Non-Fermi Liquid Effective Field Theory

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Motivation

- Matter at high baryon density exists in nature

  low energy excitations determine physical properties:

  specific heat, transport propereties, emissivity, ...

- Low energy degrees are composite (at any density)

  study effective degrees of freedom in a regime

  where we can make the connection QCD $\rightarrow$ EFT

Why Effective Field Theories?

- weak coupling expansion $\neq$ loop expansion

  organize perturbation theory, resum logarithms, etc.

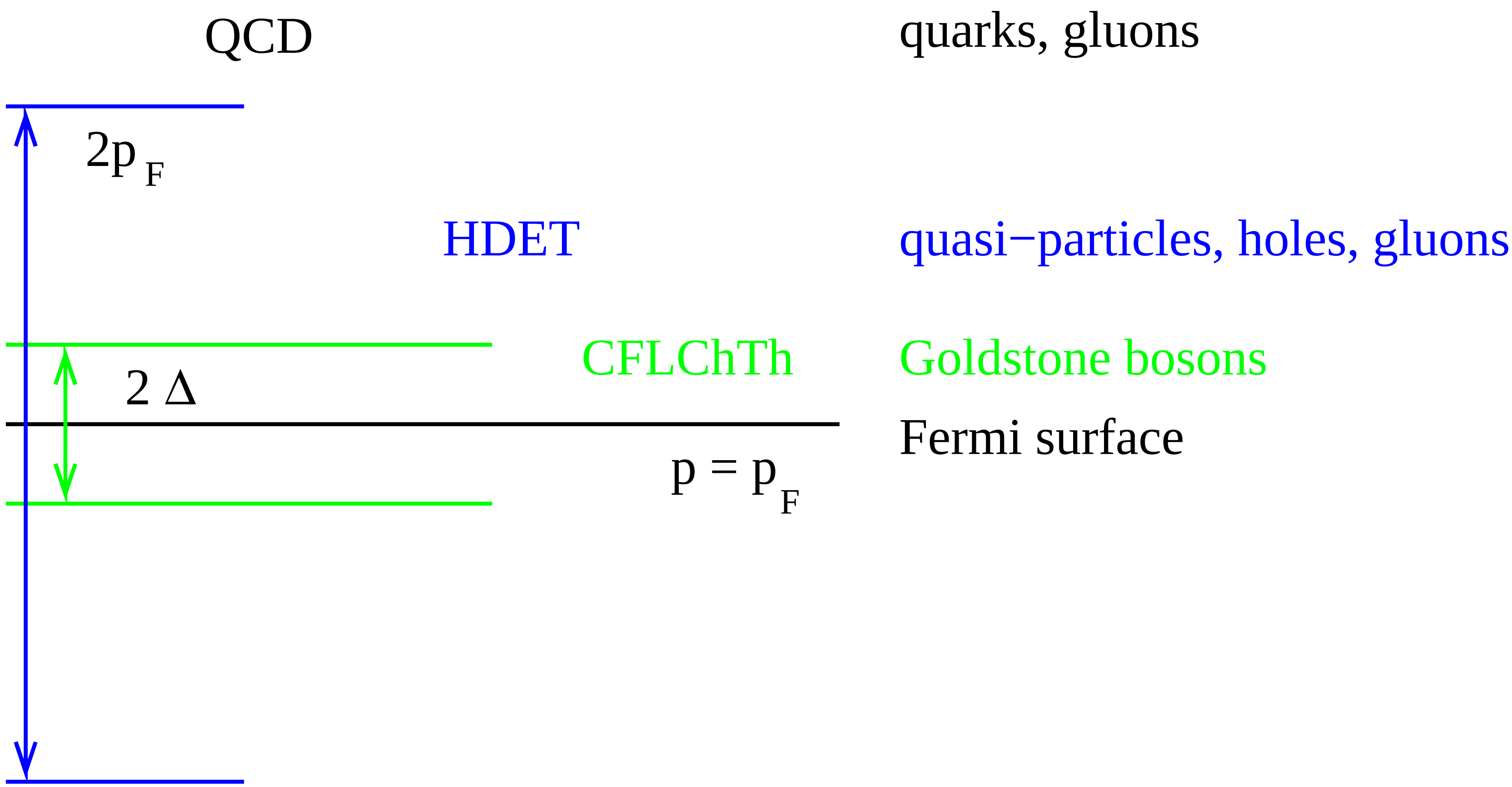
- reduce confusion

  gauge invariance, off-shell behavior, etc.

