Transport in the Unitary Fermi Gas

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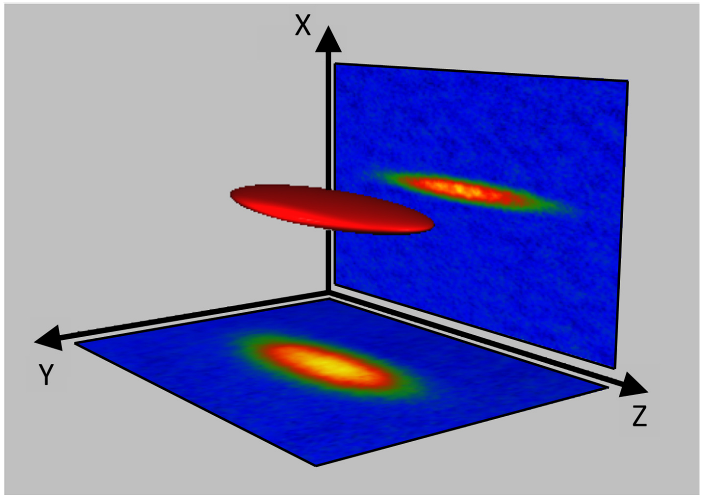
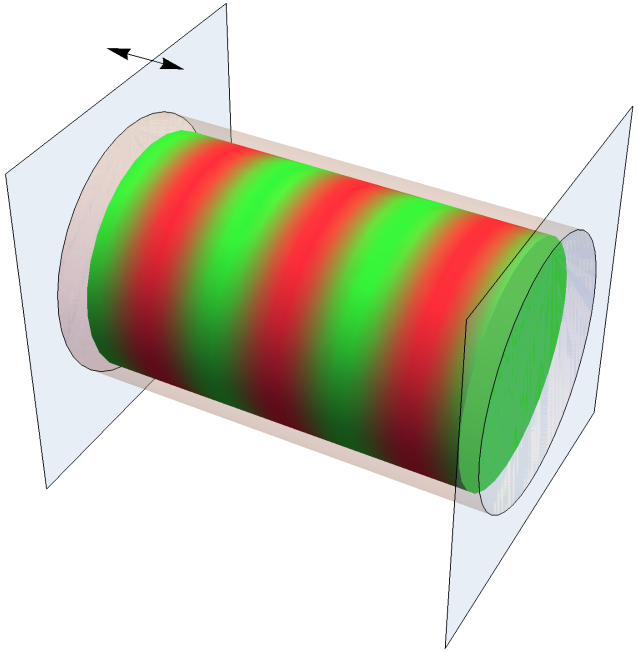




Why study transport in (nearly perfect) quantum fluids?

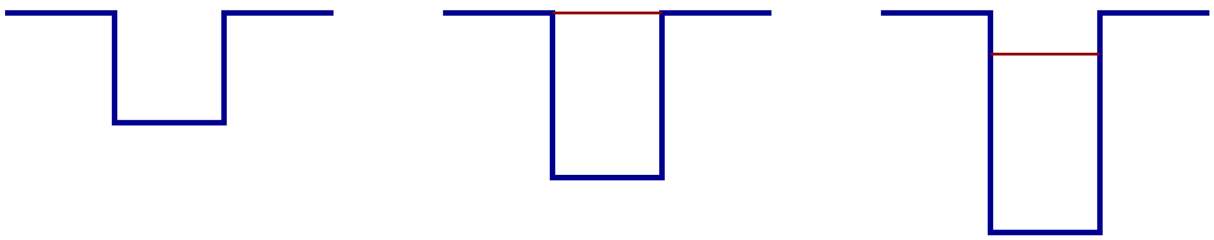
Transport without quasi-particles? Model system for QGP, neutron matter, strange metals, etc.

Fluid dynamics on the edge: Small systems, role of non-hydrodynamic modes, importance of fluctuations.

Impressive progress in experimental control: Box potentials, linear response, local density and temperature measurements.

Non-relativistic fermions in unitarity limit

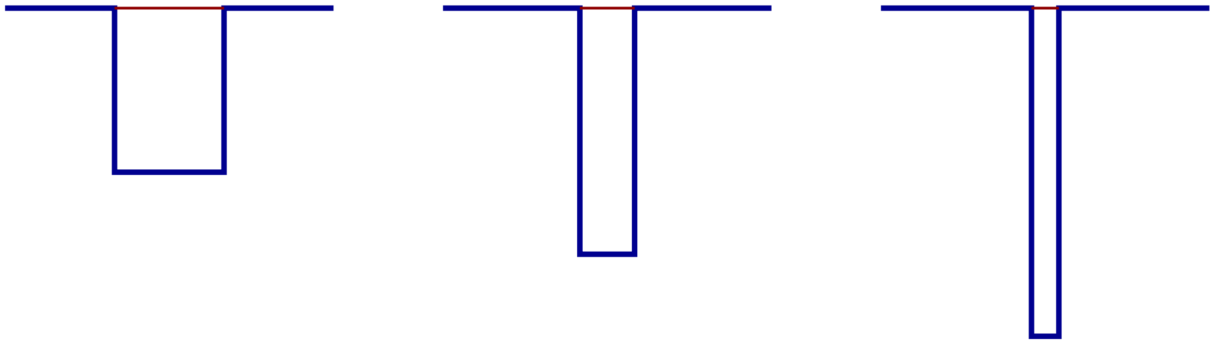
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Non-relativistic fermions in unitarity limit

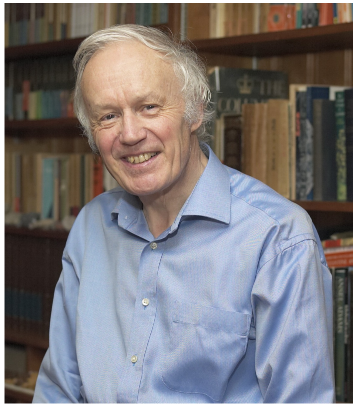
Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r} \exp(-r/a)$$

Effective theories for fluids (Unitary Fermi Gas, $T > T_F$)



$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad \omega < T \frac{s}{\eta}$$

Fluid dynamics

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \vec{\jmath}^{\,\rho} = 0 \qquad \frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla} \vec{\jmath}^{\,\epsilon} = 0$$

$$\frac{\partial \pi_i}{\partial t} + \nabla_j \Pi_{ij} = 0 \qquad \vec{\jmath}^{\,\rho} \equiv \rho \vec{v} = \vec{\pi}$$

