Ultracold Atoms as Quantum Simulators for Neutron Matter

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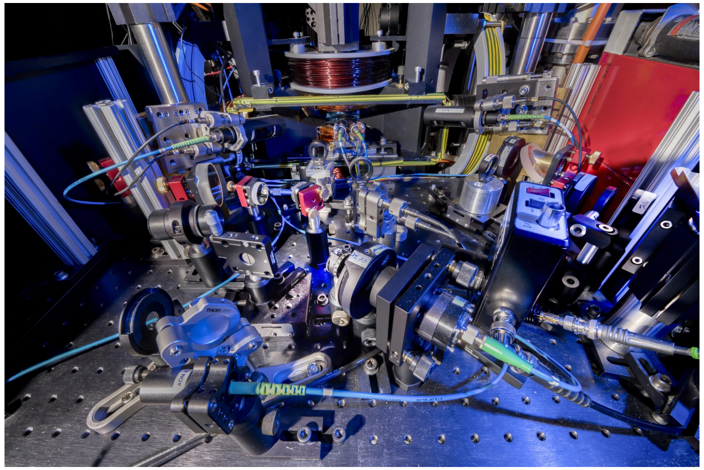
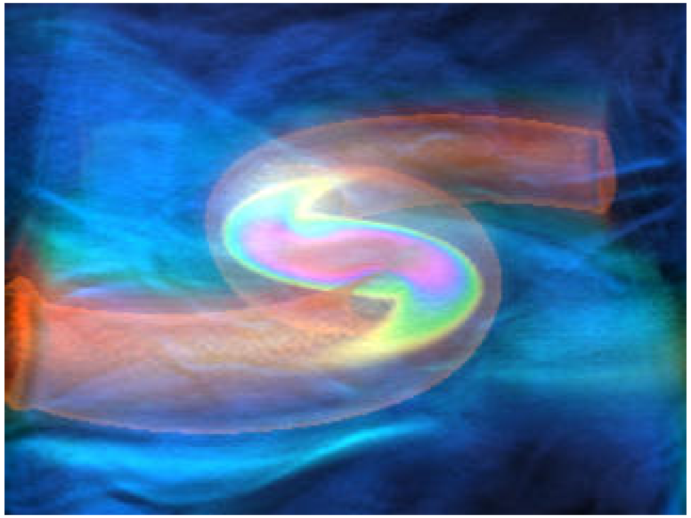




DOE Quantum Horizons, T.S., S. König (NCSU), M. Zwierlein (MIT).

Why quantum simulators?

"Hard" quantum many body problems: Sign problems in dense matter and real time response functions. Possible strategies

Find improved algorithms for classical computers.

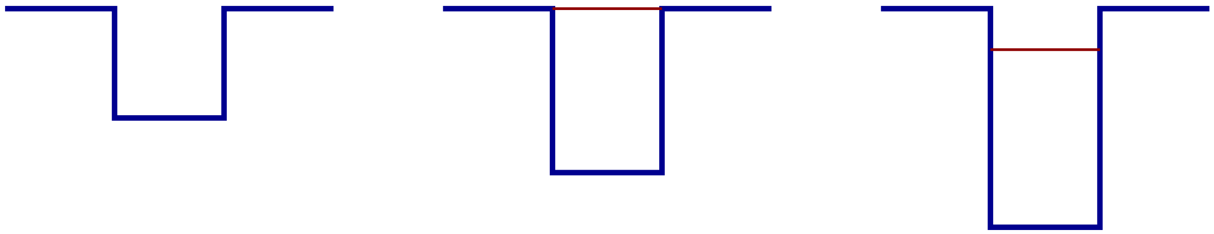
Develop algorithms for future general purpose quantum computers, and explore test problems on present and near-future hardware.

Implement Hamiltonian in special purpose analog quantum simulators.

The ultimate dream is H_{QCD}. Here, we will consider a simpler case, dilute neutron matter.