- effects of perturbations, external fields

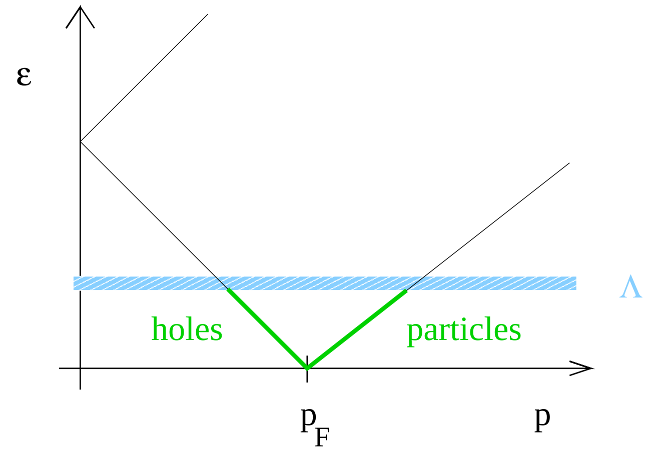
  quark masses, electron chemical potentials, etc.

Effective Field Theories



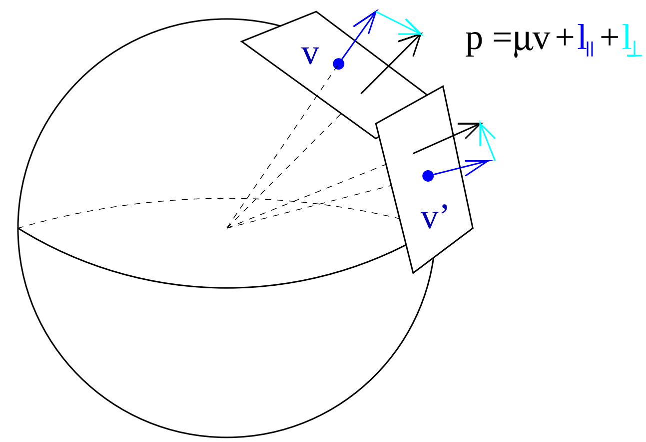
High Density Effective Theory

quasi-particles (holes)



$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

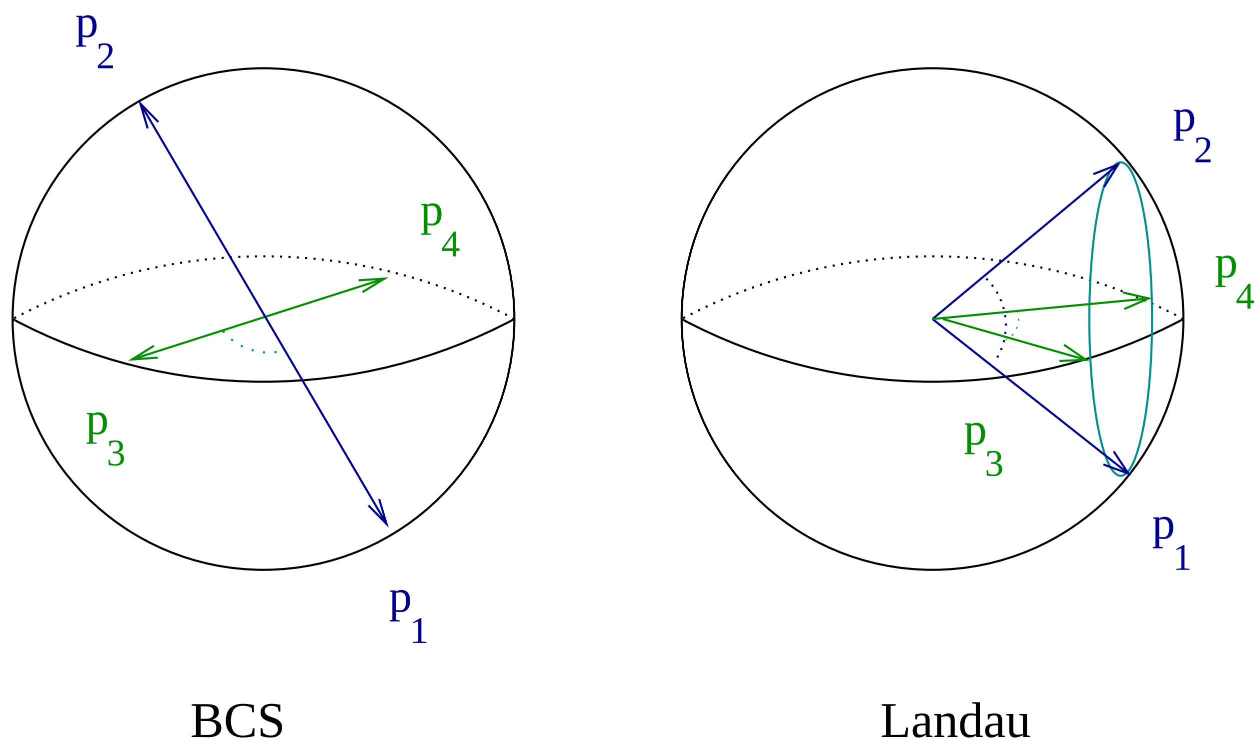
effective lagrangian for ψ_{v+}

$$\mathcal{L} = \sum_v \psi_v^\dagger (iv\cdot D)\psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + O(1/\mu)$$