Constitutive relations: Gradient expansion for currents. Building blocks:

$$\sigma_{ij} = \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3} \delta_{ij} \nabla \cdot v \right) \qquad \langle \sigma \rangle = \sigma_{ii}$$

Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j - \eta \sigma - \zeta \langle \sigma \rangle + O(\partial^2)$$

Energy current

$$\jmath_i^{\epsilon} = (\mathcal{E} + P) v_i - \eta \sigma_{ij} v_j + \kappa \nabla_i T$$

Scale invariant fluid

Stress tensor: Ideal fluid dynamics

$$\Pi^0_{ij} = P g_{ij} + \rho v_i v_j , \qquad P = \frac{2}{3}\mathcal{E}$$

First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij} - \zeta g_{ij}\langle\sigma\rangle \qquad \zeta = 0$$

Linear response: Couple to $g_{ij}(x,t)$

$$\sigma_{ij} = \left(\nabla_i v_j + \nabla_j v_i + \dot{g}_{ij} - \frac{2}{3} g_{ij}\langle\sigma\rangle\right) \qquad \langle\sigma\rangle = \nabla\cdot v + \frac{\dot{g}}{2g}$$

Son (2007)

Simple application: Kubo formula

Consider background metric $g_{ij}(t,x) = \delta_{ij} + h_{ij}(t,x)$. Linear response

$$\delta\Pi^{xy} = -\frac{1}{2} G_R^{xyxy} h_{xy}$$

Harmonic perturbation $h_{xy} = h_0 e^{-i\omega t}$

$$G_R^{xyxy} = P - i\eta\omega + \ldots$$

Kubo relation: $$\eta = -\lim_{\omega\to 0}\left[\frac{1}{\omega}\mathrm{Im}G_R^{xyxy}(\omega,0)\right]$$

Gradient expansion: $\omega \leq \frac{P}{\eta} \simeq \frac{s}{\eta}T$.

Kinetic theory

Microscopic picture: Quasi-particle distribution function $f_p(x,t)$

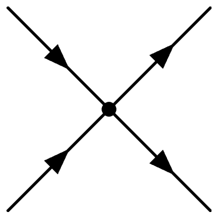
$$\rho(x,t) = \int d\Gamma_p \sqrt{g} m f_p(x,t) \qquad \pi_i(x,t) = \int d\Gamma_p \sqrt{g} p_i f_p(x,t)$$

$$\Pi_{ij}(x,t) = \int d\Gamma_p \sqrt{g} p_i v_j f_p(x,t)$$

Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \frac{p^i}{m} \frac{\partial}{\partial x^i} - \left(g^{il} \dot{g}_{lj} p^j + \Gamma^i_{jk} \frac{p^j p^k}{m} \right) \frac{\partial}{\partial p^i} \right) f_p(t,x,) = C[f]$$

$$C[f] =$$



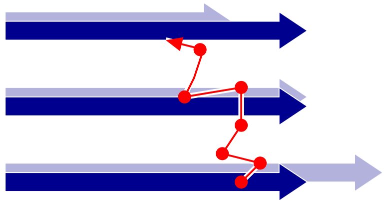
Solve order-by-order in Knudsen number $Kn = l_{mfp}/L$

Kinetic theory: Knudsen expansion

Chapman-Enskog expansion $f = f_0 + \delta f_1 + \delta f_2 + \ldots$

Gradient exp. $\delta f_n = O(\nabla^n)$

$\equiv$ Knudsen exp. $\delta f_n = O(Kn^n)$



First order result

Bruun, Smith (2005)

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij} \qquad \delta^{(1)}j_i^{\epsilon} = -\kappa\nabla_i T \qquad \eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \qquad \kappa = \frac{2}{3}c_P\eta$$

Second order result

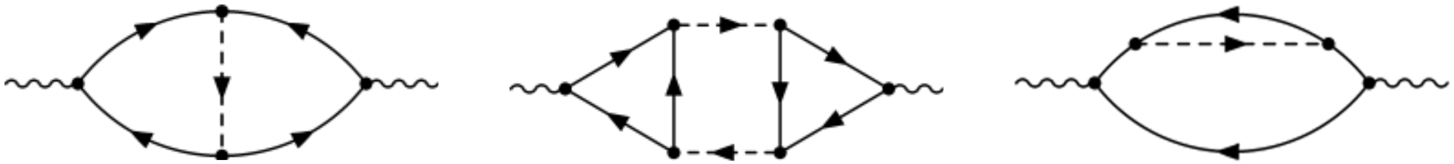
Chao, Schaefer (2012), Schaefer (2014)

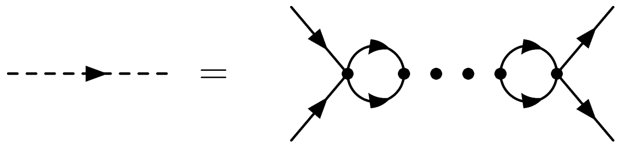
$$\delta^{(2)}\Pi^{ij} = \frac{\eta^2}{P}\left[\langle D\sigma^{ij}\rangle + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] + \frac{\eta^2}{P}\left[\frac{15}{14}\sigma^{\langle i}{}_k\sigma^{j\rangle k} - \sigma^{\langle i}{}_k\Omega^{j\rangle k}\right] + O(\kappa\eta\nabla^i\nabla^j T)$$