Non-relativistic fermions in unitarity limit

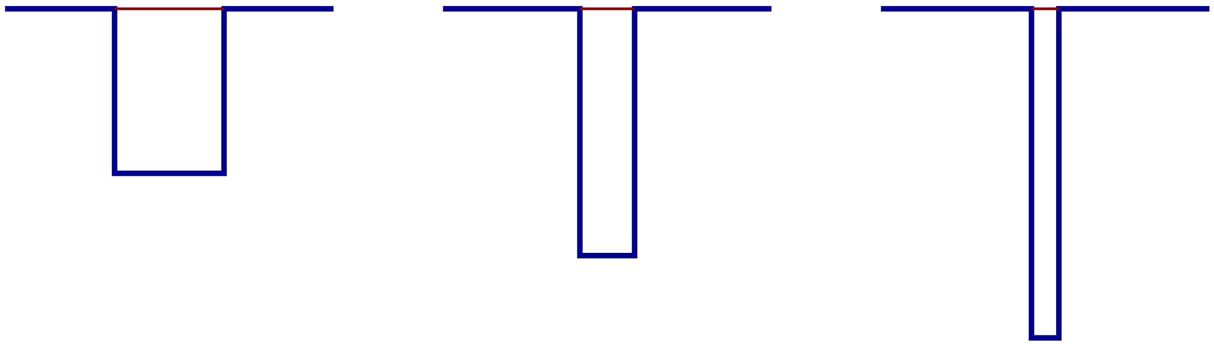
Two body interaction: Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Non-relativistic fermions in unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r}\exp(-r/a)$$

Fermi Gas at Unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$ (DR: $C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

$\phi \sim \psi_\uparrow \psi_\downarrow$ auxiliary "pair" or "dimer" field.

Fermi Gas at Unitarity: Computational Aspects

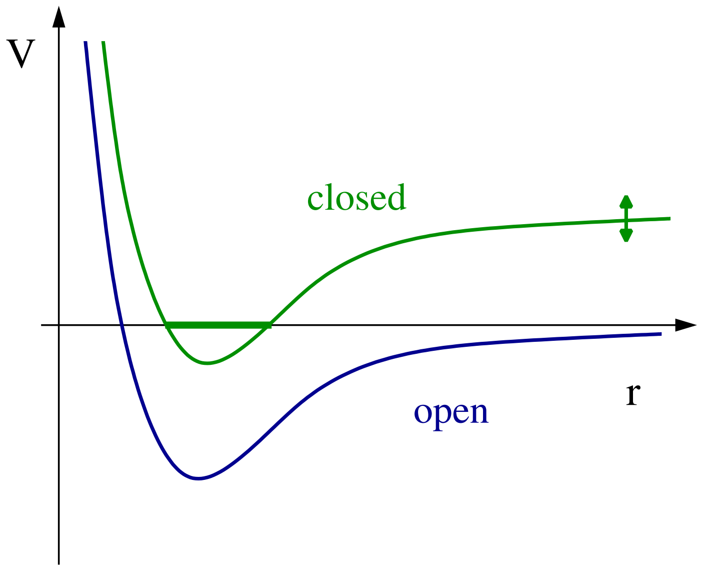
For $C_0 > 0$ this is an attractive Hubbard model. No sign problem in imaginary time path integral simulations. (But: Many body system require significant computational resources.)

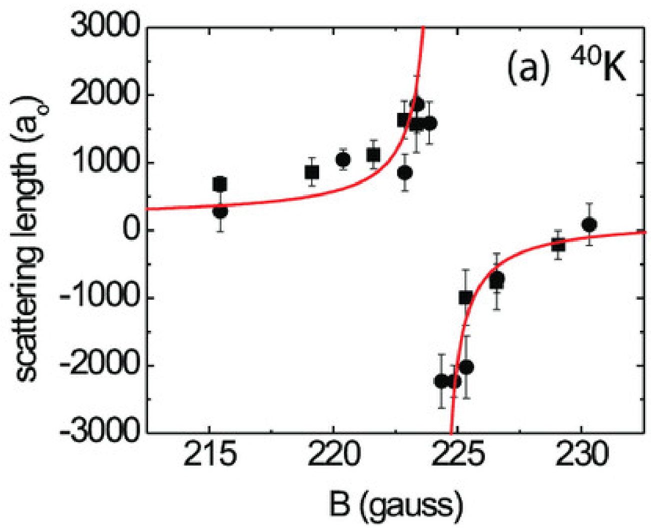
There is a sign problem in the repulsive model, or if there is a non-zero polarization $P = \psi^\dagger \sigma_3 \psi$.

Only have access to imaginary time correlation functions. Continuation to real time is hard.