Four Quark Operators

quark-quark scattering

$$(v_1, v_2) \to (v_3, v_4)$$

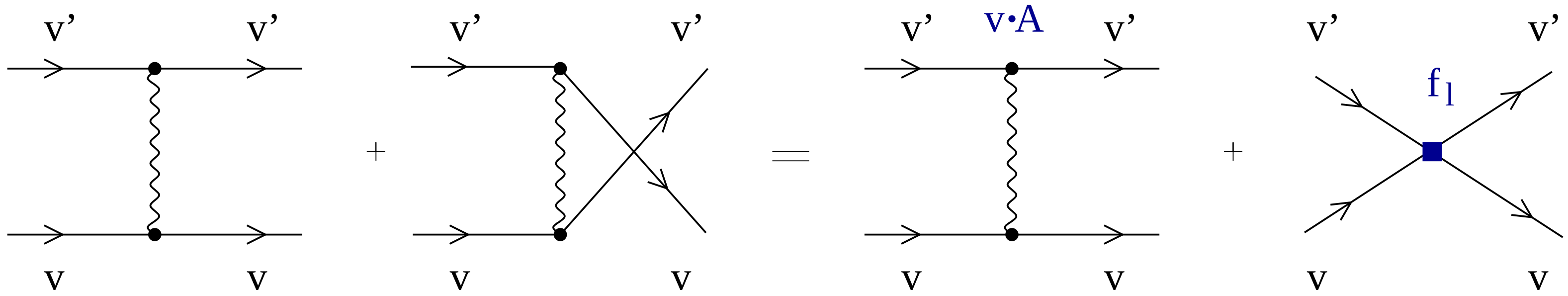


$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'}(\vec{v}\cdot\vec{v}') \Big(\psi_v \Gamma \psi_{-v}\Big)\Big(\psi^\dagger_{v'} \Gamma' \psi^\dagger_{-v'}\Big),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'}(\vec{v}\cdot\vec{v}') \Big(\psi_v \Gamma \psi_{v'}\Big)\Big(\psi^\dagger_{\tilde{v}} \Gamma' \psi^\dagger_{\tilde{v}'}\Big)$$

Four Fermion Operators: Matching

- match scattering amplitudes on Fermi surface: forward scattering



- color-flavor-spin symmetric terms

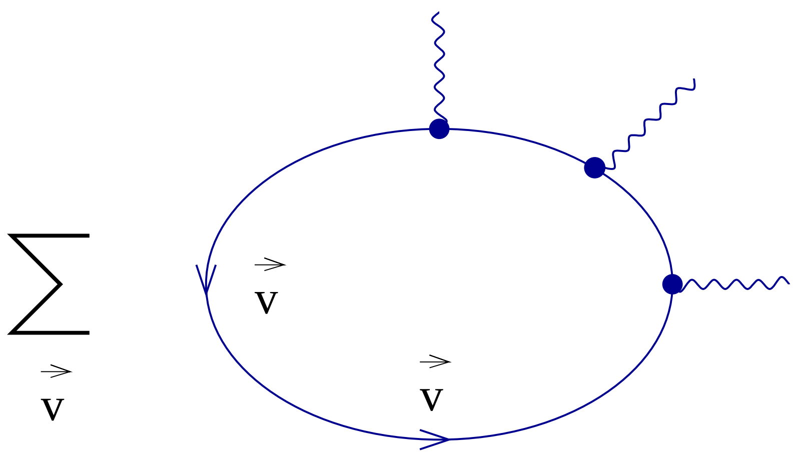
$$f_0^s = \frac{C_F}{4N_cN_f}\frac{g^2}{p_F^2}, \qquad f_i^s = 0 \;\; (i > 1)$$

Power Counting

- naive power counting

$$\mathcal{L} = \hat{\mathcal{L}}\left(\psi, \psi^\dagger, \frac{D_{||}}{\mu}, \frac{D_\perp}{\mu}, \frac{\bar{D}_{||}}{\mu}, \frac{m}{\mu}\right)$$

