)\frac{\nu k^2}{\omega^2+(\nu k^2)^2} \qquad \textit{shear}$$

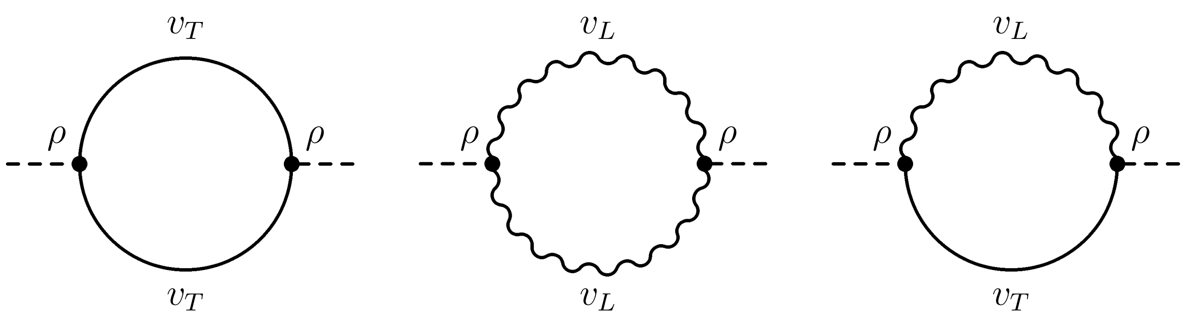
$$\langle \delta v_i^L \delta v_j^L\rangle_{\omega,k} = \frac{2T}{\rho}\hat{k}_i\hat{k}_j\frac{\omega k^2\Gamma}{(\omega^2-c_s^2k^2)^2+(\omega k^2\Gamma)^2} \qquad \textit{sound}$$

$v = v_T + v_L$: $\nabla\cdot v_T = 0$, $\nabla\times v_L = 0$ $\qquad$ $\nu = \eta/\rho$, $\Gamma = \frac{4}{3}\nu + \ldots$

Hydro Loops: "Breakdown" of second order hydro

Correlation function in hydrodynamics

$$G_S^{xyxy} = \langle \{\Pi^{xy}, \Pi^{xy}\} \rangle_{\omega,k} \simeq \rho_0^2 \langle \{v_x v_y, v_x v_y\} \rangle_{\omega,k}$$



Match to response function in $\omega \to 0$ (Kubo) limit

$$G_R^{xyxy} = P + \delta P - i\omega[\eta + \delta\eta] + \omega^2 \left[\eta\tau_\pi + \delta(\eta\tau_\pi)\right]$$

with

$$\delta P \sim T\Lambda^3 \qquad \delta\eta \sim \frac{T\rho\Lambda}{\eta} \qquad \delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \frac{T\rho^{3/2}}{\eta^{3/2}}$$

Hydro Loops: RG and "breakdown" of 2nd order hydro

Cutoff dependence can be absorbed into bare parameters. Non-analytic terms are cutoff independent.

Fluid dynamics is a "renormalizable" effective theory.

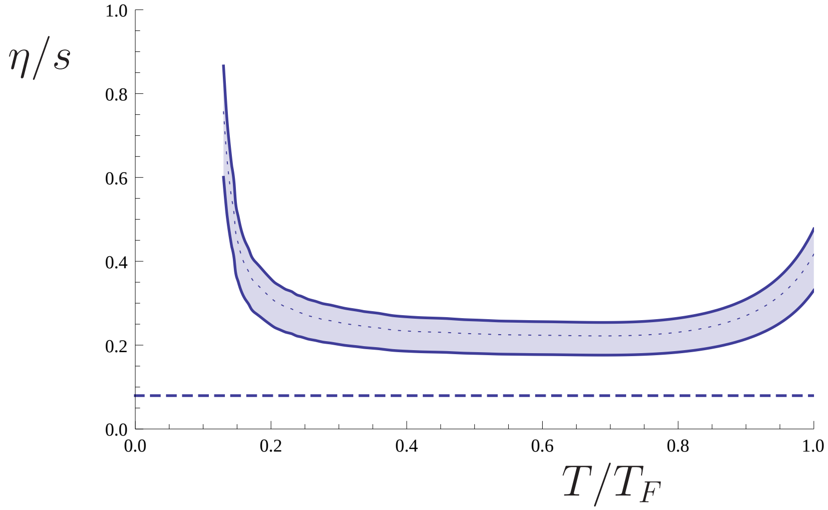
Small η enhances fluctuation corrections: $\delta\eta \sim T\left(\frac{\rho}{\eta}\right)^2\left(\frac{P}{\rho}\right)^{1/2}$

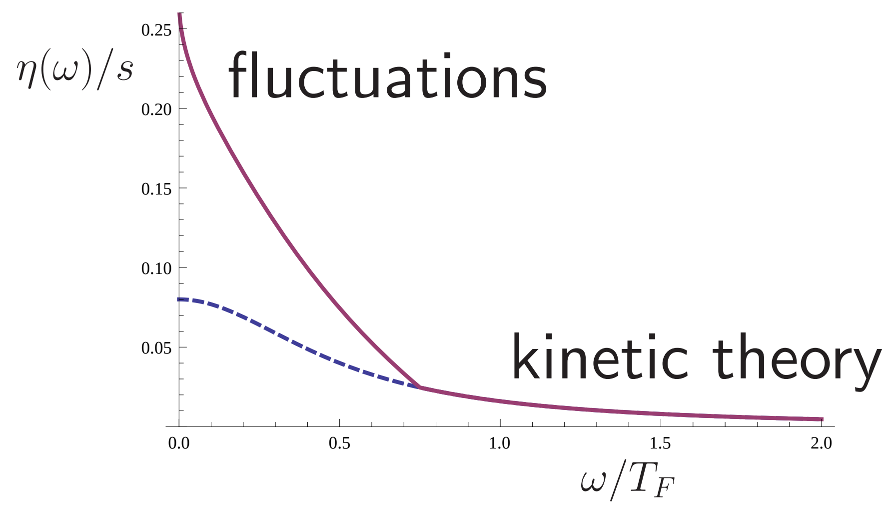
Small η leads to large $\delta\eta$: There must be a bound on η/n.

