Perfect Fluidity

in Cold Atomic Gases?

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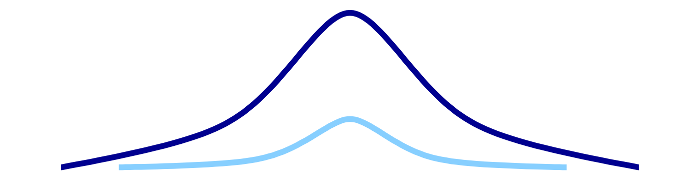
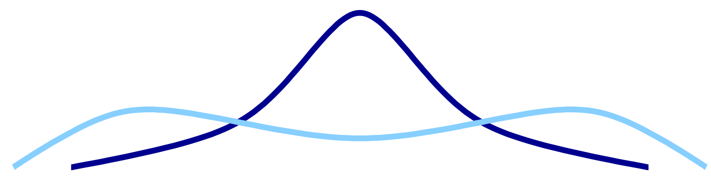
"RHIC serves the perfect liquid"

Hydrodynamics

Long-wavelength, low-frequency dynamics of conserved or spontaneoulsy broken symmetry variables.



$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water $(\rho, \epsilon, \vec{\pi})$

Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

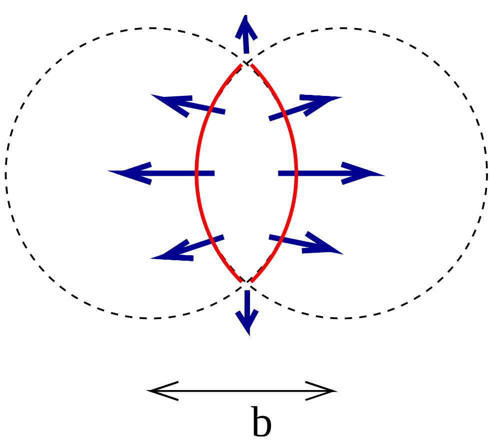
Energy momentum tensor

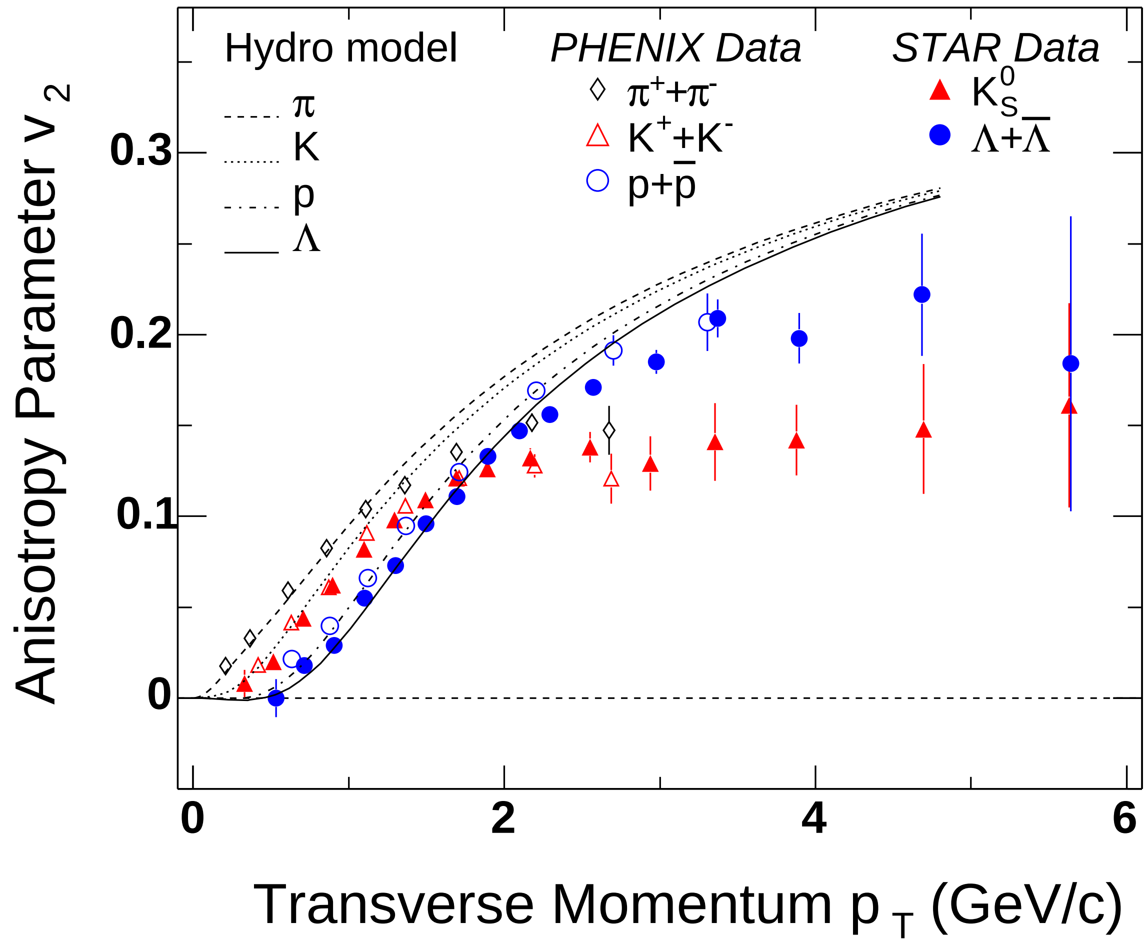
$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k \right) + \ldots$$