relaxation time $\tau_\pi = \eta/P$

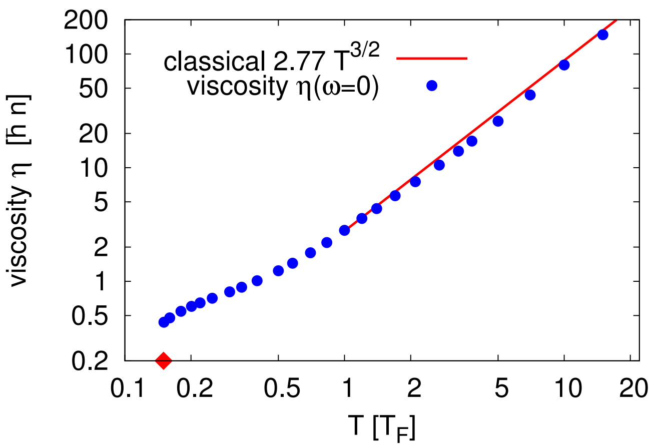
Quantum Field Theory

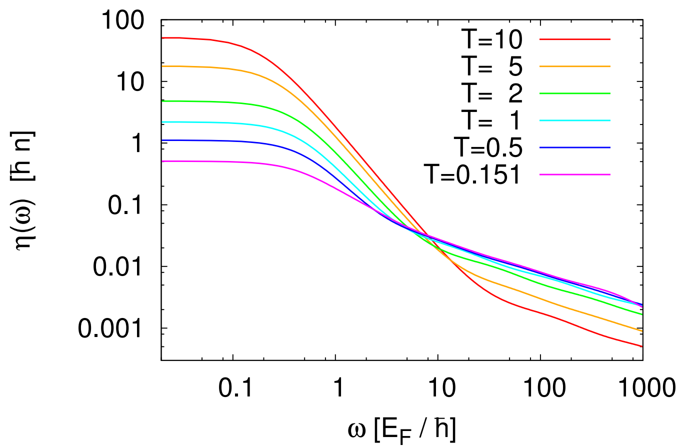
The diagrammatic content of the Boltzmann equation is known: Kubo formula with "Maki-Thompson" + "Azlamov-Larkin" +"Self-energy"





Limits subtle ($\omega \to 0$ and $n\lambda^3 \to 0$ don't commute). Can be used to extrapolate Boltzmann result to $T \sim T_F$





Enss, Zwerger (2011), see also Levin (2014)

Short time behavior: OPE

Operator product expansion (OPE)

$$\eta(\omega) = \sum_n \frac{\langle \mathcal{O}_n \rangle}{\omega^{(\Delta_n - d)/2}} \qquad \mathcal{O}_n(\lambda^2 t, \lambda x) = \lambda^{-\Delta_n} \mathcal{O}_n(t, x)$$

Leading operator: Contact density (Tan)

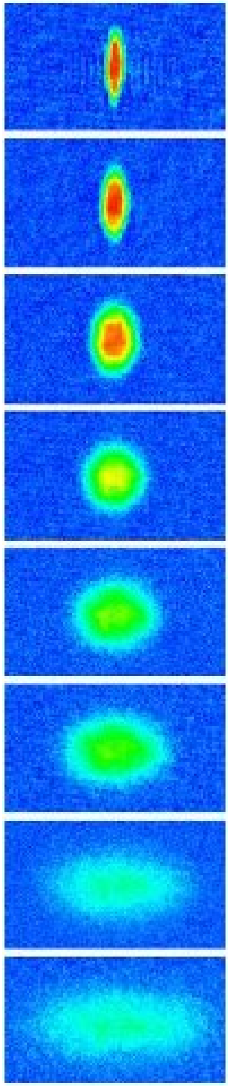
$$\mathcal{O}_{\mathcal{C}} = C_0^2 \psi\psi\psi^\dagger\psi^\dagger = \Phi\Phi^\dagger \qquad \Delta_{\mathcal{C}} = 4$$

$\eta(\omega) \sim \langle \mathcal{O}_{\mathcal{C}} \rangle / \sqrt{\omega}$. Asymptotic behavior + analyticity gives sum rule

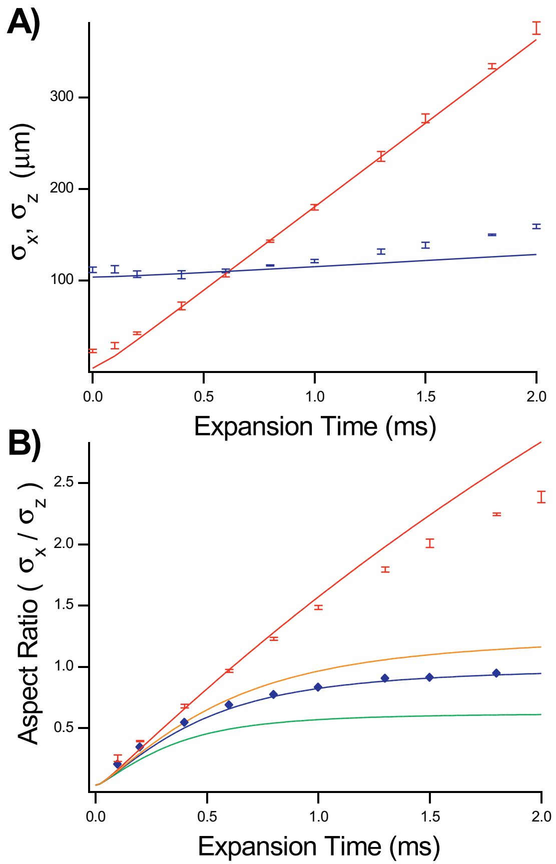
$$\frac{1}{\pi} \int dw \left[\eta(\omega) - \frac{\langle \mathcal{O}_{\mathcal{C}} \rangle}{15\pi\sqrt{m\omega}} \right] = \frac{\mathcal{E}}{3}$$

Randeria, Taylor (2010), Enss, Zwerger (2011), Hoffman (2013)

Experiments: Elliptic flow



O'Hara et al. (2002)