Relaxation time diverges: $\delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}}\left(\frac{\rho}{\eta}\right)^{3/2}$

2nd order hydro without fluctuations inconsistent.

Fluctuation induced bound on η/s





$$(\eta/s)_{min} \simeq 0.2$$

spectral function

non-analytic $\sqrt{\omega}$ term

Schaefer, Chafin (2012), see also Kovtun, Moore, Romatschke (2011)

V. Effective field theory?

Consider $T = 0$ systems with broken $U(1)$ (superfluids).

Goldstone mode φ. Impose exact gauge & Galilei invariance

$$\mathcal{L} = P(X) + O(\nabla X) \qquad\qquad X = \mu - \partial_0\varphi - \frac{(\nabla\varphi)^2}{2m}$$

Standard EFT: Expand in gradients

$$\mathcal{L} = f^2\left[(\partial_0\varphi)^2 - c_s^2(\nabla\varphi)^2\right] + g(\partial_0\varphi)(\nabla\varphi)^2 + \ldots$$

Define hydrodynamic variables

$$n = P'(X) \qquad\qquad v_s = \frac{1}{m}\nabla\varphi$$

Use Euler equation for φ & Gibbs-Duhem $\nabla P = n\nabla\mu$

$$\partial_0 n + \frac{1}{m}\nabla(n\nabla\varphi) = 0 \qquad\qquad \partial_0 v_s + \frac{1}{2}\nabla\, v_s^2 = -\frac{1}{m}\nabla\mu$$

Superfluid hydrodynamics

Remarks

(Quantum) loop corections?

Calculable, but highly suppressed.

Can this be extended to $T \neq 0$?

Not directly, fluid dynamics is irreversible.

Can construct generating functional for linearized hydro. Simple example: Diffusion $\partial_0 n = D\nabla^2 n$ (model B)

$$Z = \int DnD\psi\, e^{iS}, \qquad S = \int dxdt \left[\psi\partial_0 n - \psi D\nabla^2 n + iD\psi\chi T(\nabla\psi)^2\right]$$

Analytic structure? Higher order terms?

VI. Kinetic theory

Microscopic picture: Quasi-particle distribution function $f_p(x,t)$

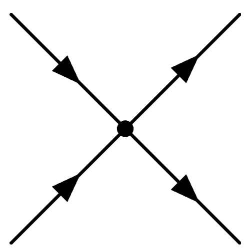
$$\rho(x,t) = \int d\Gamma_p \sqrt{g} m f_p(x,t) \qquad \pi_i(x,t) = \int d\Gamma_p \sqrt{g} p_i f_p(x,t)$$

$$\Pi_{ij}(x,t) = \int d\Gamma_p \sqrt{g} p_i v_j f_p(x,t)$$

Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \frac{p^i}{m} \frac{\partial}{\partial x^i} - \left(g^{il} \dot{g}_{lj} p^j + \Gamma^i_{jk} \frac{p^j p^k}{m} \right) \frac{\partial}{\partial p^i} \right) f_p(t,x,) = C[f]$$

$$C[f] =$$



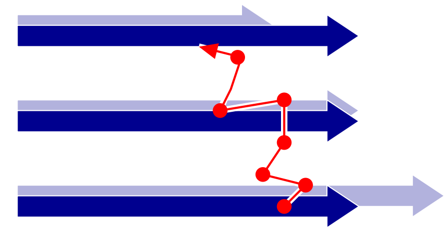
Solve order-by-order in Knudsen number $Kn = l_{mfp}/L$

Kinetic theory: Knudsen expansion

Chapman-Enskog expansion $f = f_0 + \delta f_1 + \delta f_2 + \ldots$

Gradient exp. $\delta f_n = O(\nabla^n)$

$\equiv$ Knudsen exp. $\delta f_n = O(Kn^n)$



First order result

Bruun, Smith (2005)

$$\delta^{(1)}\Pi^{ij} = -\eta\sigma^{ij} \qquad\qquad \eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2}$$

Second order result

Chao, Schaefer (2012), Schaefer (2014)

$$\begin{aligned}\delta^{(2)}\Pi^{ij} \;&=\; \frac{\eta^2}{P}\left[\langle D\sigma^{ij}\rangle + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] \\ &\quad + \frac{\eta^2}{P}\left[\frac{15}{14}\sigma^{\langle i}{}_k\sigma^{j\rangle k} - \sigma^{\langle i}{}_k\Omega^{j\rangle k}\right] + O(\kappa\eta\nabla^i\nabla^j T)\end{aligned}$$

relaxation time $\tau_\pi = \eta/P$

Knudsen vs fugacity expansion

Knudsen expansion

$$\delta f^{(1)} \sim \frac{\eta}{\rho T^2} p^i p^j \sigma_{ij} \qquad \delta f^{(2)} \sim O(\eta^2 \sigma^2)$$

Fugacity expansion

$$\eta = (n\lambda^3)\left\{\eta_0 + \eta_1 (n\lambda^3) + \ldots\right\}$$

Analog of virial expansion for equilibrium properties.