Experimental realization: Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"

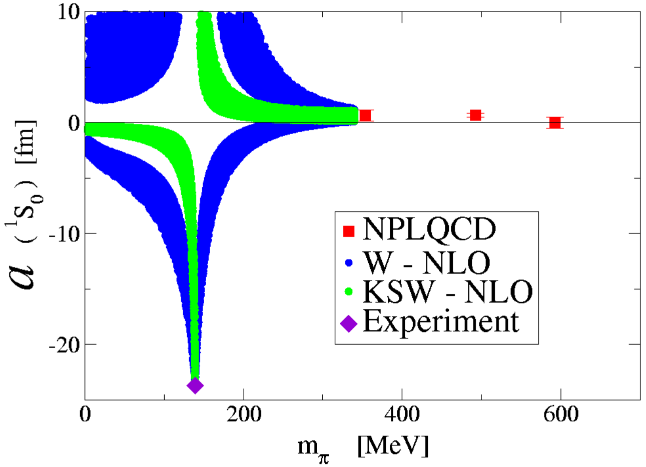




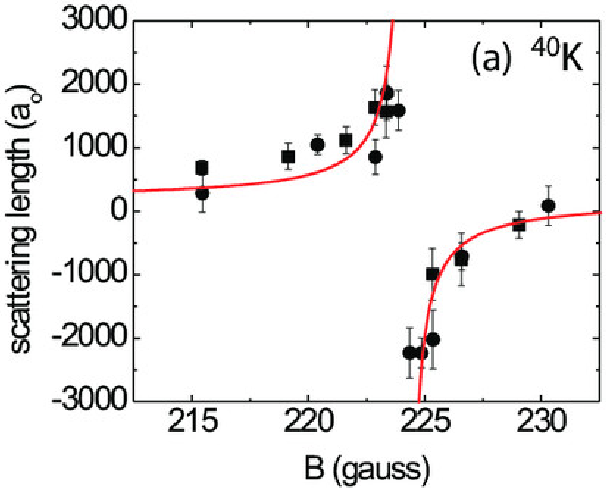
Feshbach resonance $$a(B) = a_0\Big(1 + \frac{\Delta}{B - B_0}\Big)$$

Universality: From neutrons to atoms

Neutron Matter



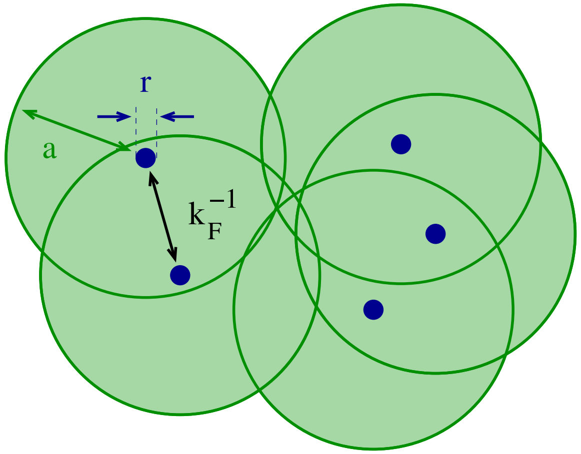
^{40}K Feshbach resonance



What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$

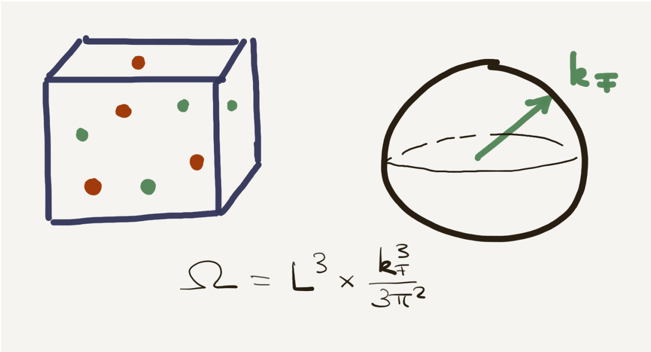


Outline

1. Equation of state: From trapped atoms to neutron stars.
2. Transport: Viscosity from elliptic flow.
3. Transport: Linear response.
4. Outlook: External fields, OTOCs, etc.