Hydrodynamic expansion

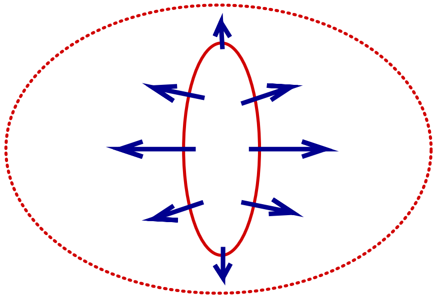
converts

coordinate space

anisotropy

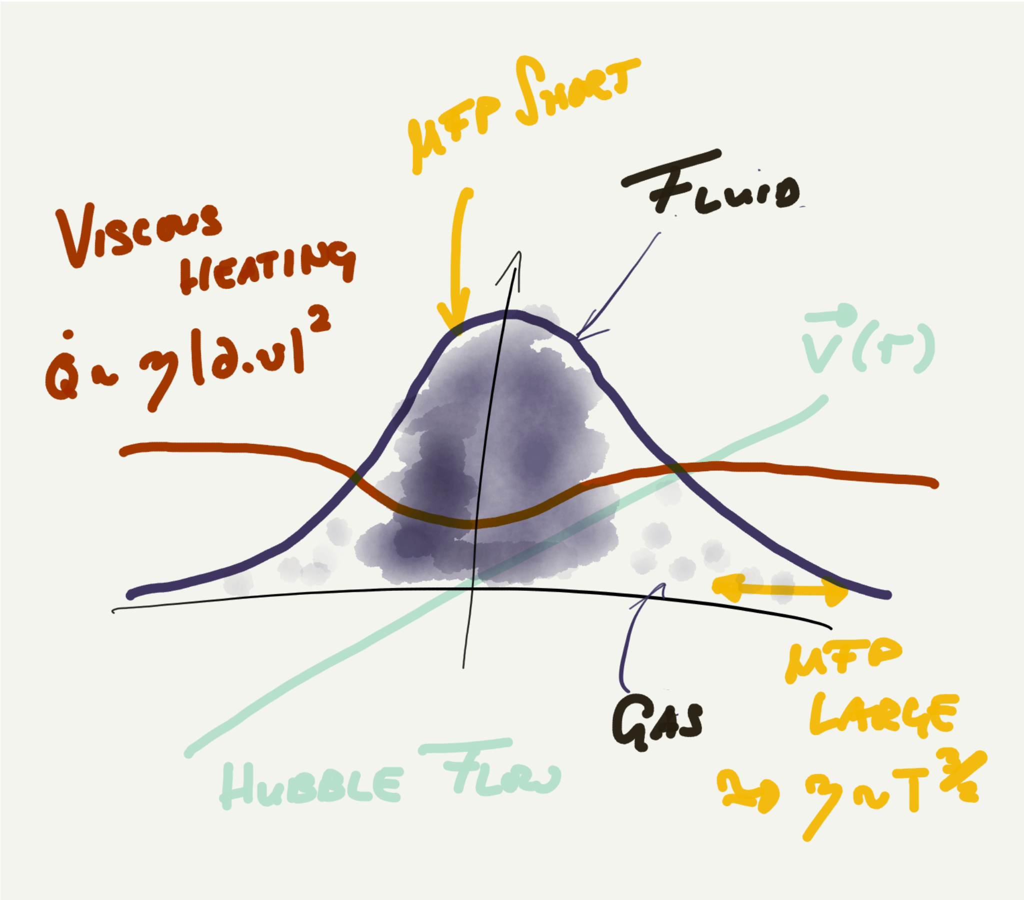
to momentum space

anisotropy



Determination of $\eta(n, T)$

Measurement of $A_R(t, E_0)$ determines $\eta(n, T)$. But:



Fluid dynamics breaks down in the dilute corona. Causes large artifacts.

Not a fundamental problem. Corona described by Boltzmann equation near ballistic limit.

Possible Solutions

Combine hydrodynamics & Boltzmann equation. Not straightforward.

Hydrodynamics + non-hydro degrees of freedom ($\mathcal{E}_a$; $a = x, y, z$)

$$\frac{\partial \mathcal{E}_a}{\partial t} + \vec{\nabla} \cdot \vec{j}^{\,\epsilon}_a = -\frac{\Delta P_a}{2\tau} \qquad \Delta P_a = P_a - P$$

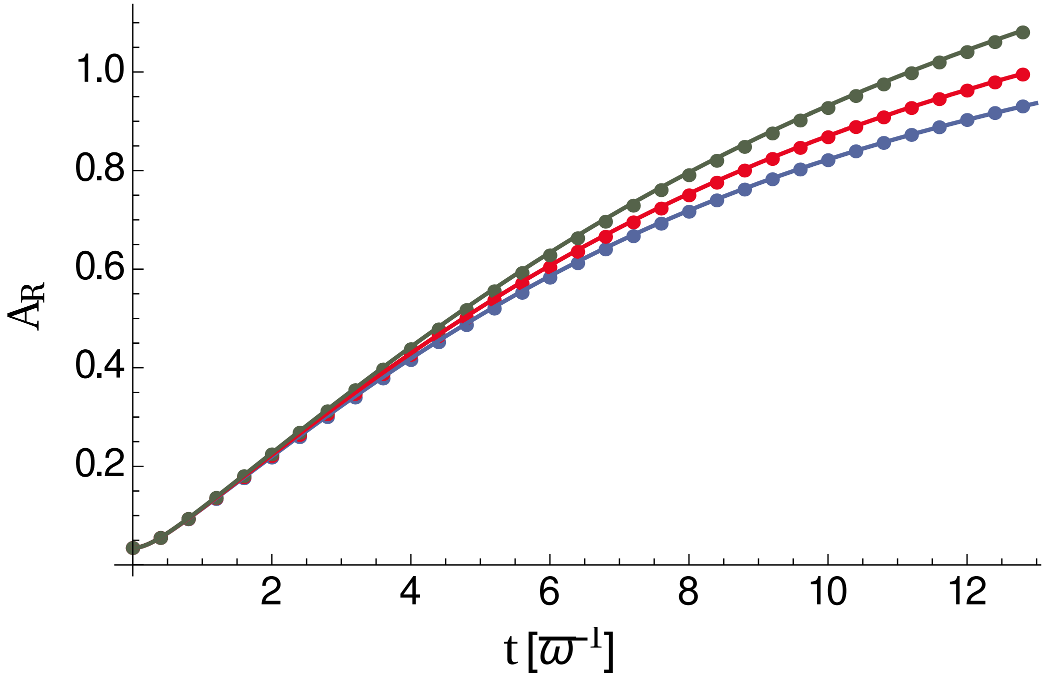
$$\frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla} \cdot \vec{j}^{\,\epsilon} = 0 \qquad \mathcal{E} = \sum_a \mathcal{E}_a$$

τ small: Fast relaxation to Navier-Stokes with $\tau = \eta/P$

τ large: Additional conservation laws. Ballistic expansion.

Anisotropic Hydrodynamics: Comparison with Boltzmann

Aspect ratio $A_R(t) = (\langle r_\perp^2 \rangle / \langle r_z^2 \rangle)^{1/2}$ $\qquad (T/T_F = 0.79, 1.11, 1.54)$



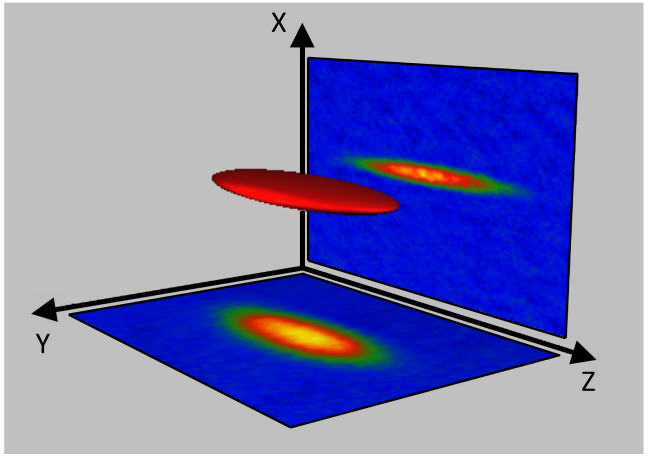
Dots: Two-body Boltzmann equation with full collision kernel