Dense gas $(n\lambda^3) \sim 1$: Kinetic theory not applicable.

Relativistic theories $(n\lambda^3) = 1$: Coupling constant expansion.

Frequency dependence, breakdown of kinetic theory

Consider harmonic perturbation $h_{xy}e^{-i\omega t+ikx}$. Use schematic collision term $C[f_p^0 + \delta f_p] = -\delta f_p/\tau$.

$$\delta f_p(\omega, k) = \frac{1}{2T} \frac{-i\omega p_x v_y}{-i\omega + i\vec{v}\cdot\vec{k} + \tau_0^{-1}} f_p^0 \, h_{xy} \,.$$

Leads to Lorentzian line shape of transport peak

$$\eta(\omega) = \frac{\eta(0)}{1+\omega^2\tau_0^2}$$

Pole at $\omega = i\tau_0^{-1}$ ($\tau_0 = \eta/(sT)$) controls range of convergence of gradient expansion.

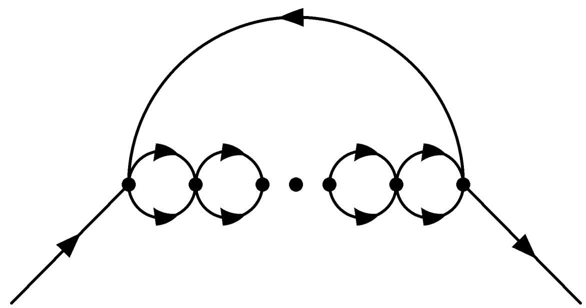
High frequency behavior misses short range correlations for $\omega > T$.

Bulk viscosity and conformal symmetry breaking

Conformal symmetry breaking (thermodynamics)

$$1 - \frac{2\mathcal{E}}{3P} = \frac{\langle \mathcal{O}_{\mathcal{C}} \rangle}{12\pi maP} \sim \frac{1}{6\pi} n\lambda^3 \frac{\lambda}{a}$$

How does this translate into $\zeta \neq 0$? Momentum dependent $m^*(p)$.



$$Im\,\Sigma(k) \sim zT\sqrt{\frac{T}{\epsilon_k}}Erf\left(\sqrt{\frac{\epsilon_k}{T}}\right) \ll T$$

$$Re\,\Sigma(k) \sim zT\frac{\lambda}{a}\sqrt{\frac{T}{\epsilon_k}}F_D\left(\sqrt{\frac{\epsilon_k}{T}}\right)$$

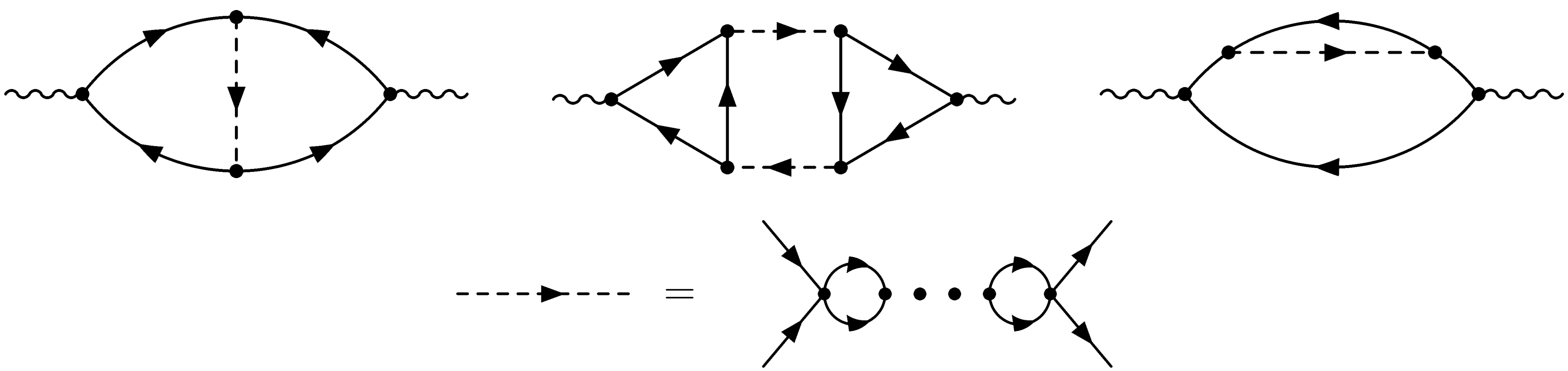
Bulk viscosity

$$\zeta = \frac{1}{24\sqrt{2}\pi}\lambda^{-3}\left(\frac{z\lambda}{a}\right)^2$$

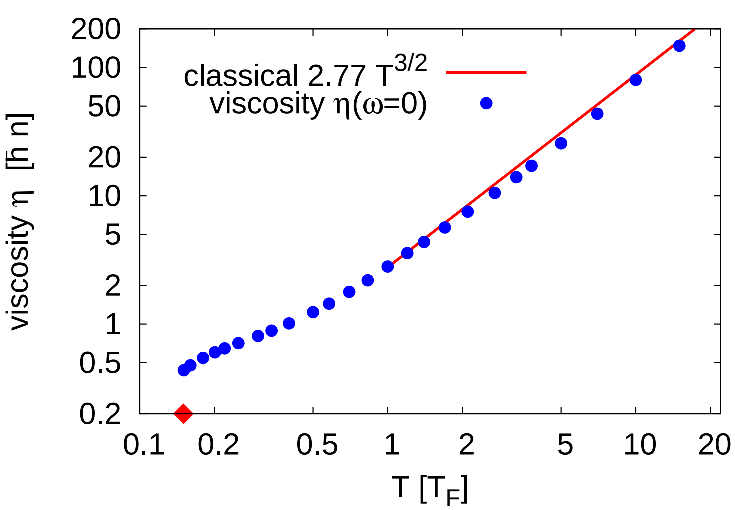
$$\zeta \sim \left(1 - \frac{2\mathcal{E}}{3P}\right)^2 \eta$$