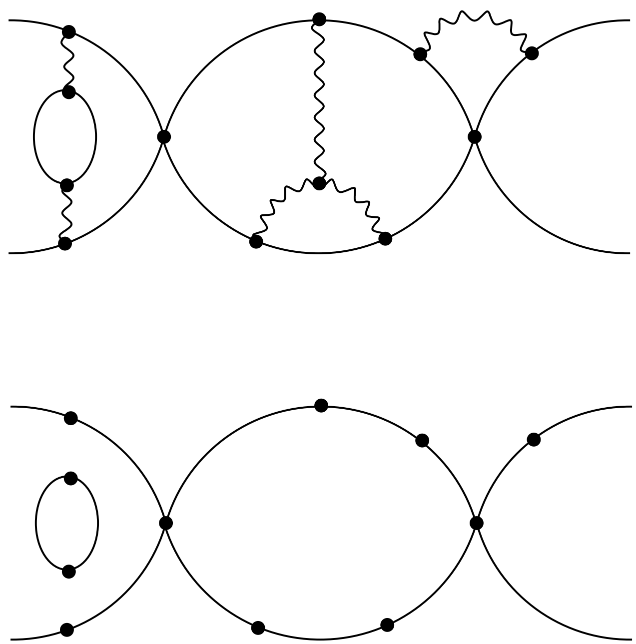
- problem: hard loops



$$\frac{1}{2\pi}\sum_{\vec{v}}\int\frac{d^2 l_\perp}{(2\pi)^2} = \frac{\mu^2}{2\pi^2}\int\frac{d\Omega}{4\pi}.$$

Modified Power Counting: $\mathcal{A} \sim l^\delta$

$$\delta = \sum_i \left[(k-4)V_k^S + (k-2-f_k)V_k^H \right] + E_Q + 4 - 2N_C$$



V_k^S	soft vertices of $O(l^k)$
V_k^H	hard vertices of $O(l^k)$
f_k	number of fermion fields in V_k^H
E_Q	external quark lines
N_C	connected graphs in hard graph

- quark loops in gluon n-pt fcts blow up at $l \sim g\mu$
- four quark operators are leading order
- six quark operators (etc) are suppressed

Effective Theory for $l \sim g\mu$

$$\mathcal{L} = \psi_v^\dagger (iv\cdot D)\psi_v + f_0^s(\psi_v^\dagger \psi_v)(\psi_{v'}^\dagger \psi_{v'}) - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v\cdot D)^2} G^b_{\mu\beta}$$

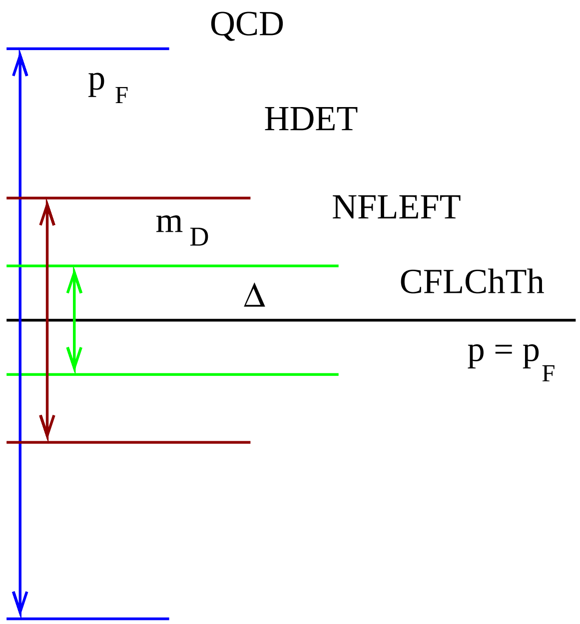
- transverse gauge boson propagator

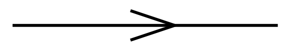
$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0|/|\vec{k}|},$$

- scaling of gluon momenta

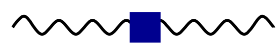
$$|\vec{k}| \sim k_0^{1/3} \eta^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Effective Theory for $l \sim g\mu$

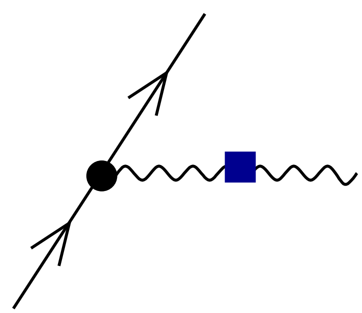




$$\frac{i}{p_0 - p_{||} - p_\perp^2/(2\mu)}$$



$$\frac{ik_\perp}{k_\perp^2 - i\eta k_0}$$

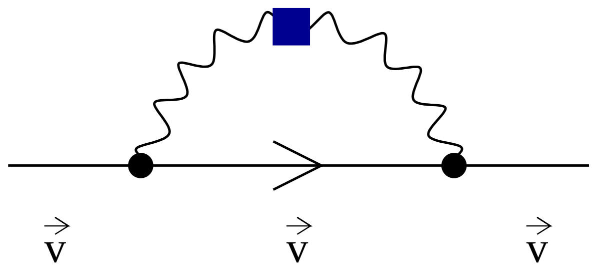


$$ig + \ldots$$

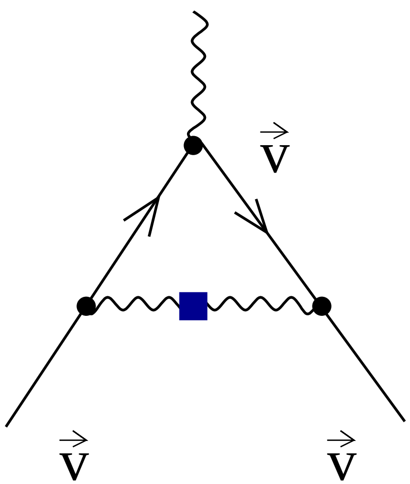
Quasi-Quarks at Large Density:

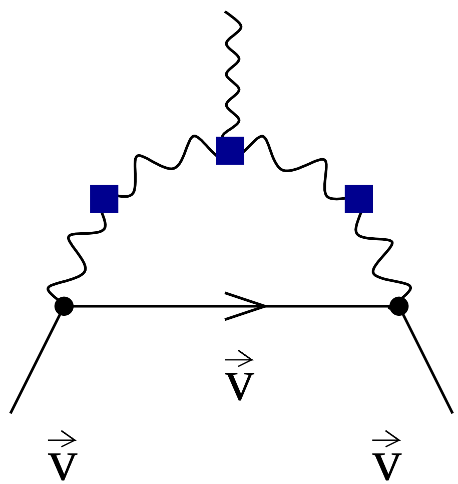
Non-Fermi Liquid Effects

Loop Corrections



$$\Sigma(\omega) \simeq \frac{g^2 C_F}{12\pi^2}\omega \log\left(\frac{\Lambda}{\omega}\right)$$





$$\Gamma_\alpha = \frac{g^3 C_F v_\alpha}{12\pi^2} \log\left(\frac{\Lambda}{\omega}\right)$$

time like

$$\Gamma_\alpha = O(g^3)$$

space like

"Migdal's Theorem" for QCD

- self energy has to be resummed for $\omega \sim \Lambda \exp(-9\pi^2/g^2)$
- coupling has no logs $(k \gg k_4!)$

Renormalization Group

renormalized parameters

$$\psi_{0,v} = Z^{1/2}\psi_v, \quad v_{0,F} = Z_F v_F, \quad g_0 = \frac{Z_g}{Z Z_F} g, \quad \alpha = \frac{g^2 v_F}{4\pi},$$

one loop calculation ($\beta = \frac{\partial \alpha}{\partial \log \Lambda}$, $\gamma = \frac{\partial \log Z}{\partial \log \Lambda}$, $\gamma_F = \frac{\partial \log Z_F}{\partial \log \Lambda}$)

$$\gamma(\alpha) = -\gamma_F(\alpha) = \frac{4\alpha}{9\pi}, \qquad \beta(\alpha) = -\gamma_F(\alpha)\alpha \qquad \text{IR free !!}$$

RG equation

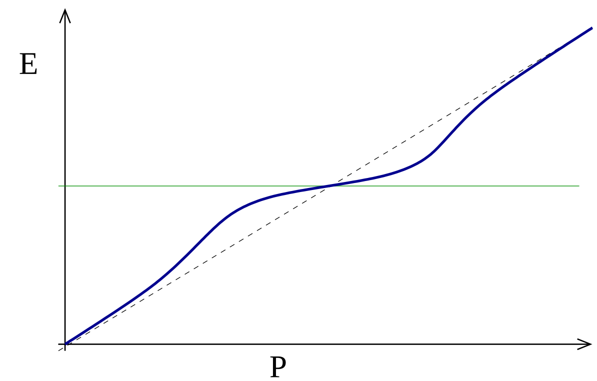
$$\left\{ \Lambda \frac{\partial}{\partial \Lambda} + \beta(\alpha)\frac{\partial}{\partial \alpha} - \gamma_F(\alpha) l_i \frac{\partial}{\partial l_i} + \frac{n}{2}\gamma(\alpha) \right\} G^{(n)}(\omega_i, l_i, \alpha) = 0,$$

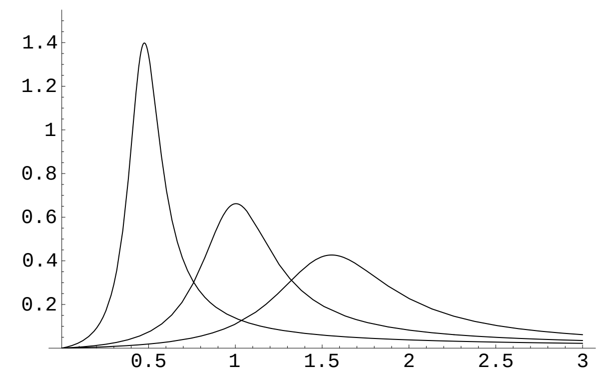
$\gamma \ll 1$: RG equation can be solved exactly

$$S^{-1}(\omega, l) = \omega \left(1 + \gamma \log\left(\frac{\Lambda}{\omega}\right)\right) - v_F l$$

no terms $\alpha^2 \log^2(\omega)$, etc.