1. Equation of state

Free fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

$$\mathcal{E} = E/V \sim (N/V)^{5/3}$$

Unitarity limit ($a \to \infty$, $r \to 0$). No expansion parameters.

$$\frac{E}{N} = \xi\,\frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (George Bertsch, 1998): Determine ξ.

Is $\xi > 0$ (is the system stable)?

How to measure ξ with trapped atoms

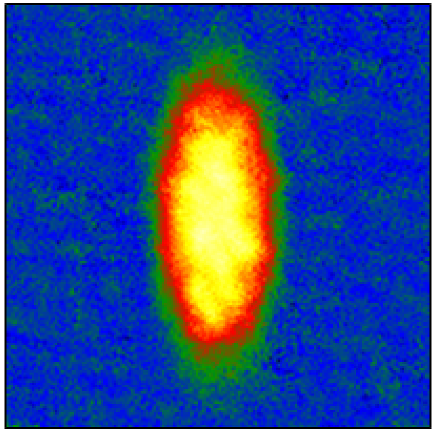
Trapped gas in hydrostatic equilibrium

$$\frac{1}{n}\vec{\nabla}P = -\vec{\nabla}V_{ext} \qquad P = \frac{2}{3}\mathcal{E}$$

Pressure determines size of the cloud ($V_{ext} = \frac{1}{2}m\omega^2 x^2$).

$$r(a=0) = \sqrt{\frac{2E_F}{m\omega^2}} \qquad r(a=\infty) = \xi^{1/4} r(0)$$

Cloud size can be measured with a CCD camera and a ruler

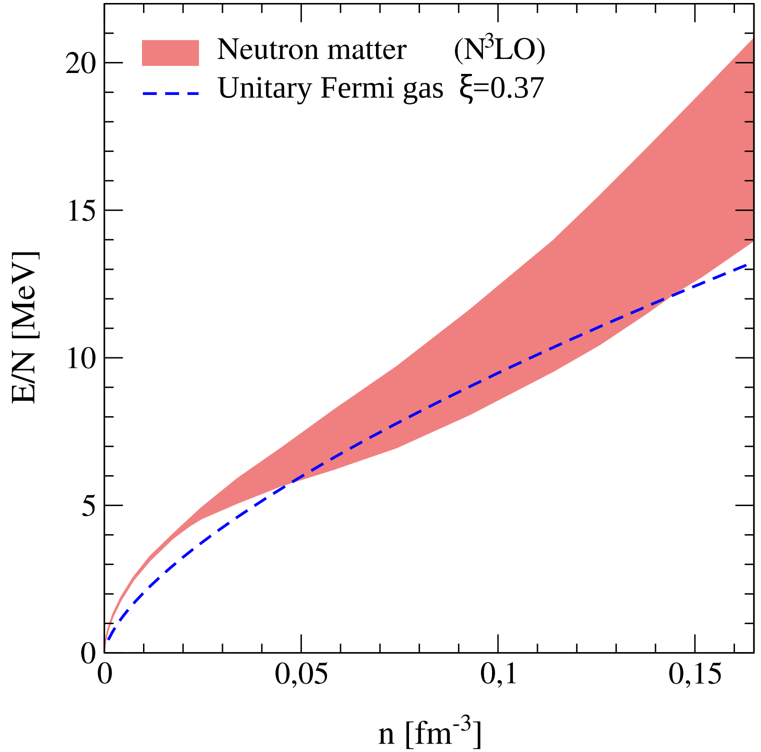


modern value

$$\xi = 0.37(5)$$

(MIT, Sommer et al.)