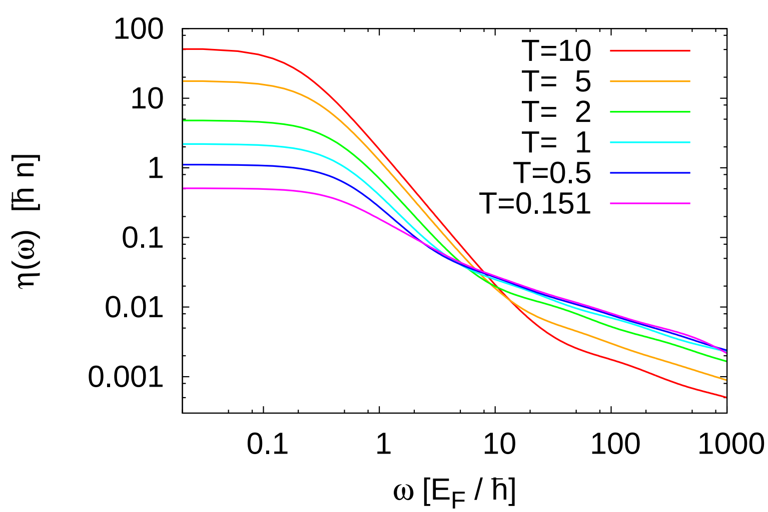
VII. Quantum Field Theory

The diagrammatic content of the Boltzmann equation is known: Kubo formula with "Maki-Thompson" + "Azlamov-Larkin" +"Self-energy"



Can be used to extrapolate Boltzmann result to $T \sim T_F$





Enss, Zwerger (2011)

Short time behavior: OPE

Operator product expansion (OPE)

$$\eta(\omega) = \sum_n \frac{\langle \mathcal{O}_n \rangle}{\omega^{(\Delta_n - d)/2}} \qquad \mathcal{O}_n(\lambda^2 t, \lambda x) = \lambda^{-\Delta_n} \mathcal{O}_n(t, x)$$

Leading operator: Contact density (Tan)

$$\mathcal{O}_\mathcal{C} = C_0^2 \psi\psi\psi^\dagger\psi^\dagger = \Phi\Phi^\dagger \qquad \Delta_\mathcal{C} = 4$$

$\eta(\omega) \sim \langle \mathcal{O}_\mathcal{C} \rangle / \sqrt{\omega}$. Asymptotic behavior + analyticity gives sum rule

$$\frac{1}{\pi} \int dw \left[\eta(\omega) - \frac{\langle \mathcal{O}_\mathcal{C} \rangle}{15\pi\sqrt{m\omega}} \right] = \frac{\mathcal{E}}{3}$$

Randeria, Taylor (2010), Enss, Zwerger (2011), Hoffman (2013)

VIII. Holography

DLCQ idea: Light cone compactification of relativistic theory in d+2

$$p_\mu p^\mu = 2p_+p_- - p_\perp^2 = 0 \qquad p_- = \frac{p_\perp^2}{2p_+} \qquad p_+ = \frac{2n+1}{L}$$

Galilean invariant theory in d+1 dimensions.

String theory embedding: Null Melvin Twist

$$AdS_{d+3} \xrightarrow{\text{NMT}} Schr_d^2$$

$$Iso(AdS_{d+3}) = SO(d+2,2) \supset Schr(d)$$

Son (2008), Balasubramanian et al. (2008)

Other ideas: Horava-Lifshitz (Karch, 2013)

Schrödinger Metric

Coordinates $(u, v, \vec{x}, r)$, periodic in v, $\vec{x} = (x, y)$

$$ds^2 = \frac{r^2}{k(r)^{2/3}} \left\{ \left[\frac{1 - f(r)}{4\beta^2} - r^2 f(r) \right] du^2 + \frac{\beta^2 r_+^4}{r^4} dv^2 - [1 + f(r)]\, du\, dv \right\}$$

$$+ k(r)^{1/3} \left\{ r^2\, d\vec{x}^2 + \frac{dr^2}{r^2 f(r)} \right\}$$

Fluctuations $\delta g_x^y = e^{-i\omega u}\chi(\omega, r)$ satisfy $(u = (r_+/r)^2)$