reactive dissipative

Elliptic Flow

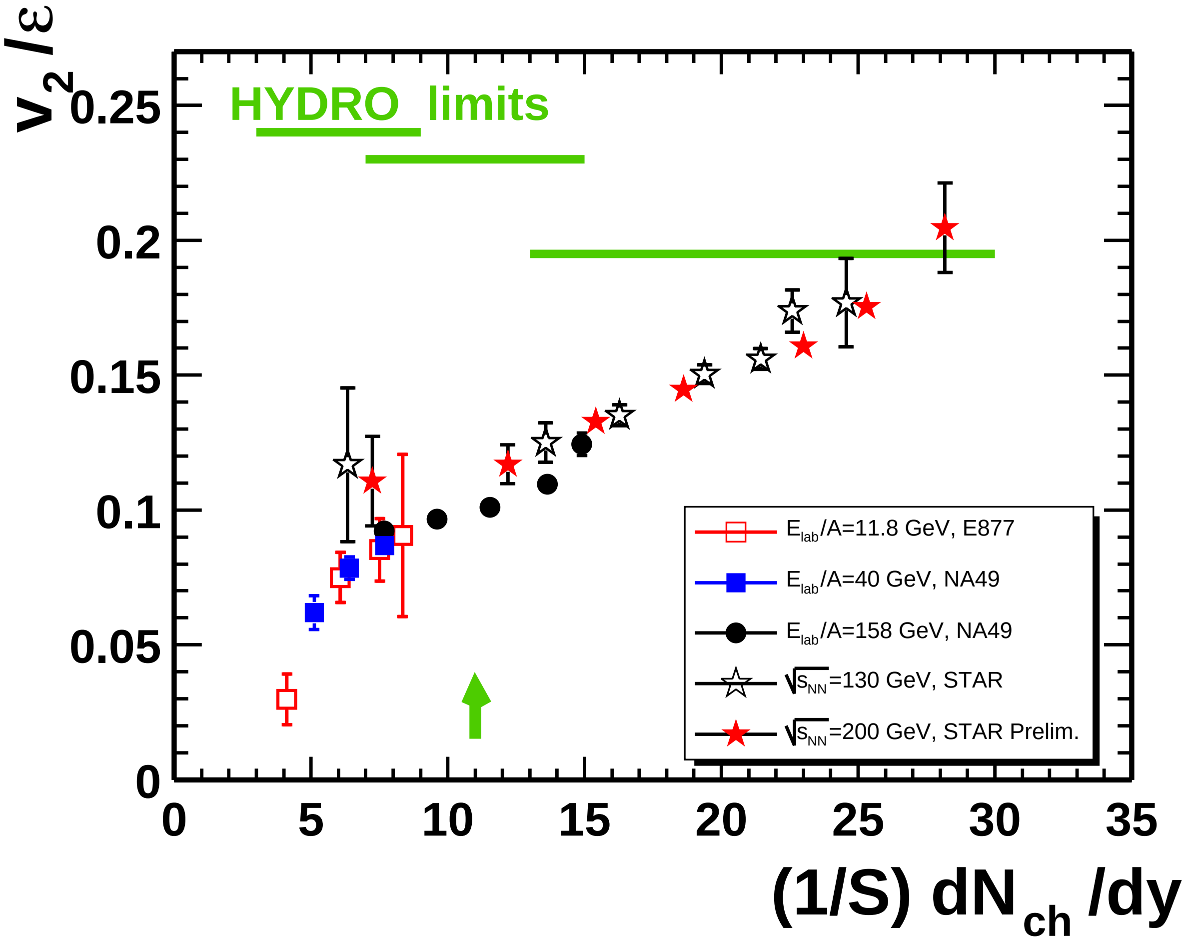
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

Elliptic Flow II



Requires "perfect" fluidity ($\eta/s < 0.1$?)

(s)QGP saturates (conjectured) universal bound $\eta/s = 1/(4\pi)$?

Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

$$\eta \sim \frac{1}{3} n \bar{v} m l = \frac{2}{3} n \left(\frac{1}{2} m \bar{v}^2 \right) \frac{l}{\bar{v}} = \frac{2}{3} n E \tau_{mft}$$

Entropy density: $s \sim k_B n$. Uncertainty relation implies

$$\frac{\eta}{s} \sim \frac{n E \tau_{mft}}{k_B n} \sim \frac{E \tau_{mft}}{k_B} \geq \frac{\hbar}{k_B}$$

Validity of kinetic theory as $E\tau \sim \hbar$?

Why η/s? Why not η/n?

Holographic Duals at Finite Temperature

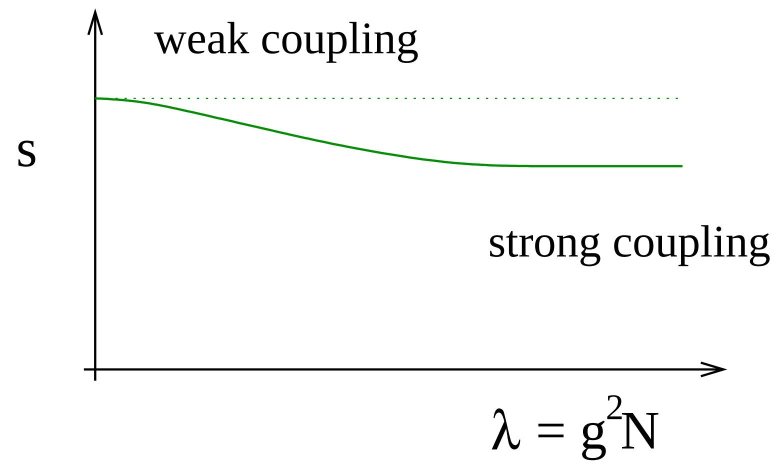
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon

Strong coupling limit

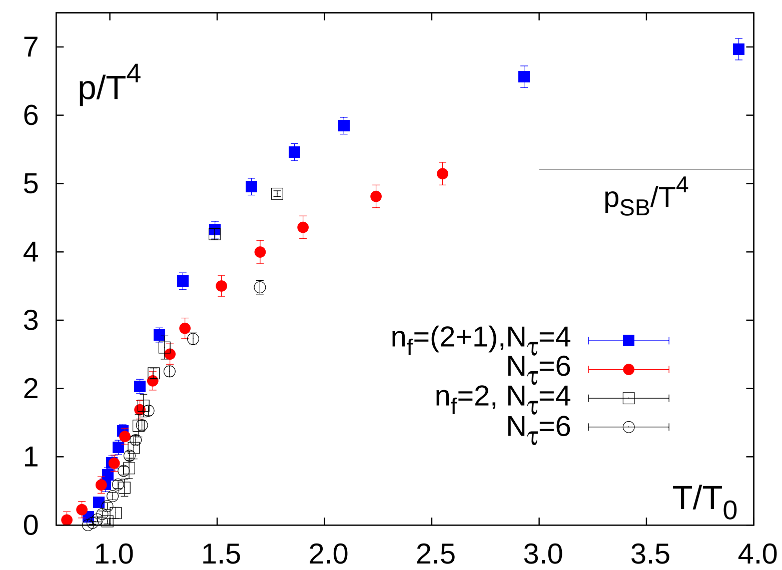
$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