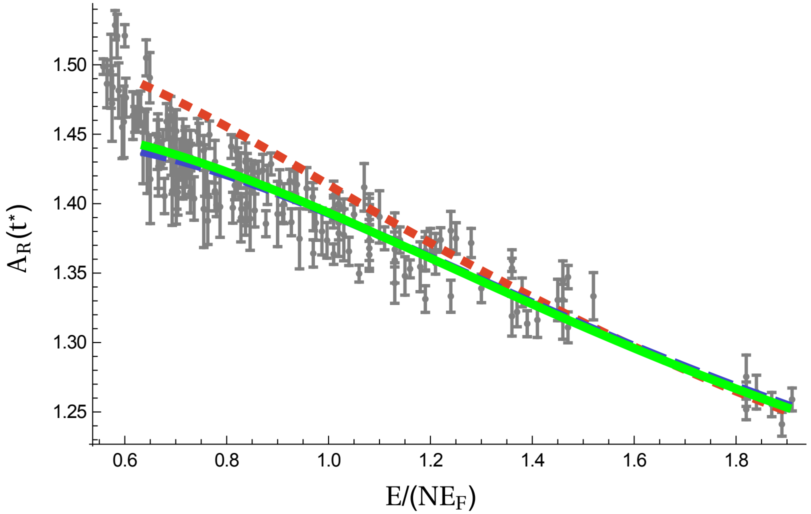
Lines: Anisotropic hydro with η fixed by Chapman-Enskog

High temperature (dilute) limit: Perfect agreement!

Boltzmann, Pantel et al (2015); AVH1 hydro code, M. Bluhm & T.S. (2015)

Anisotropic fluid dynamics analysis



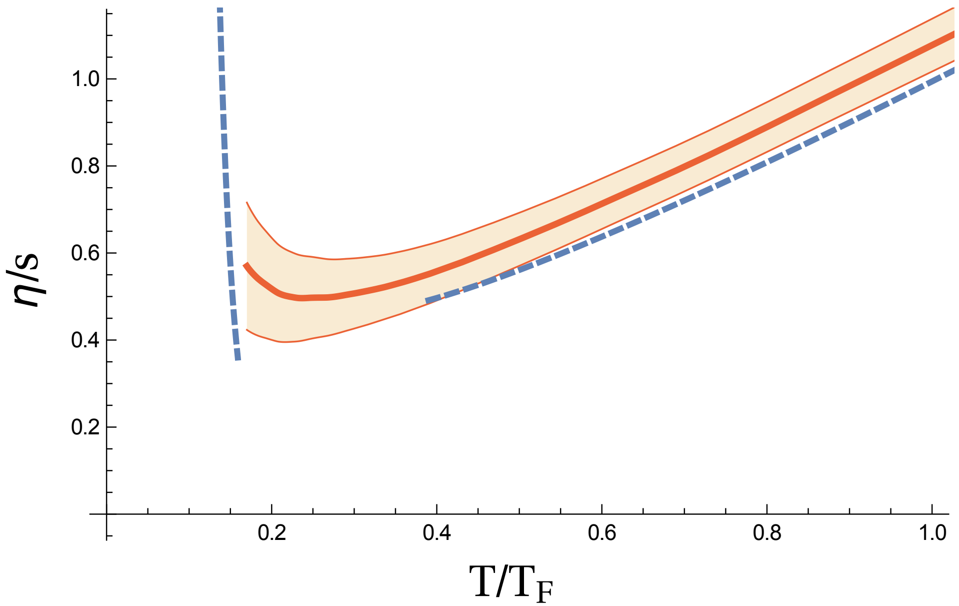


$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

Grey, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0 (mT)^{3/2} \left\{ 1 + \eta_2 n\lambda^3 + \eta_3 (n\lambda^3)^2 + \ldots \right\}$$

Reconstruct η/s (normal fluid)



Left: η/s (Red band) $T_c \sim 0.17 T_F$. Kinetic theory at low and high T (blue dashed)

$$\eta(T \gg T_c) = (0.265 \pm 0.02)(mT)^{3/2} \qquad \eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

$$\eta/s|_{T_c} = 0.56 \pm 0.20$$

Superfluid hydrodynamics

Spontaneous symmetry breaking: $\langle\Psi\rangle = v_0 e^{i\theta}$.

Goldstone boson is a new hydro mode: $\vec{v}_s = \frac{\hbar}{m}\vec{\nabla}\theta$

$$\partial_t \vec{v}_s + \frac{1}{2}\vec{\nabla}(v_s^2) = -\vec{\nabla}\mu$$

Momentum density: $\vec{\pi} = \rho_n \vec{v}_n + \rho_s \vec{v}_s$

$$\rho = \rho_n + \rho_s \qquad \rho_n = \frac{1}{2}\frac{\partial P}{\partial w^2} \qquad \vec{w} = \vec{v}_n - \vec{v}_s$$

Stress tensor and energy current

$$\begin{aligned} \Pi_{ij} &= P\delta_{ij} + \rho_n v_{n,i} v_{n,j} + \rho_s v_{s,i} v_{s,j} \\ \vec{j}^{\epsilon} &= sT\vec{v}_n + \left(\mu + \frac{1}{2}v_s^2\right)\vec{\pi} + \rho_n \vec{v}_n \vec{v}_n \cdot \vec{w} \end{aligned}$$

Superfluid hydrodynamics

Dissipative stresses

$$\delta\Pi_{ij} = -\eta\left(\nabla_i v_{n,j} + \nabla_j v_{n,i} - \frac{2}{3}\delta_{ij}\vec{\nabla}\cdot\vec{v}_n\right)$$

$$-\,\delta_{ij}\Big(\zeta_1\vec{\nabla}\left(\rho_s\left(\vec{v}_s-\vec{v}_n\right)\right) + \zeta_2\left(\vec{\nabla}\cdot\vec{v}_n\right)\Big)$$

Equation of motions for v_s: $\dot{v}_s + \frac{1}{2}\nabla(v_s{}^2) = -\nabla(\mu + H)$ with

$$H = -\zeta_3\vec{\nabla}\left(\rho_s\left(\vec{v}_s-\vec{v}_n\right)\right) - \zeta_4\vec{\nabla}\cdot\vec{v}_n$$