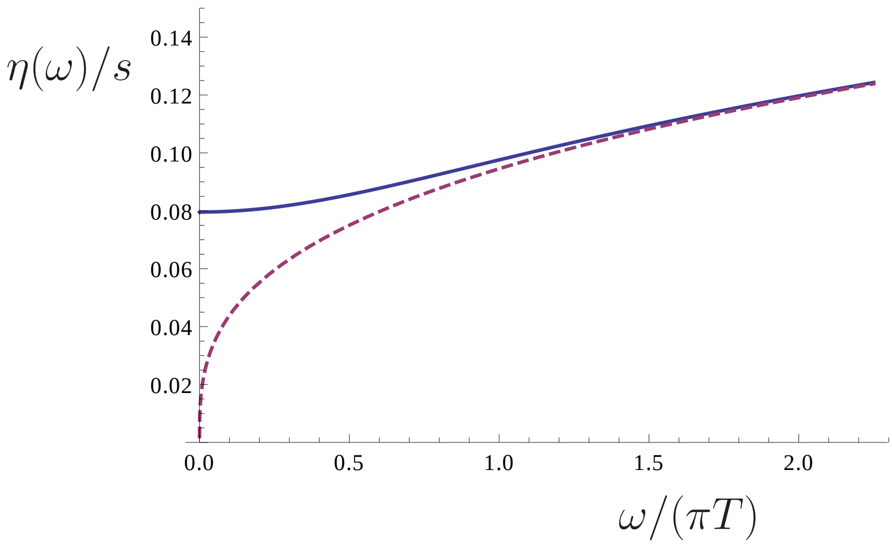
$$\chi''(\omega, u) - \frac{1 + u^2}{f(u)u}\chi'(\omega, u) + \frac{u}{f(u)^2}\mathfrak{w}^2\chi(\omega, u) = 0$$

Retarded correlation function

$$G_R(\omega) = \frac{\beta r_+^3 \Delta v}{4\pi G_5} \left. \frac{f(u)\chi'(\omega, u)}{u\chi(\omega, u)} \right|_{u \to 0} .$$

Adams et al. (2008), Herzog et al. (2008)

Spectral function



$$\eta(0)/s = 1/(4\pi)$$

$$\eta(\omega \to \infty) \sim \omega^{1/3}$$

Kubo relation (incl. τ_π): $G_R(\omega) = P - i\eta\omega + \tau_\pi \eta \omega^2 + \kappa_R k^2$

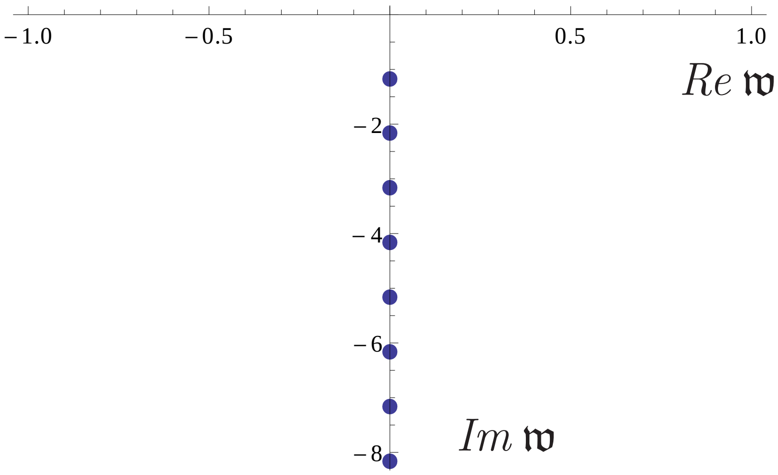
$$\tau_\pi T = -\frac{\log(2)}{2\pi} \qquad AdS_5 : \ \tau_\pi T = \frac{2 - \log(2)}{2\pi}$$

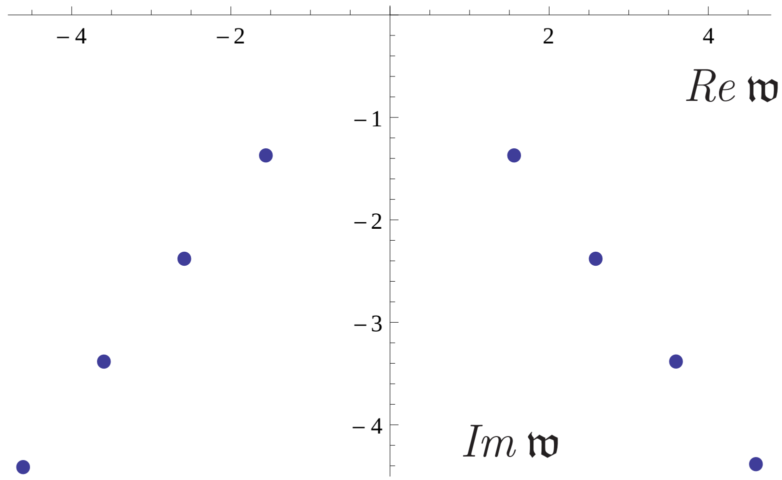
Range of validity of fluid dynamics: $\omega < T$

Sch_2: Cannot be matched to relaxation type hydro?

Schaefer (2014), BRSSS (2008)

Quasi-normal modes





Sch_2^2 AdS_5

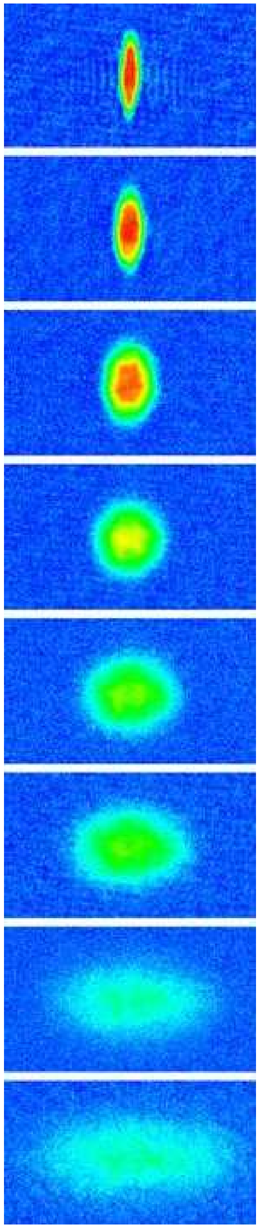
QNM's are stable, $\mathrm{Im}\,\lambda < 0$.