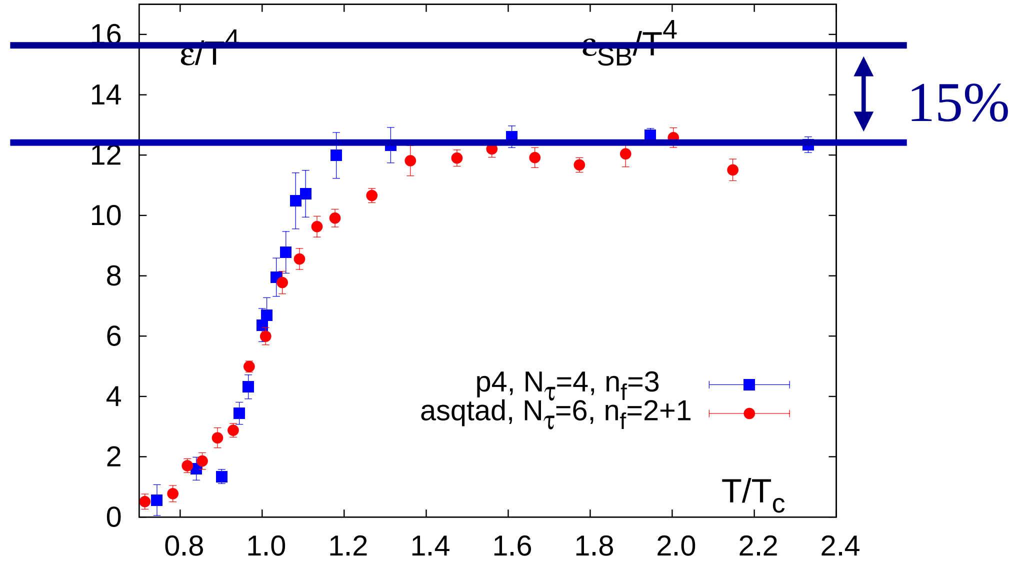
Gubser and Klebanov



Extended to transport properties by Policastro, Son and Starinets

Quark Gluon Plasma Equation of State (Lattice)





Compilation by F. Karsch (SciDAC)

Holographic Duals: Transport Properties

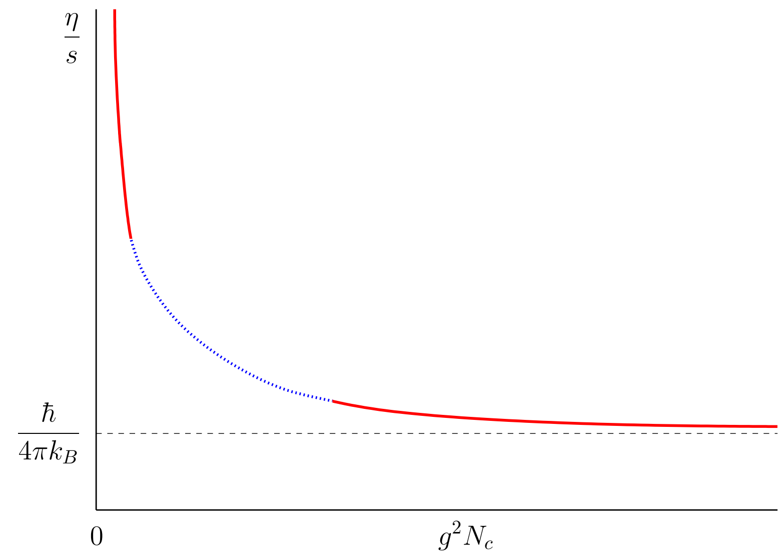
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

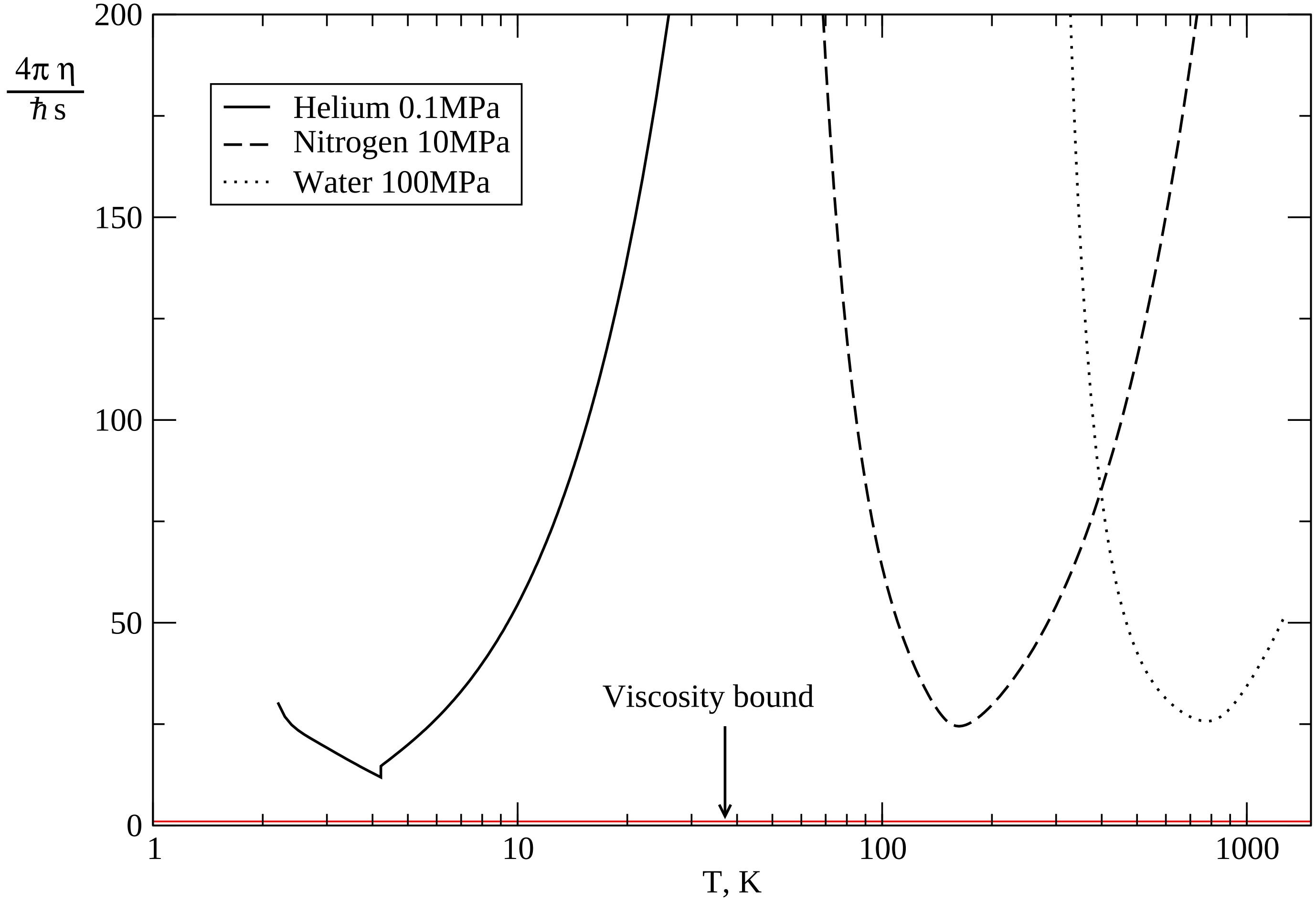
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets



Strong coupling limit universal? Provides lower bound for all theories?

Viscosity Bound: Common Fluids



Viscosity Bound: Counter Examples?

non relativistic systems: can make S/N large

$$\frac{\eta}{s} = \frac{1}{\log(N_s)} \frac{c\sqrt{mT}}{a^2 n}$$

modified conjecture: applies to systems that can be embedded in a relativistic (gauge?) theory

T. Cohen: Consider heavy-light mesons in QCD with $N_F = N_c \to \infty$

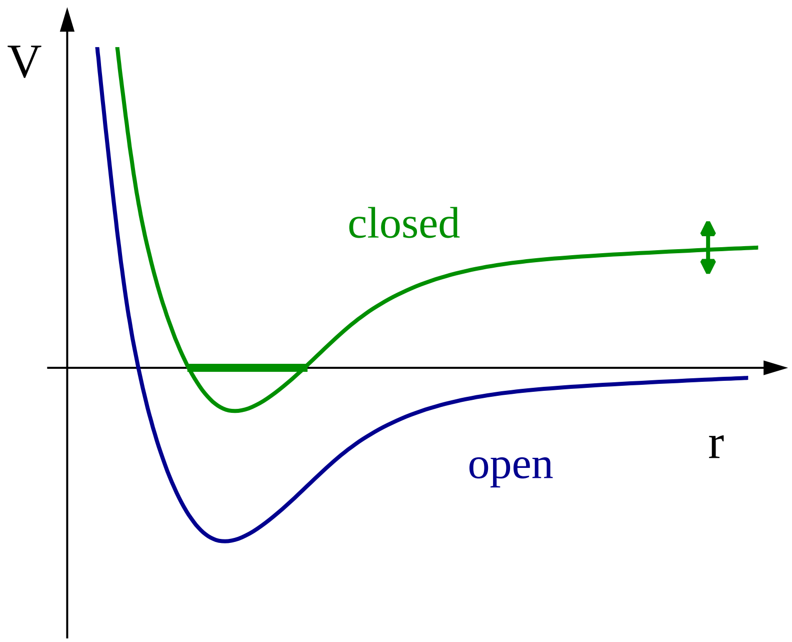
$$m_Q = m_Q^0 N_F \qquad n = \frac{n_0}{\log(N_F)}, \qquad T = \frac{T_0}{N_F \log(N_F)^{1/2}}$$

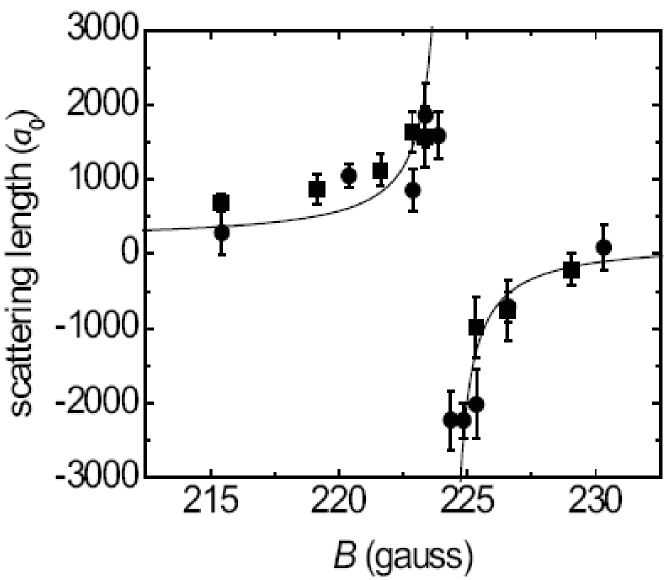
$$\frac{\eta}{s} \sim \frac{1}{\log(N_s)}$$

stable fluid? s well defined?

Designer Fluids

Atomic gas with two spin states: "$\uparrow$" and "$\downarrow$"





Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$ $$\sigma = \frac{4\pi}{k^2}$$

Why are these systems interesting?