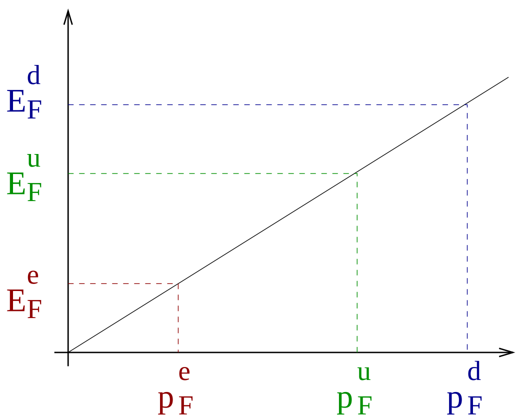
- quasi-particle velocity vanishes as
  $$v \sim \log(\Lambda/\omega)^{-1}$$
- anomalous term in the specific heat
  $$c_v \sim \gamma T \log(T)$$
- enhanced corrections to the gap
  $$\log(\mu/\Delta) = \log(\mu/\Delta_0)(1 - O(\gamma g))$$

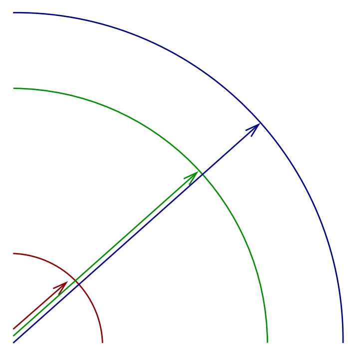




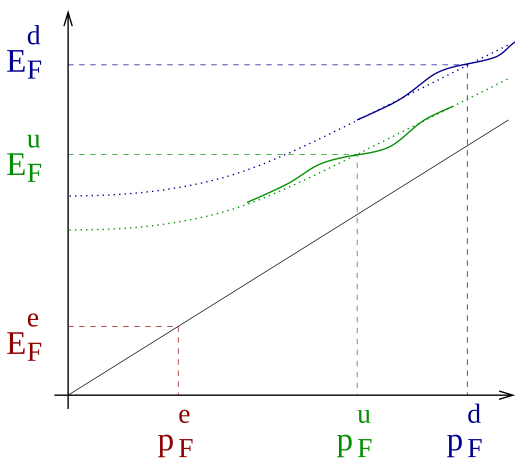
Neutrino Emission

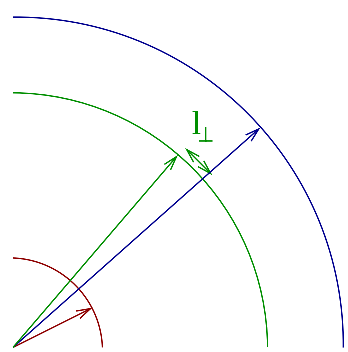
Quark Direct URCA: $d \to u + e^- + \bar{\nu}, \;\; u + e^- \to d + \bar{\nu}$





$$\epsilon \sim G_F T^7$$





$$\epsilon \sim G_F \alpha_s^3 T^6 \log^2(T)$$

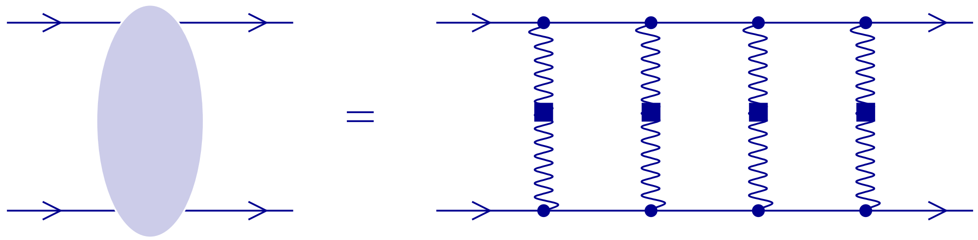
Quasi-Baryons at Large Density:

CFL Phase

Superconductivity

quark-quark scattering

$(\mu \gg \Lambda_{QCD})$



gap equation: double logarithmic behavior

$$\Delta(p_0) = \frac{g^2}{18\pi^2} \int dq_0 \left\{ \log\left(\frac{b_M}{|p_0 - q_0|} \right) + \ldots \right\} \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

collinear log BCS log

$$\Rightarrow \qquad \Delta_0 = 512\pi^4 \mu g^{-5} \exp\left(-\frac{\pi^2 + 4}{8} \right) \exp\left(-\frac{3\pi^2}{\sqrt{2}g} \right)$$