Neutron matter equation of state

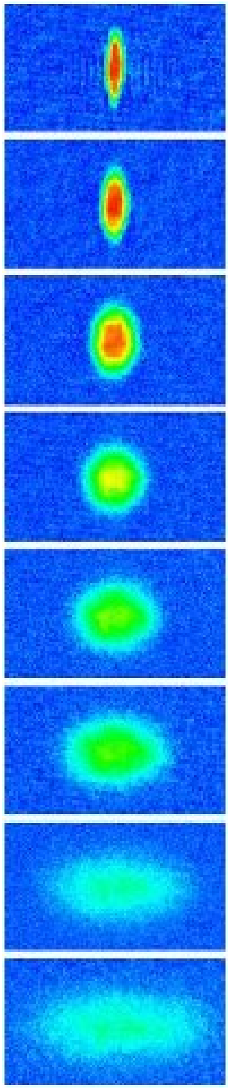


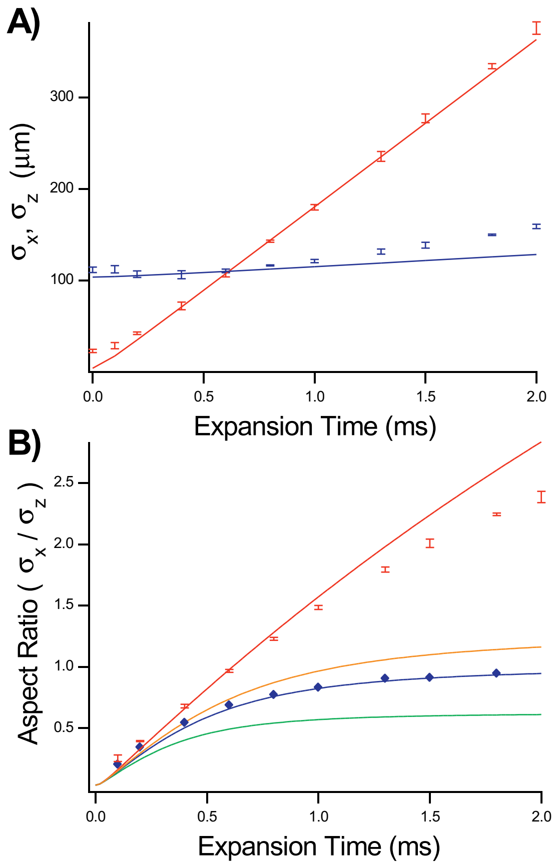
$n \lesssim 0.1\,\mathrm{fm}^{-3}$: Unitary gas with a^{-1}, r corrections.

$n \gtrsim 0.1\,\mathrm{fm}^{-3}$: Repulsive 2-body, 3-body forces.

$n \gtrsim 0.2\,\mathrm{fm}^{-3}$: New degrees of freedom.

2. Elliptic flow in the unitary Fermi gas





Hydrodynamic

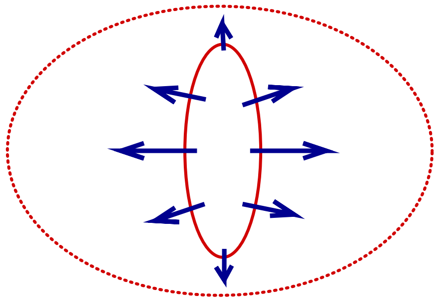
expansion converts

coordinate space

anisotropy

to momentum space

anisotropy



O'Hara et al. (2002)

Fluid dynamics

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \vec{j}^{\rho} = 0 \qquad \frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial \pi_i}{\partial t} + \nabla_j \Pi_{ij} = 0 \qquad \vec{j}^{\rho} \equiv \rho \vec{v} = \vec{\pi}$$

Scale invariance: Ideal fluid dynamics

$$\Pi^0_{ij} = P g_{ij} + \rho v_i v_j , \qquad P = \frac{2}{3}\mathcal{E}$$

First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta \sigma_{ij} - \zeta g_{ij} \langle \sigma \rangle \qquad \zeta = 0$$

$$\sigma_{ij} = (\nabla_i v_j + \nabla_j v_i - 2/3 \delta_{ij} \nabla \cdot v) \qquad \langle \sigma \rangle = \sigma_{ii}$$

Microscopic Theory

$P = P(\mathcal{E})$ fixed by conformal symmetry. $P(\mu, T)$ can be computed from euclidean data

$$P = \log Z(\mu, T) \qquad Z = \int D\psi D\psi^\dagger e^{-S_E}$$

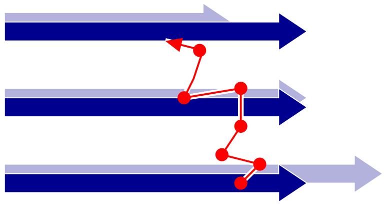
But: Transport coefficients determined by Kubo relations

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\int dt d^3x\, e^{-i(\omega t - kx)}\,\Theta(t)\langle[\Pi_{xy}(0), \Pi_{xy}(t,x)]\rangle$$

Requires real time data.