System is intrinsically quantum mechanical

cross section saturates unitarity bound

System is scale invariant at unitarity. Universal thermodynamics

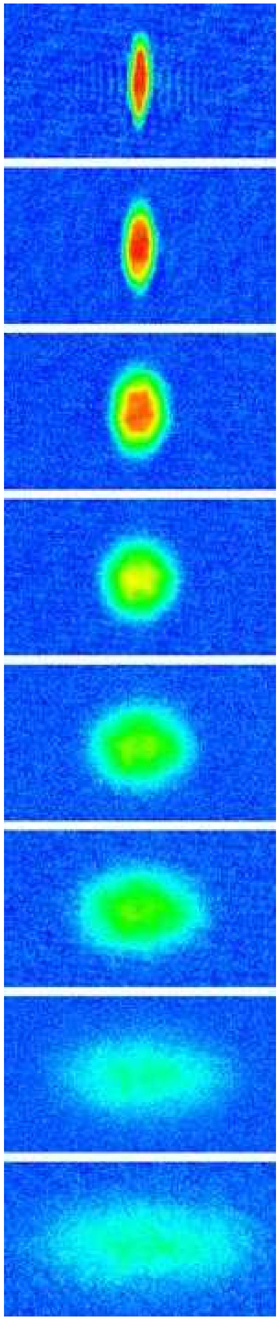
$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5}\left(\frac{k_F^2}{2M}\right)$$

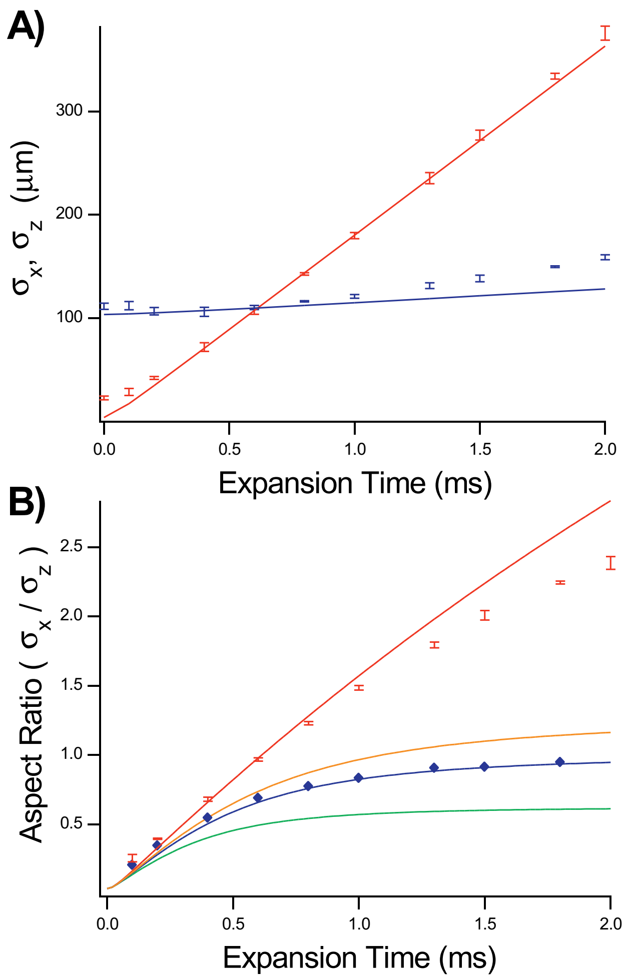
System is strongly coupled but dilute

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

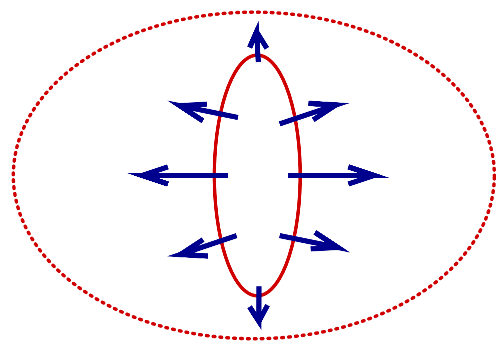
Strong elliptic flow observed experimentally

Elliptic Flow



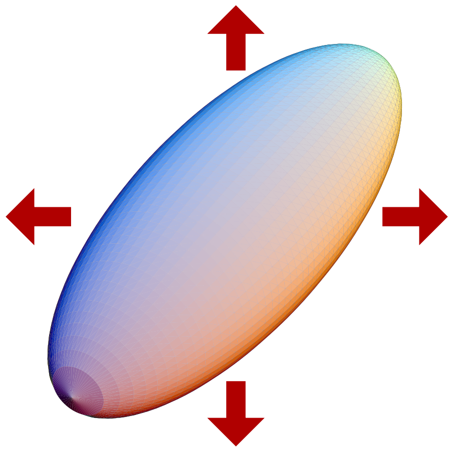


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

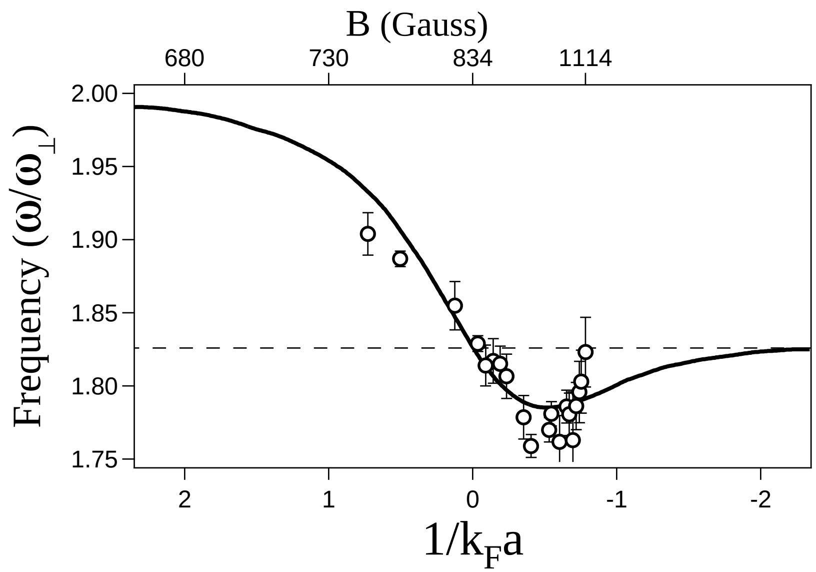


Collective Modes

Radial breathing mode



Kinast et al. (2005)