CFL Phase

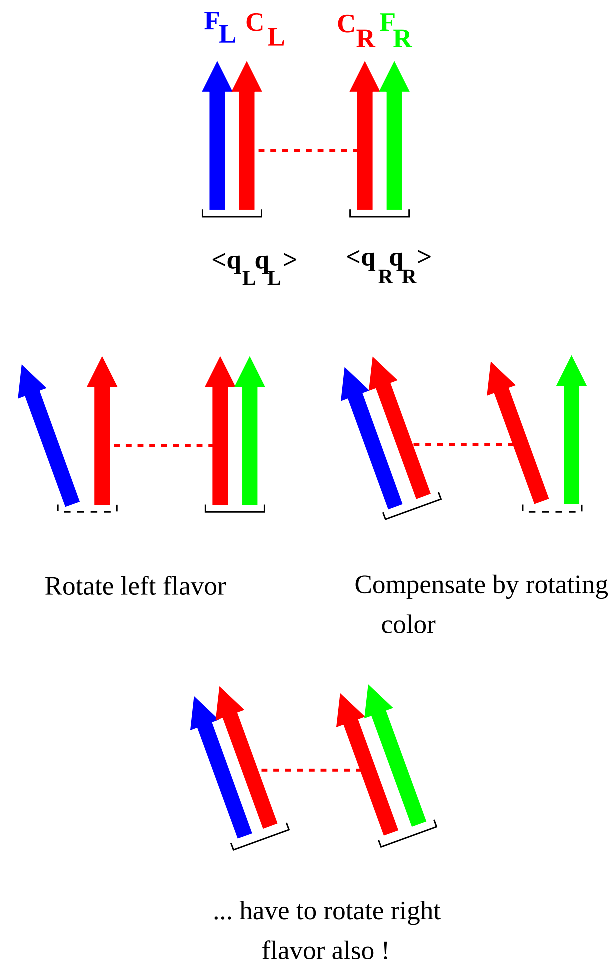
- Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$
$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

- symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

- all quarks and gluons acquire a gap



$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

EFT in the CFL Phase

consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\}$$
$$+ (L\leftrightarrow R, X\leftrightarrow Y)$$

$$\psi_L \to L\psi_L C^T,\ X \to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = 1\!\!1$$

quark loops generate a kinetic term for X, Y

$$\mathcal{L} = -\frac{f_\pi^2}{2}\left\{\mathrm{Tr}\left((X^\dagger D_0 X)^2 + (Y^\dagger D_0 Y)^2\right)\right\} + \ldots$$

integrate out gluons, identify low energy fields

$$\Sigma = XY^\dagger \ \ [8]+[1]\ \mathrm{GBs}, \qquad N_L = \xi(\psi_L X^\dagger)\xi^\dagger \ \ [8]+[1]\ \mathrm{Baryons}$$

effective theory: CFL(B)χPTh

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2 \mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left\{\mathcal{A}_\mu, N\right\}\right) \\
& - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}
$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

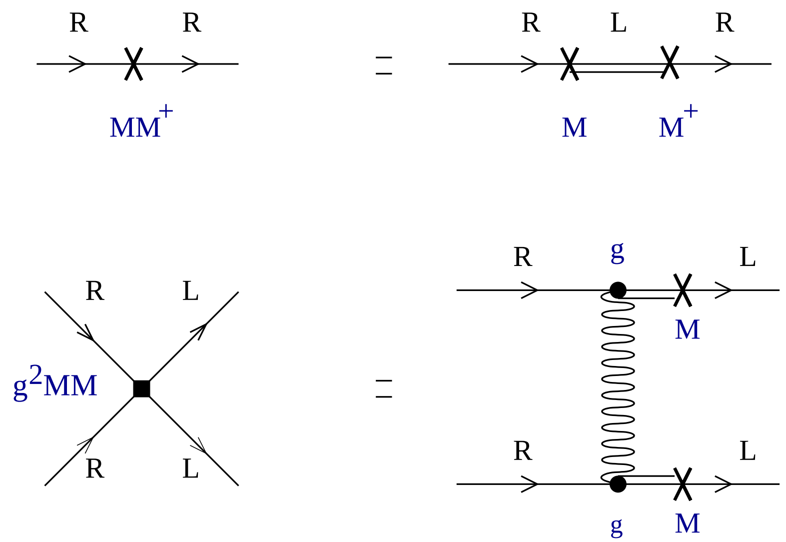
$$
\begin{aligned}
\mathcal{V}_\mu &= -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \\
\mathcal{A}_\mu &= -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi
\end{aligned}
$$

$$
f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}
$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$

$$+ \frac{C}{\mu^2}(\psi_R^\dagger M\lambda^a\psi_L)(\psi_R^\dagger M\lambda^a\psi_L)$$