Conformal symmetry: $\zeta_1 = \zeta_2 = \zeta_4 = 0$

Son (2007)

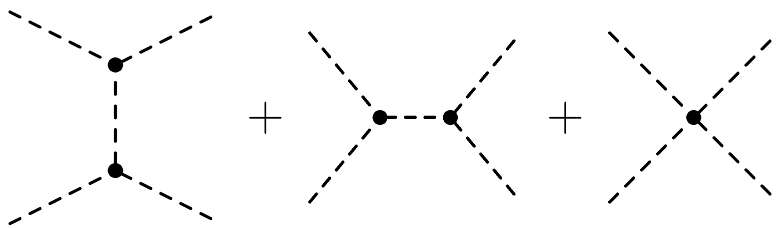
Low T: Phonons

Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$. Effective Lagrangian

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

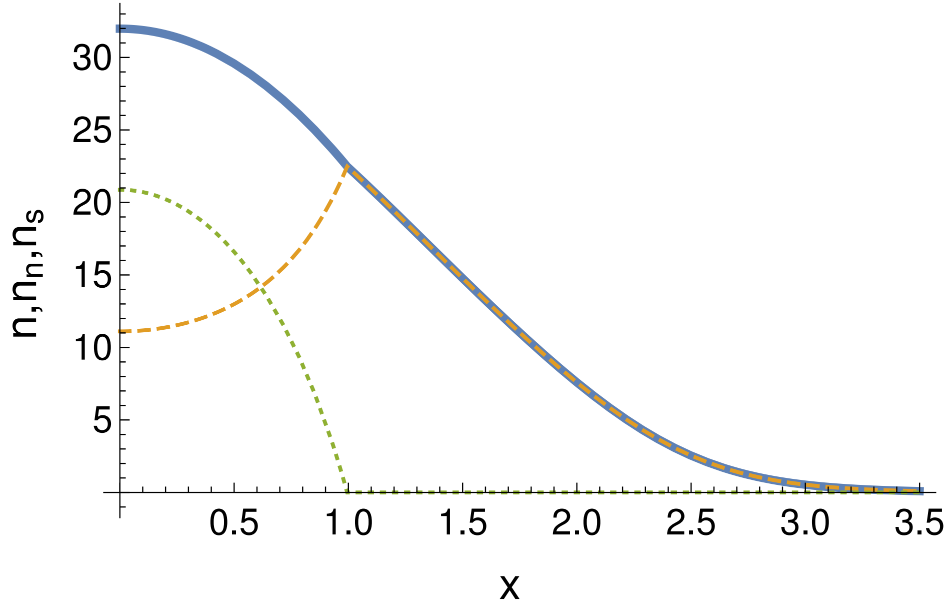
$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



Thermal conductivity is subtle, because quasi-particles with $E_p \sim c_s p$ do not contribute. The dominant process is phonon splitting, made possible by non-linear terms in the dispersion relation.

$$\kappa = \frac{128}{3\pi}\frac{\gamma^2}{g_3^2}\frac{T^2}{c_s^2}D_H = \frac{256\sqrt{2}}{25\pi^3\xi^2 m}(mT)^{3/2}\left(\frac{T}{T_F}\right)^2 D_H$$

Two-fluid hydro for an expanding cloud

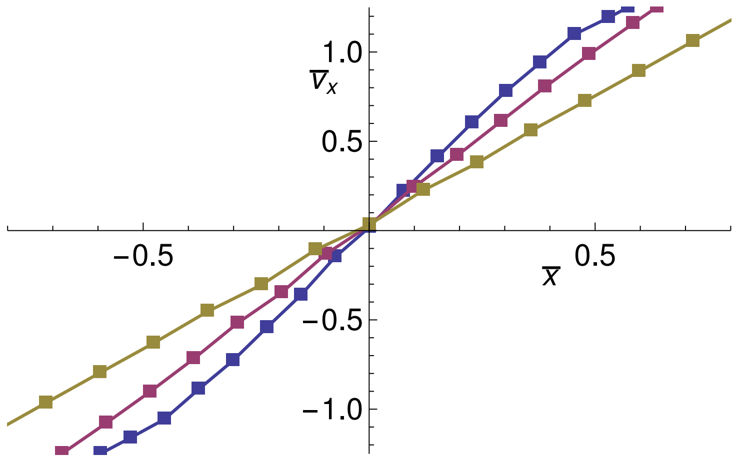


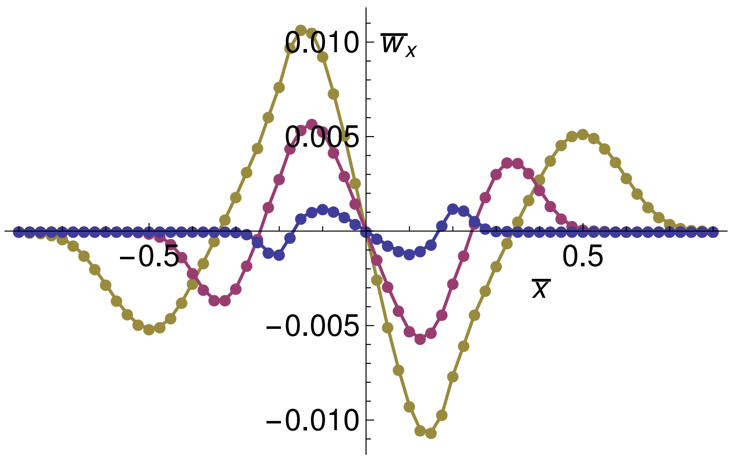
$\rho = \rho_s + \rho_n$ (solid), ρ_n (dashed), ρ_s (dotted)