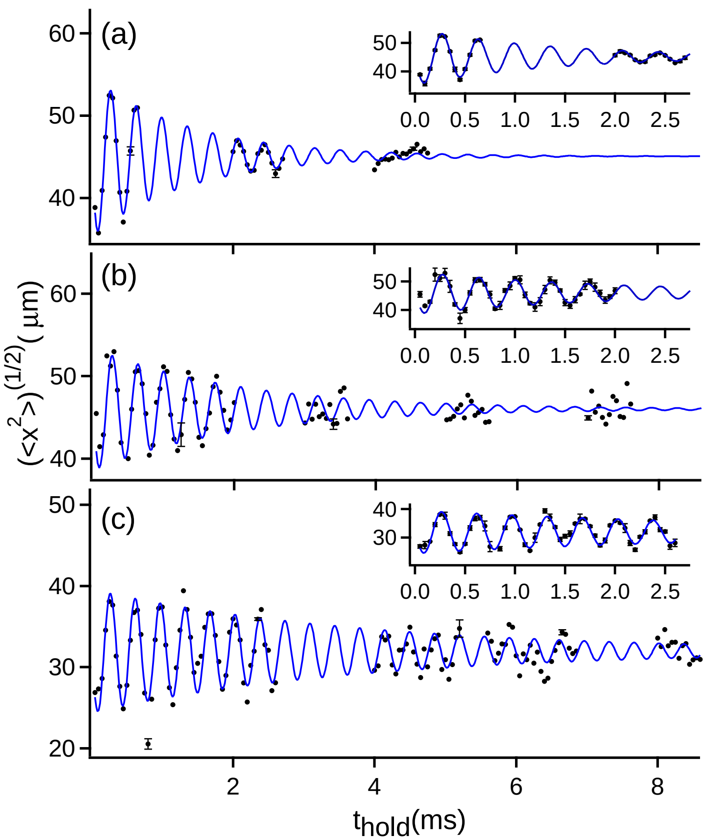
Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

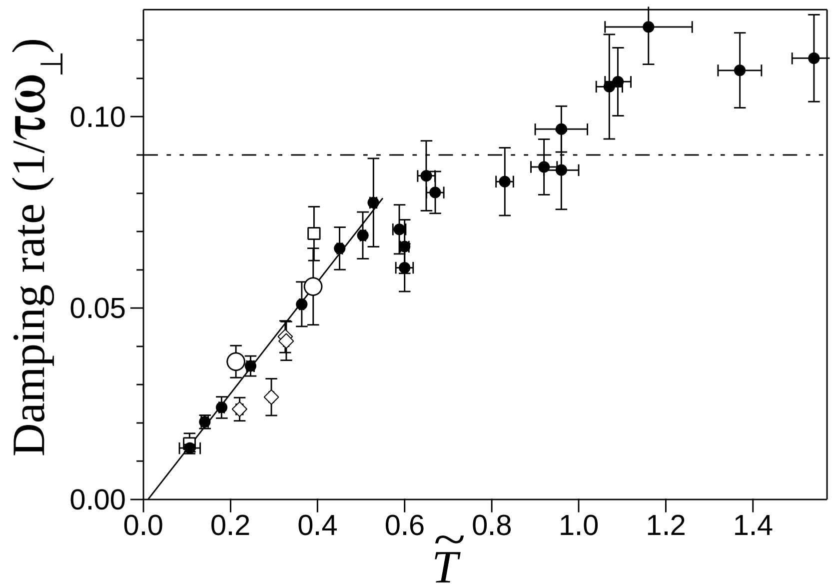
$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{1}{mn}\vec{\nabla}P - \frac{1}{m}\vec{\nabla}V$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

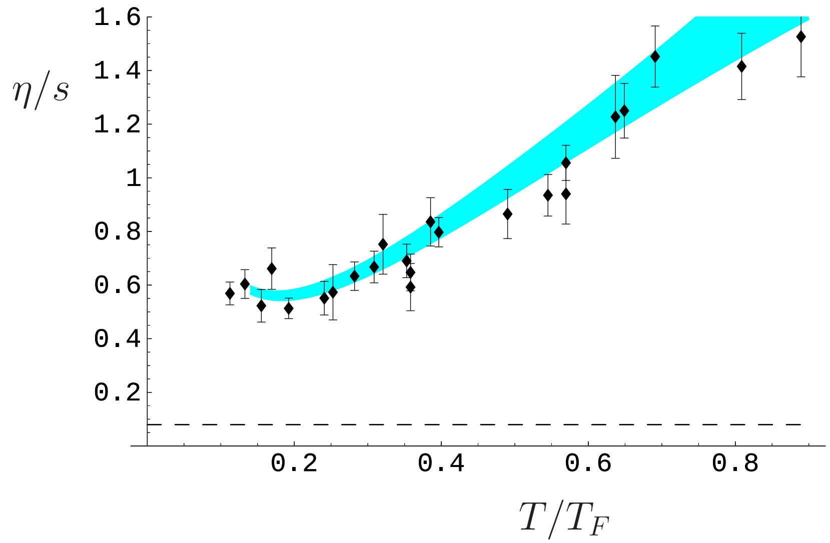
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \zeta \int d^3x\, (\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x\, (\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