Shear viscosity: Theory

Kinetic theory: Momentum transport by diffusion of atoms

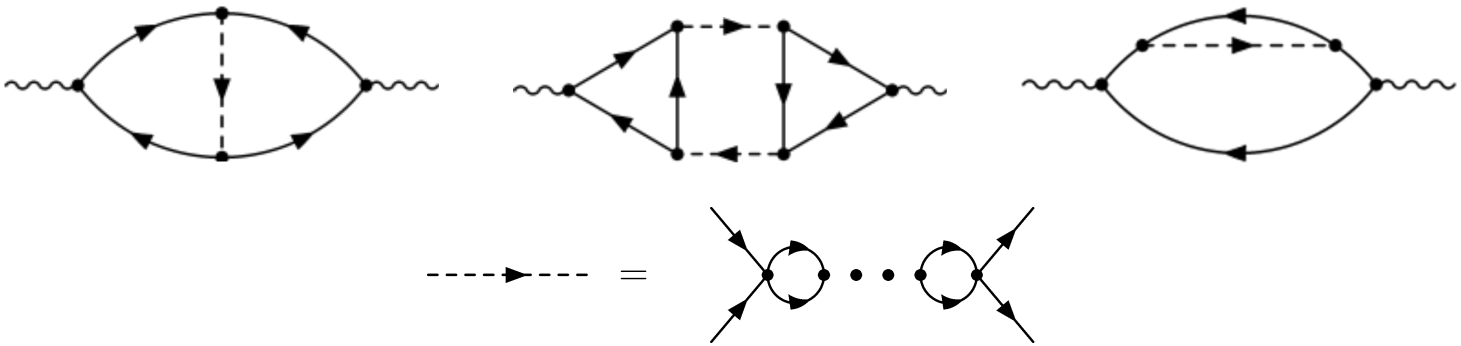


$$\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \quad (T \gtrsim T_F)$$

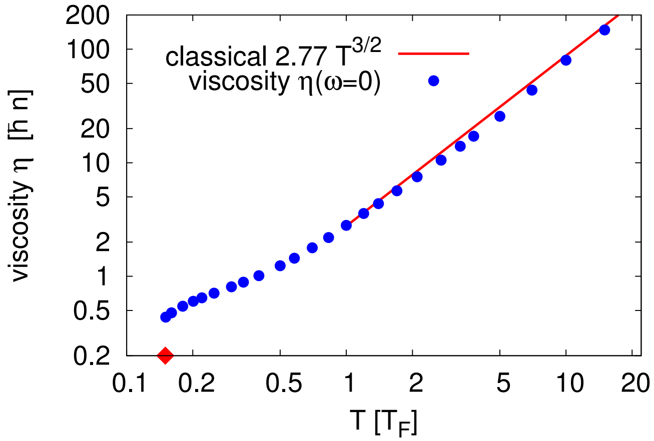
QFT: Diagrammatic content of Boltzmann equation is known.