Pole at $\omega \sim iT$ limits convergence of fluid dynamics.

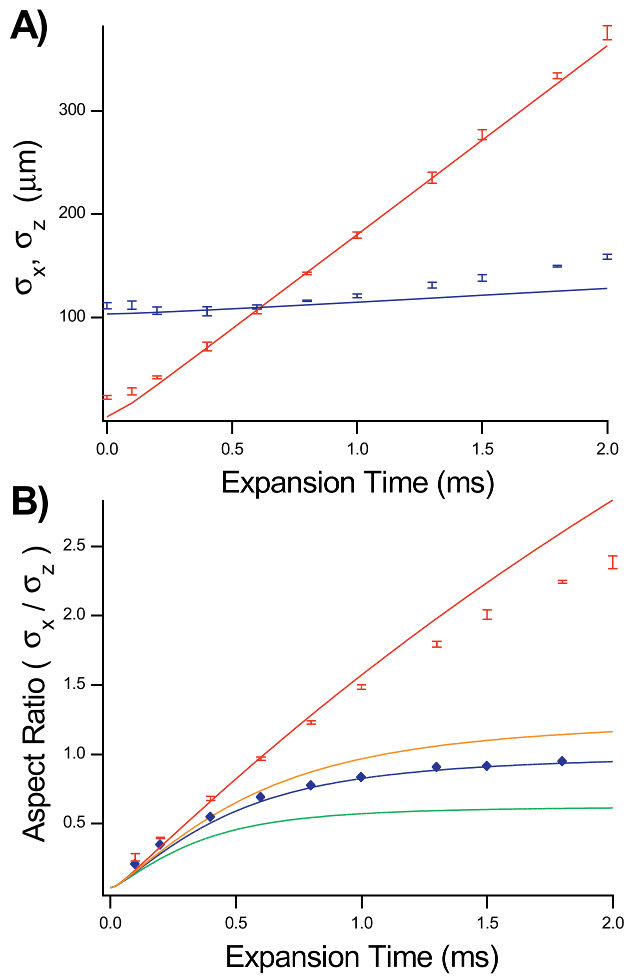
Also: Gradient expansion only asymptotic.

Schaefer (2014), Starinets (2002), Heller (2012)

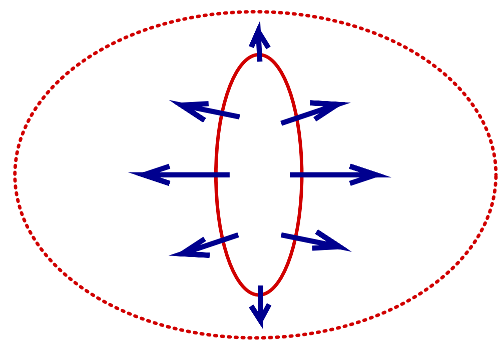
IX. Experiments: Flow and Collective Modes



O'Hara et al. (2002)

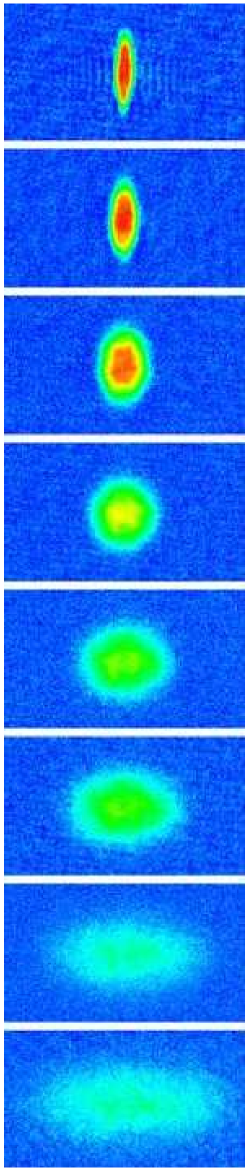


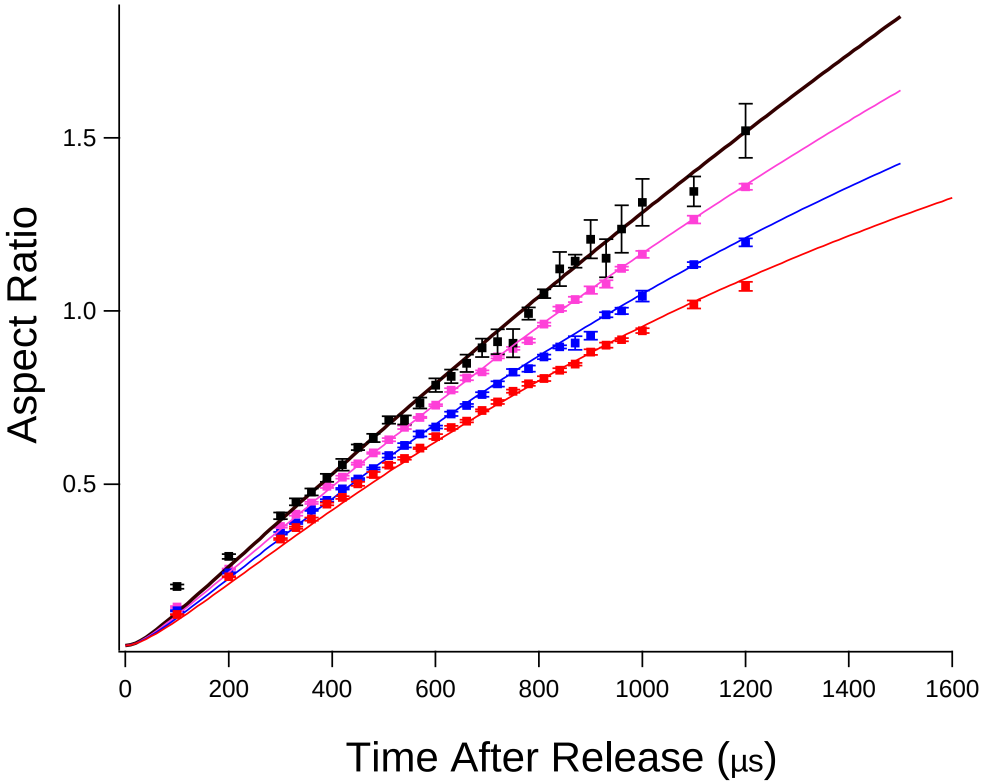
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Elliptic flow: High T limit