Gibbs-Duhem relation

$$dP = n d\mu_s + s dT + \frac{\rho_n}{2} dw^2$$

Two-fluid hydro for an expanding cloud



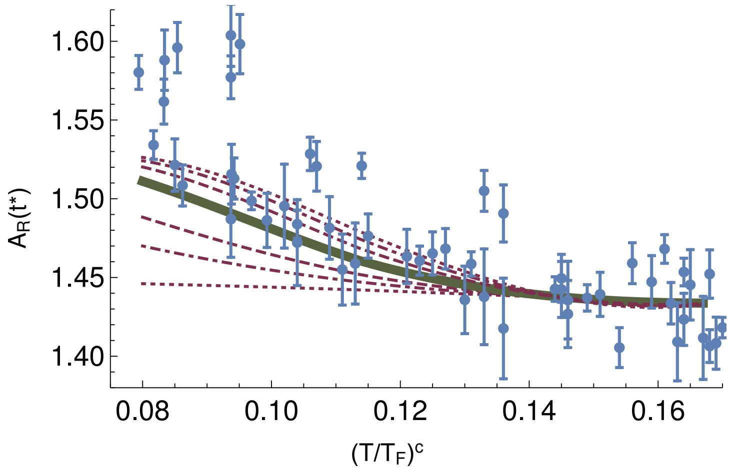


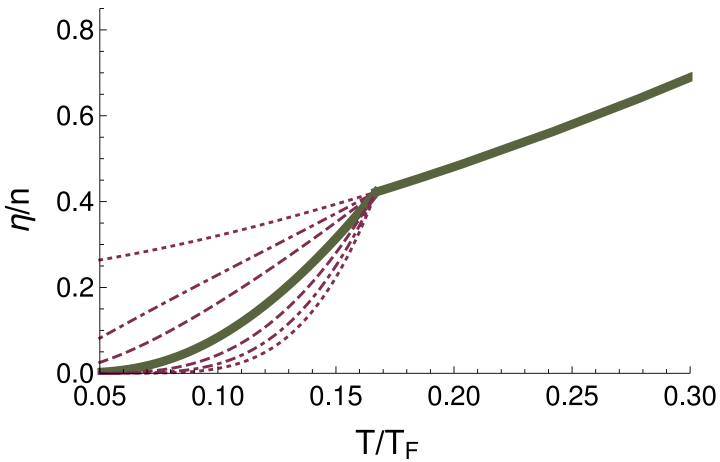
Average fluid velocity $v_x(x,t)$. Superfluid $w_x(x,t) = v_x^n(x,t) - v_x^s(x,t)$

Superfluid $\vec{w} = \vec{v}^n - \vec{v}^s$ can be computed perturbatively.

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\vec{w} = -\frac{s}{\rho_n}\vec{\nabla}T + O(w^2)\,.$$

Two-fluid hydro analysis of expanding cloud





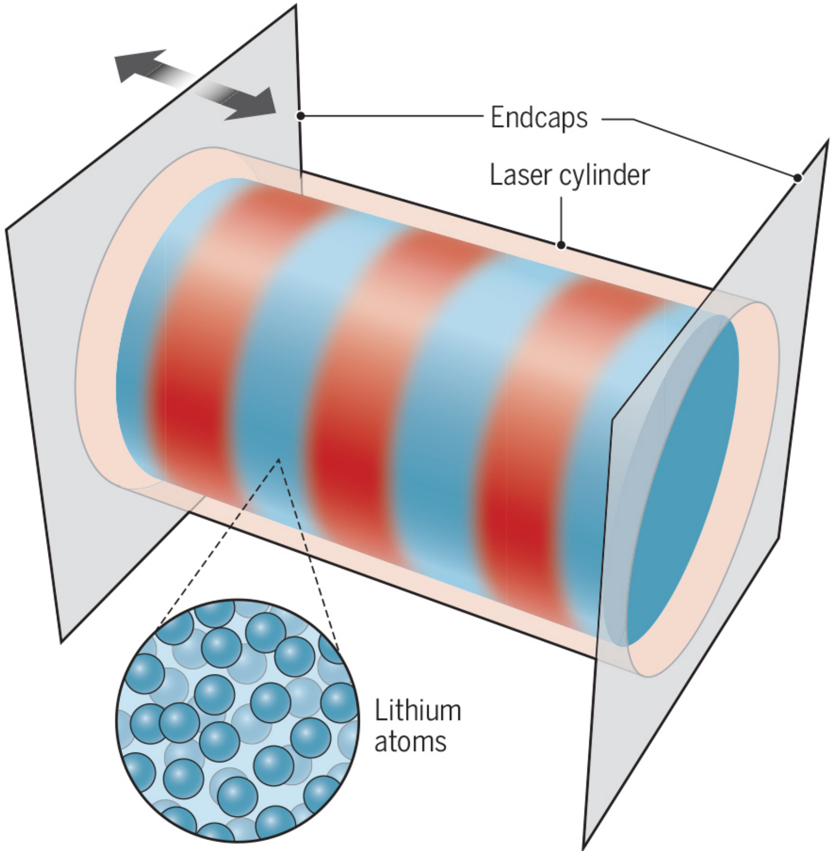
A_R in low T regime.

Small η corresponds to large A_R.

Fits for $\eta(T < T_c)$:

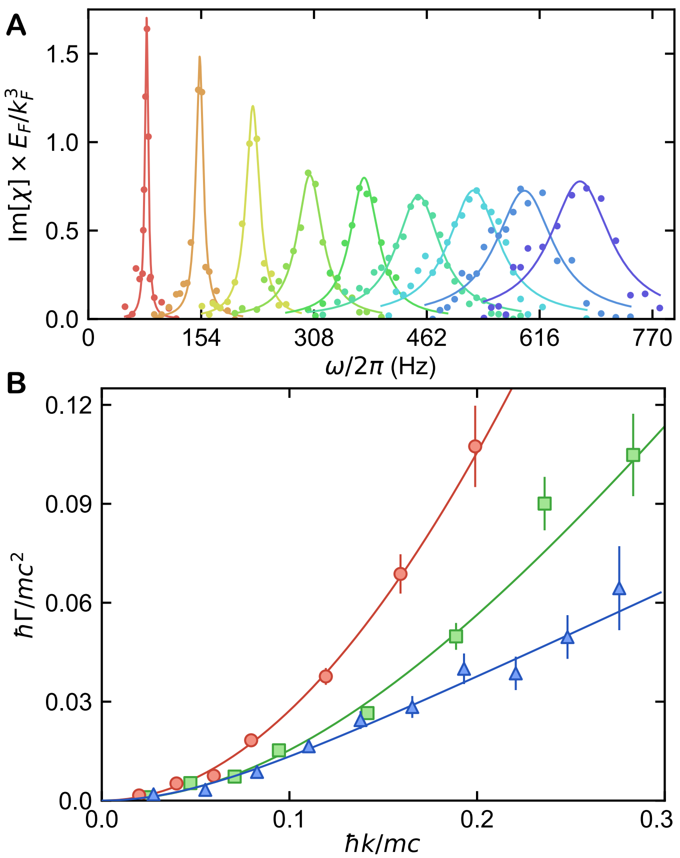
$$\eta \simeq \eta_0 \exp\left[-2\frac{T_c - T}{T}\right]$$

Sound attenuation (MIT)

