Simulating Stochastic Fluids

Why? EFT for Cor. Fct

$\rightsquigarrow$ implement FD – Rel

Critical Dynamics
($\rightsquigarrow$ QCD crit. point, chiral dynamics)

How?

1) Landau – Lifshitz

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = T_0^{\mu\nu} + \ldots + \Xi^{\mu\nu}$$

$$\langle \Xi^{\mu\nu} \Xi^{\alpha\beta} \rangle = \eta T \Delta^{\mu\nu\alpha\beta} \delta^4(x - x')$$

2) Stochastic Eff. Actions

(MSR, SK – EFT)

3) Hydro Kinetics

$$G(\vec{x}_1, \vec{x}_2; t) = \langle \phi(x_1, t) \phi(x_2, t) \rangle$$

$$\xrightarrow{WT} W_2(\vec{x}, t; p)$$

$$\partial_0 W_2 = -\gamma p^2 (W_2 - W_2^0)$$

4) Hydro+ : Hydro Kin +

$$S[m, W_2]$$

GOAL: a) CRIT. BEHAVIOR

b) DYN. BACKGROUNDS

$\rightsquigarrow$ STOCH. FLUID DYNAMICS (OPTION 1)

WHAT IS THE PROBLEM?

1) "INFINITE NOISE"

$$\langle [T_{xy}, T_{xy}]\theta(t)\rangle_{\omega,k} = P + i\eta + c\,\omega^{3/2}$$

$$P = \frac{\#}{a^3} \qquad \eta = \frac{\#}{a} \qquad c = \text{FINITE}$$

$$\rightsquigarrow \quad P_{PH} = P_B + \frac{\#}{a^3}$$

$$\frac{d}{da} P_{PH} = 0 \qquad \rightsquigarrow \text{RG}$$

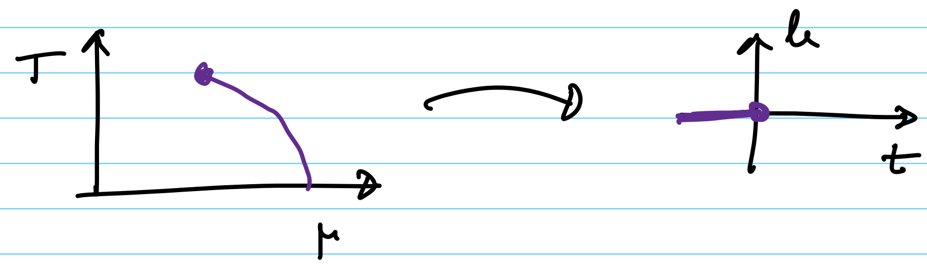
2) DISCR. AMB. ITO / STRATO

OBSERVATION

FOR TYP. a "COSM. CONSTANT" $\frac{\#}{a^3}$ IS LARGE, BUT PHYS. BACKREACTION IS SMALL

$$\xi/s = \frac{1}{4\pi} \underset{\substack{\| \\ 37}}{\frac{1}{N_{dog}}} \cdot \left(\frac{\xi}{\xi_0}\right)^3 \cdots$$

⤳ CONCENTRATE ON CRIT MODE (ADVECTED BY BACKGROUND FLOW)



IN THE RF OF THE FLUID, THESE THEORIES ARE KNOWN AS MODEL A-Z OF HH

MODEL A

$$\partial_0 \phi = -\Gamma \frac{\delta F}{\delta \phi} + \xi$$

$$\langle \xi \xi \rangle = 2 \Gamma T \, \delta^4(x-x')$$

$$F = \int d^3x \left\{ \frac{1}{2}(\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \lambda \phi^4 \right\}$$