Quantum viscosity $\eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$





$$\eta = \eta_0 (mT)^{3/2}$$

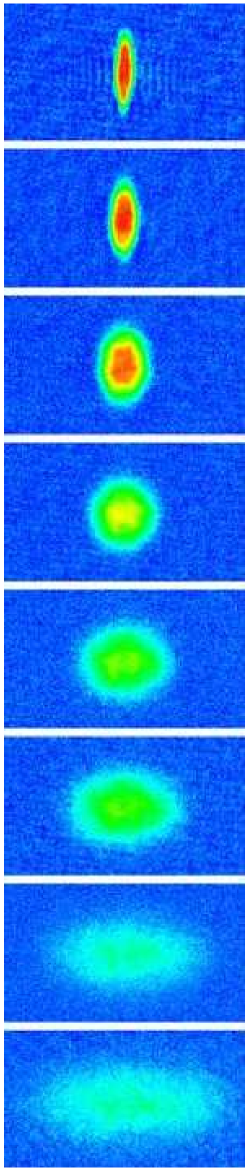
$$\tau_\pi = \eta / P$$

Cao et al., Science (2010)

fit: $\eta_0 = 0.33 \pm 0.04$

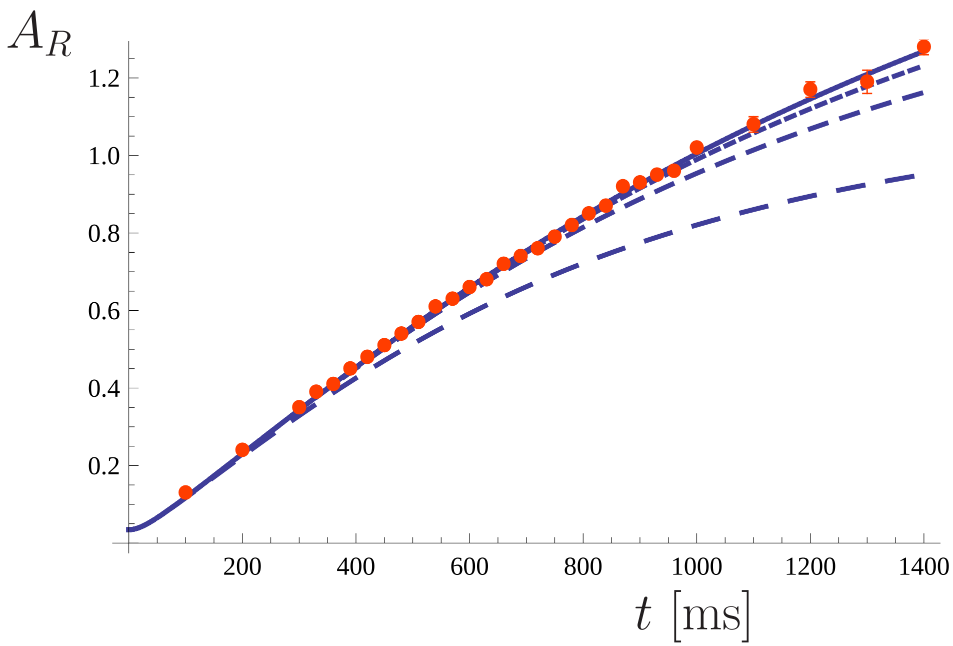
theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$

Elliptic flow: Freezeout?



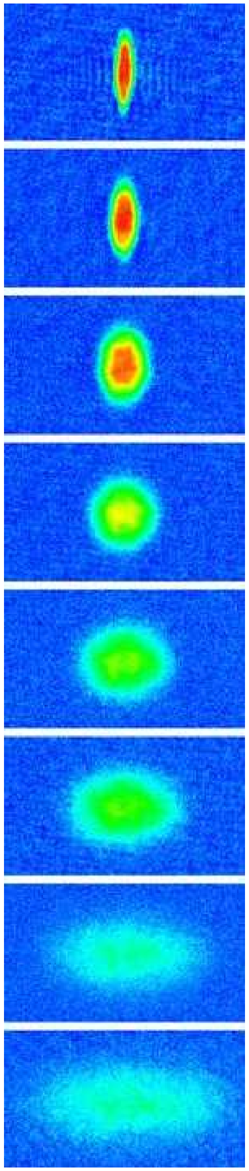
switch from hydro to (weakly collisional) kinetics