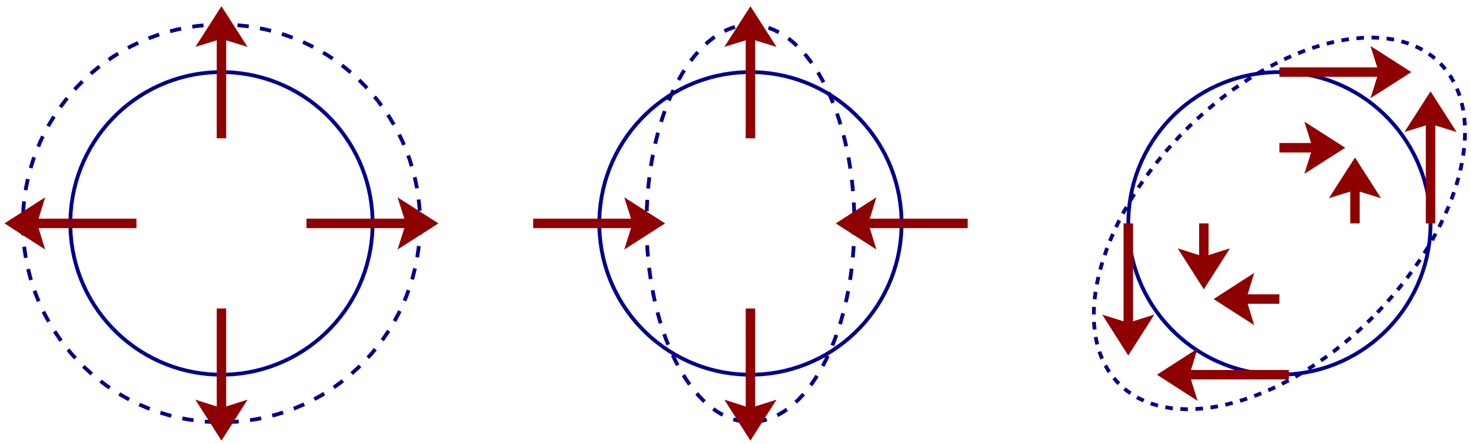
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{\bar{\omega}}{\omega_\perp}\frac{N}{S}$$



see also Bruun, Smith, Gelman et al.

Damping dominated by shear viscosity?

Study dependence on flow pattern



Study particle number scaling

viscous hydro: $\Gamma \sim N^{-1/3}$ Boltzmann: $\Gamma \sim N^{1/3}$

Role of thermal conductivity?

suppressed for scaling flows: $\delta T \sim T(\delta n/n) \sim const$

Kinetic Theory

Quasi-Particles: Kinetic Theory

$$T_{ij} = \int d^3p \, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

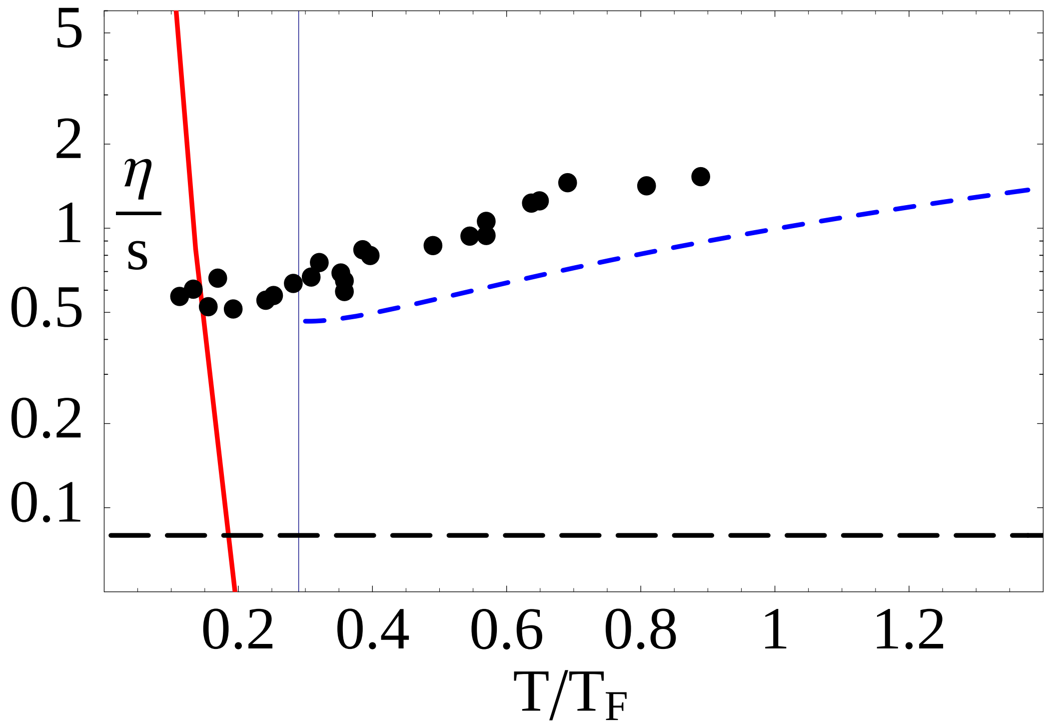
Linearized theory (Chapman-Enskog): $f_p = f_p^0 (1 + \chi_p / T)$

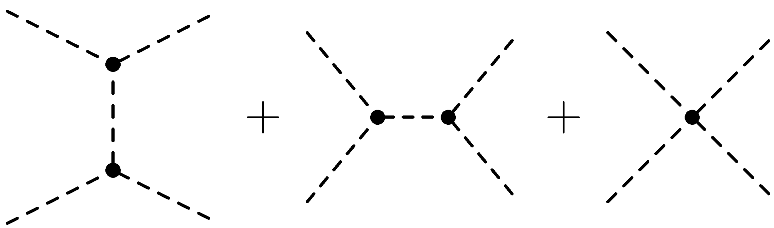
$$\eta \geq \frac{\langle \chi | X \rangle^2}{\langle \chi | C | \chi \rangle} \qquad \langle \chi | X \rangle = \int d^3p \, f_p^0 \, \chi_p \, p_{ij} v_{ij}$$

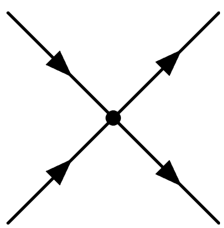
$$v_{ij} = v^2 \delta_{ij} - 3 v_i v_j$$