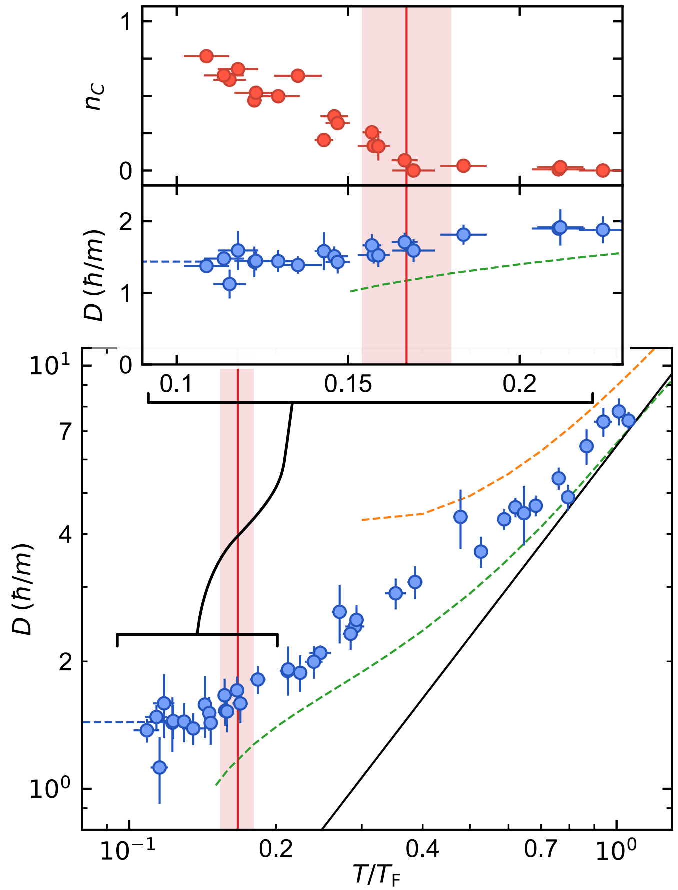


Sound attenuation (MIT)

Spectral response $\rho_k(\omega)$.



Sound diffusivity $D_s(T)$



Damping rate $\Gamma(k)$

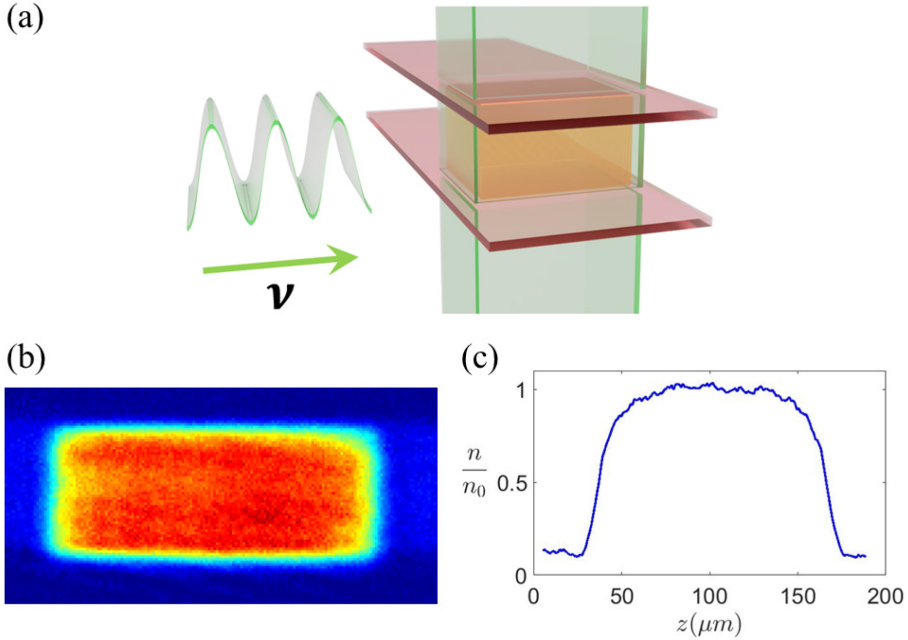
$$D_s = \frac{4\eta}{3\rho} + \frac{4\kappa T}{15P}.$$

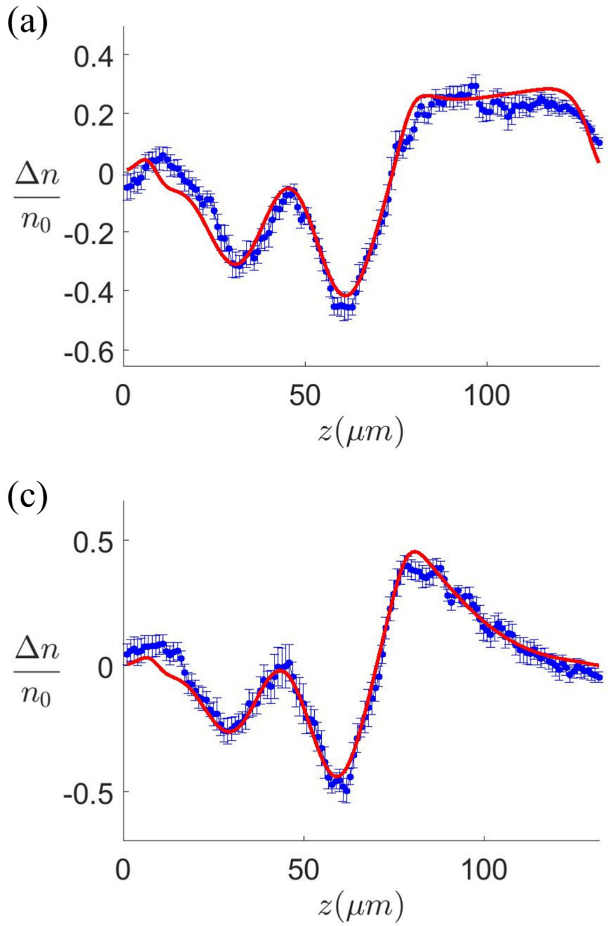
$(T/T_F = 0.36, 0.21, 0.13)$.

Patel et al., Science (2021)

Linear Response (NC State)

Baird et al., PRL 2019





$$(\kappa/\eta)(T \gg T_c) = 0.93(14)(15/4)(k_B/m)$$

Final thoughts

Unfold temperature, density dependence of η/s, D_s and κ. Puzzles remain for $T < T_c$, possibly related to breakdown of hydro in small systems.

Measure response at short distances and short times.

Measure approach to hydrodynamics. Quasi-particles or quasi-normal modes?