at scale factor $b_\perp^{fr} = 1, 5, 10, 20$



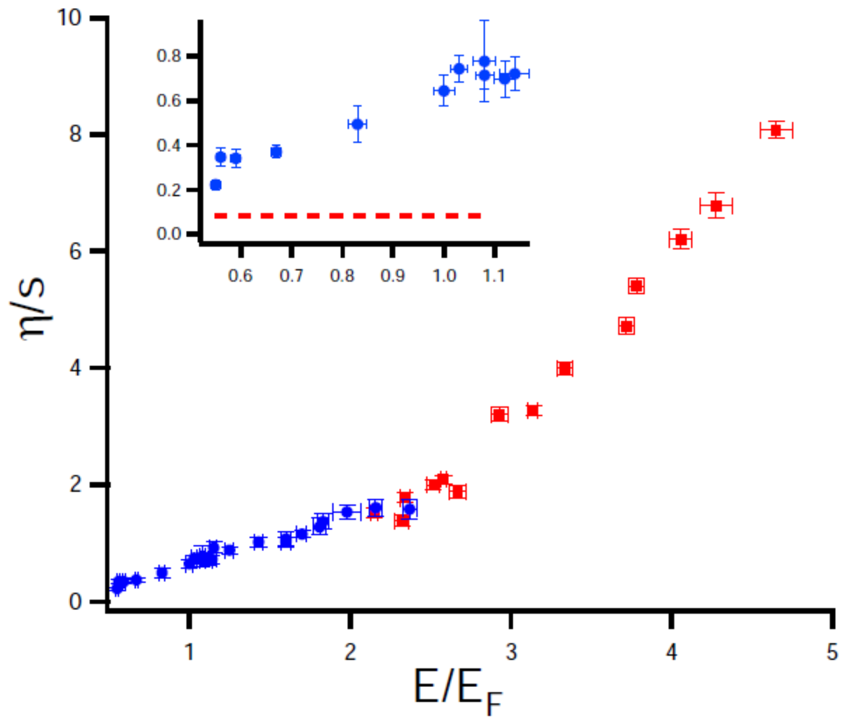
no freezeout seen in the data

Viscosity to entropy density ratio



consider both collective modes (low T)

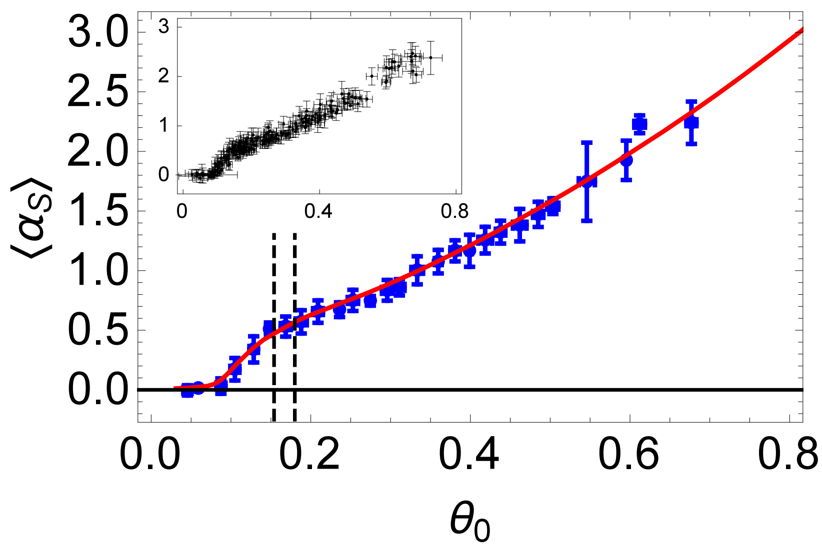
and elliptic flow (high T)

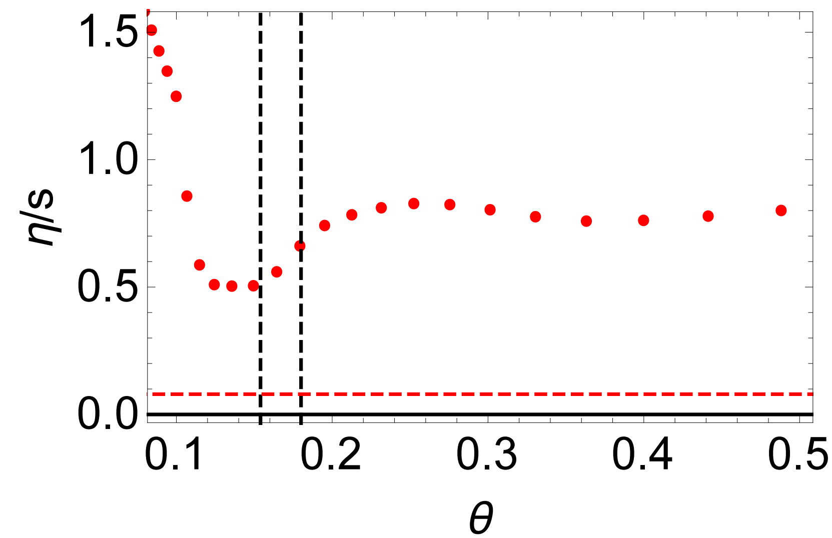


Cao et al., Science (2010)

$$\eta/s \leq 0.4$$

Viscosity to entropy density ratio (recent update)





(η/n) drops to zero
in superfluid phase

(η/s) has a minimum
near T_c

Joseph et al. (2014)

Outlook

Fluid dynamics as an E(F)T: Many interesting questions remain.

Experiment: Main issue is temperature, density dependence of η/s. How to unfold?

Need hydro codes that exit "gracefully" (LBE, anisotropic hydro, hydro+cascade)

Quasi-particles vs quasi-normal modes (kinetics vs holography) unresolved. Need better holographic models, improved lattice calculations.