MODEL B

$$\partial_0 \phi = \Gamma \nabla^2 \frac{\delta F}{\delta \phi} + \xi$$

$$\langle \xi \xi \rangle = -2 \Gamma T \, \nabla^2 \delta(x-x')$$

$$\partial_0 \psi + \vec{\nabla} \cdot \vec{j} = 0$$

MODEL H

$$\partial_0 \psi = \Gamma \nabla^2 \frac{\delta \mathcal{F}}{\delta \psi} - \nabla_i \psi \cdot \frac{\delta \mathcal{F}}{\delta \pi_T^i} + \xi$$

$$\partial_0 \pi_T^i = \eta \nabla^2 \frac{\delta \mathcal{F}}{\delta \pi_T^i} + \nabla_i \psi \frac{\delta \mathcal{F}}{\delta \psi} + \theta_i + \nabla_k \pi_T^i \frac{\delta \mathcal{F}}{\delta \pi_T^k}$$

$$\mathcal{F} = \mathcal{F}_0 + \int d^3x \, \frac{1}{2} \pi_T^2$$

CONSIDER MODEL A

$$\partial_0 \psi = - \Gamma \frac{\delta \mathcal{F}}{\delta \psi} + \xi$$

$$\psi(t+\Delta t) = \psi(t) - \Delta t \, \Gamma \left.\frac{\delta \mathcal{F}}{\delta \psi}\right|_a + (\Delta t) \left(\frac{\Gamma T}{a^3 \Delta t} \right)^{1/2} \theta$$

$$\langle \theta^2 \rangle = 1$$

METROPOLIS:

$$\Delta \psi = \sqrt{2 \Gamma T (\Delta t)} \cdot \theta$$

$$P_{acc} = e^{-\Delta \mathcal{F} / T}$$

$$\psi(t+\Delta t) = \psi(t) + \Delta\psi \qquad P_{acc}$$

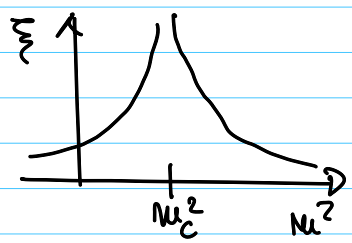
$$\psi(t+\Delta t) = \psi(t) \qquad 1 - P_{acc}$$

THEN

$$\langle (\psi(t+\Delta t) - \psi(t)) \rangle = -(\Delta t)\,\Gamma \frac{\delta F}{\delta \psi}$$

$$\langle (\psi(t+\Delta t) - \psi(t))^2 \rangle = \Delta t \, (2\Gamma T)$$

STUDY EQU. BEHAVIOR

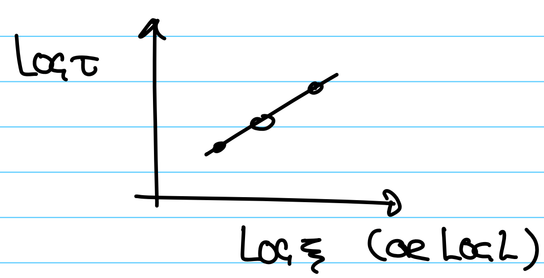


(IN PRACTICE: USE BINDER CUMULANTS)

THEN AT $m^2 = m_c^2$

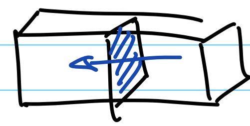
$$\langle \psi_k(0)\, \psi_{-k}(t) \rangle \sim e^{-\Gamma_k t}$$

$$\tau_k = \frac{1}{\Gamma_{\xi^{-1}}} \sim \xi^2$$



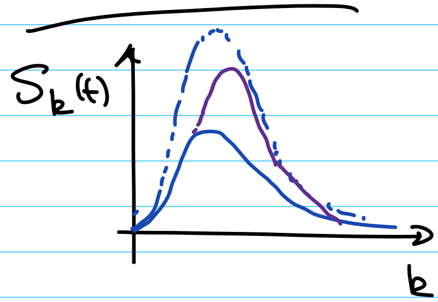
MOD A: $z \simeq 2.03$

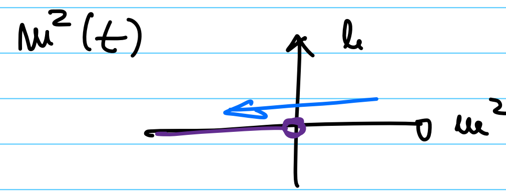
Model B



Mod B: $z = 3.91$

KZ - Scaling





Diff. Sweep Rates: Data Collapse
as a fct of $(k\, l_{KZ})$

$$\left.\frac{d\tau_R}{dt}\right|_{t=\tau_{KZ}} = 1 \qquad l_{KZ} = \xi(\tau_{KZ})$$