Low T: Phonons

High T: Atoms







$$\frac{\eta}{s} \sim \left(\frac{T_F}{T}\right)^8$$

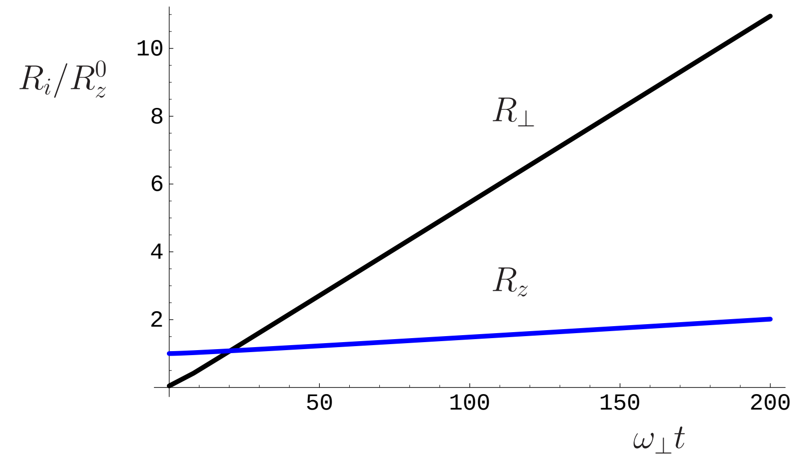
$$\frac{\eta}{s} \sim \left(\frac{T}{T_F}\right)^{3/2} \log\left(\frac{T}{T_F}\right)^{-1}$$

Elliptic Flow

Free scaling expansion

$$n(r_\perp, r_z) = \frac{1}{b_\perp^2 b_z} n_0\left(\frac{r_\perp}{b_\perp}, \frac{r_z}{b_z}\right)$$

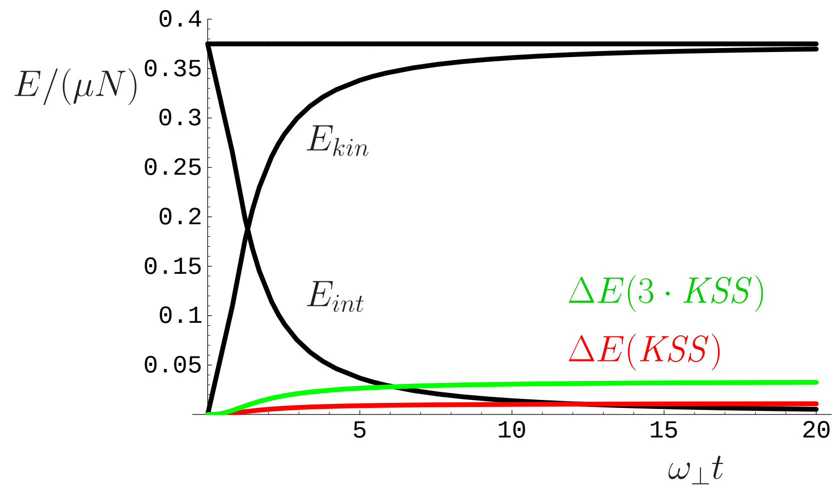
$$\ddot{b}_\perp = \frac{\omega_\perp^2}{b_\perp (b_\perp^2 b_z)^\gamma}$$



Viscous damping

$$\dot{E} = -\frac{4}{3}\left(\frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_z}{b_z}\right)^2 \int d^3x\, \eta(x)$$

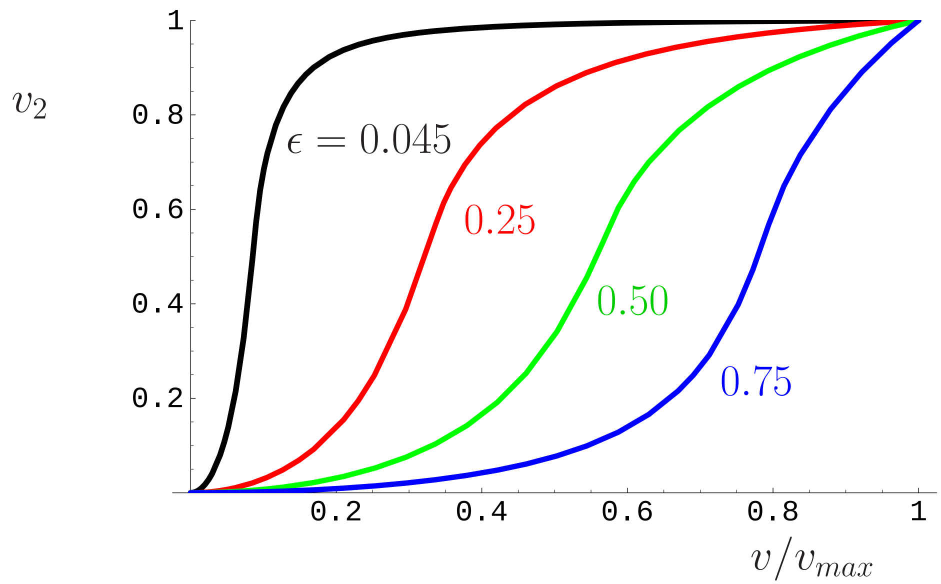
$$\Delta E = \int dt\, \dot{E} \quad \text{converges quickly}$$



Elliptic Flow (cont)

Can define $v_2 = \langle \cos(2\phi) \rangle$ as in HI collisions

$$\epsilon = \frac{\langle 2z^2 - x^2 + y^2 \rangle}{\langle z^2 + x^2 + y^2 \rangle}$$



Can also sweep to BEC regime and simulate recombination models

Final Thoughts

Cold atomic gases provide interesting, strongly coupled, model system in which to study sources of dissipation.

$$\eta/s \sim 1/2$$

Smaller than any other known liquid (except for QGP?). Since other sources of dissipation exist, this is really an upper bound.

There are reliable calculations of η/s at high T (Bruun, Smith, ...) and low T (Rupak and T.S). Extrapolate to $T \sim T_F$

Conjectured bound has a smooth non-relativistic limit. Note that the $a \to \infty$ limit can also be realized in QCD (by tuning μ, μ_e and m_q).

But: In non-relativistic systems $s \gg n$ possible

Purely field theoretic proofs?

$\mathcal{N} = 4$ SUSY YM is special because there is no phase transition. In real systems there is a phase transition as the coupling becomes large, and the new phase (confined in QCD, superfluid in the atomic system) has weakly coupled low energy excitations, and a large viscosity.

No quasi-particles in sQGP?