Mass Terms: Match HDET to CFLχTh

kinetic term: $\psi_L^\dagger X_L \psi_L + \psi_R^\dagger X_R \psi_R$

$$\begin{aligned} D_0 N &= \partial_0 N + i[\Gamma_0, N], \qquad \Gamma_0 = \mathcal{V}_0 + \frac{1}{2}\left(\xi X_R \xi^\dagger + \xi^\dagger X_L \xi\right) \\ \nabla_0 \Sigma &= \partial_0 \Sigma + i X_L \Sigma - i \Sigma X_R \end{aligned}$$

vector (axial) potentials

contact term: $(\psi_R^\dagger M \psi_L)(\psi_R^\dagger M \psi_L)$

$$\mathcal{L} = \frac{3\Delta^2}{4\pi^2}\left\{[\mathrm{Tr}(M\Sigma)]^2 - \mathrm{Tr}(M\Sigma M\Sigma)\right\}$$

meson mass terms

Phase Structure and Spectrum

phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(X_L\Sigma X_R\Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

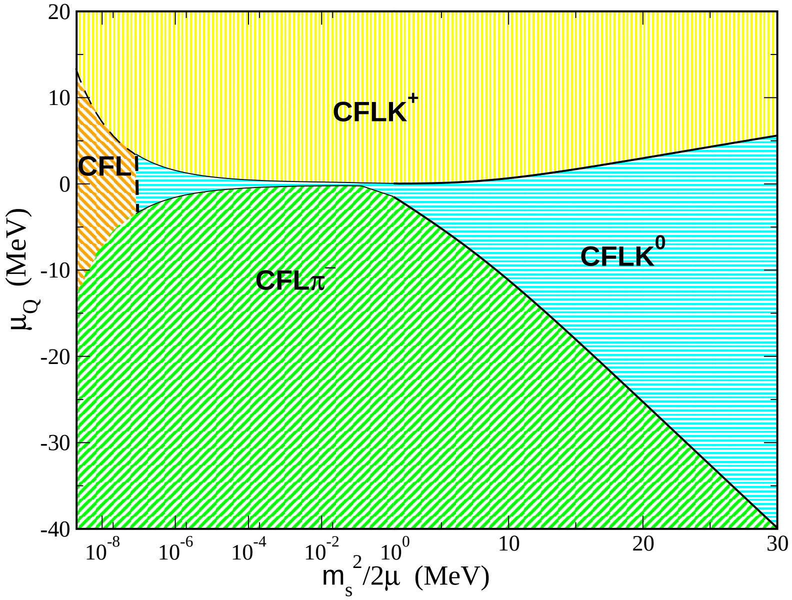
$$V(\Sigma_0) \equiv min$$

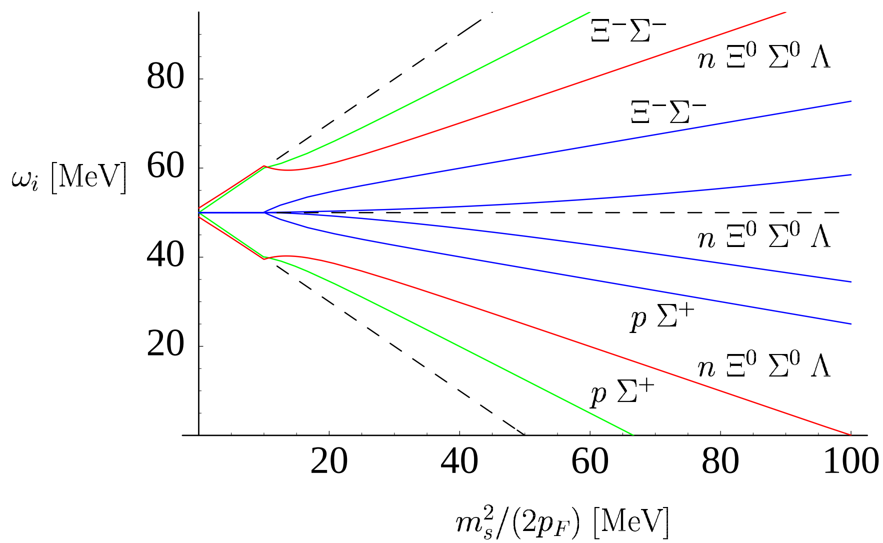
fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger iv^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger\gamma_5\rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi\frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger\frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





meson condensation: CFLK

reliable: yes!

gapless modes? (gCFLK)

reliable: not clear yet

Summary

- EFT/RG methods provide powerful tools

  phase structure and spectrum at large density

- normal phase: non-Fermi liquid behavior due to unscreened transverse gauge bosons

  perturbation theory reliable (no rainbows, etc.)

- superfluid phase: effective chiral theory with calculable coefficients

  kaon condensation, possibility of gapless modes