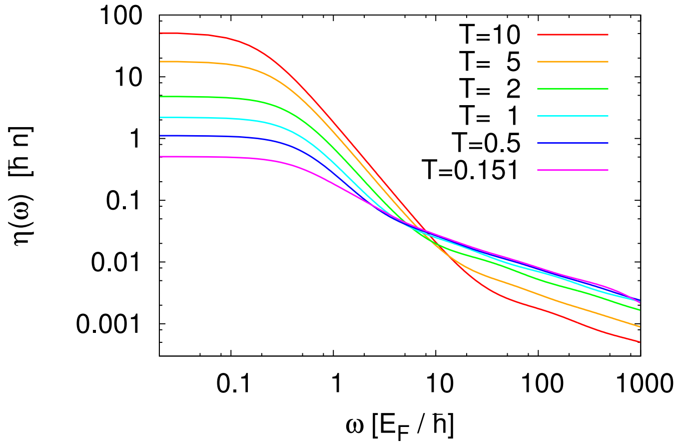
Kubo formula with

Maki-Thompson + Azlamov-Larkin + Self-energy



Can be used to extrapolate kinetic theory to $T \sim T_F$





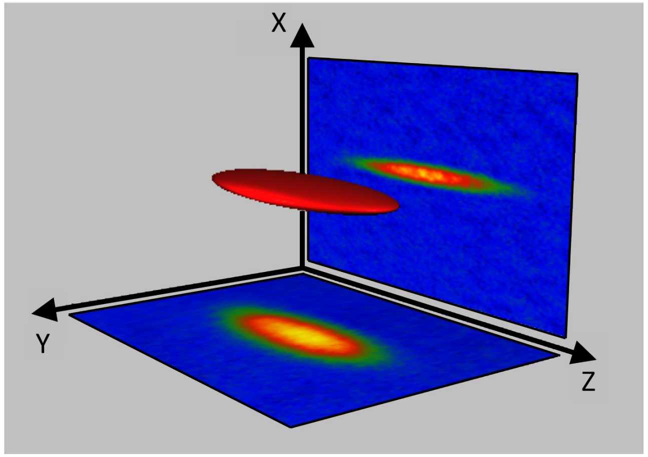
$$\eta(T \sim T_c) \sim \hbar n$$

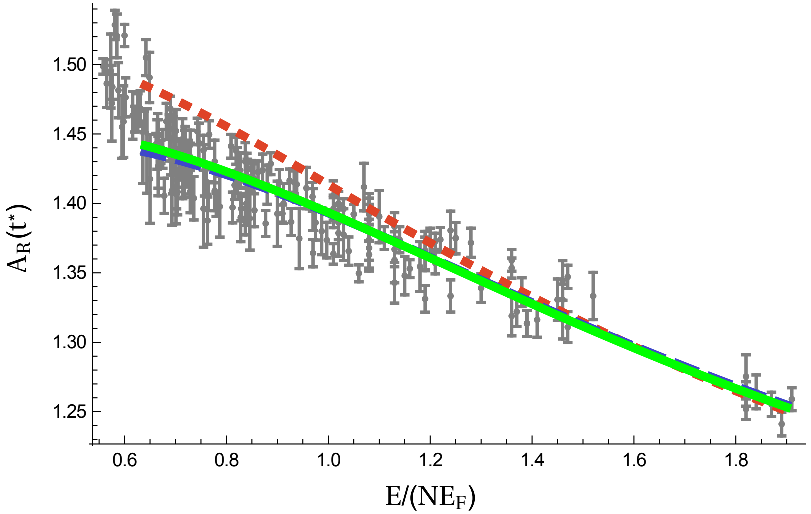
Drude peak, universal tail.

Drude peak characteristic of quasi-particle behavior.

Enss, Zwerger (2011), see also Levin (2014)

Fluid dynamics analysis



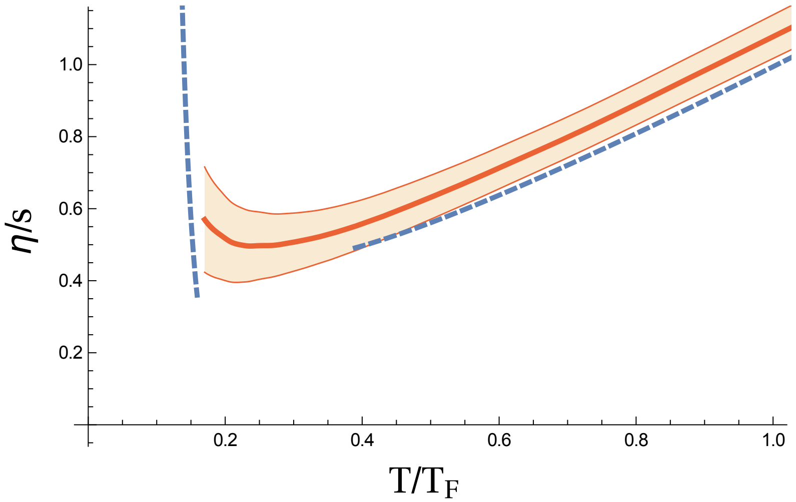


$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

Grey, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0 (mT)^{3/2} \left\{ 1 + \eta_2 n\lambda^3 + \eta_3 (n\lambda^3)^2 + \ldots \right\}$$

Reconstruct η/s (normal fluid)



$T_c \sim 0.17 T_F$. Kinetic theory at low and high T (blue dashed)

Consistency check: $T \gg T_c$

$$\eta|_{T\gg T_c} = (0.265 \pm 0.02)(mT)^{3/2}$$

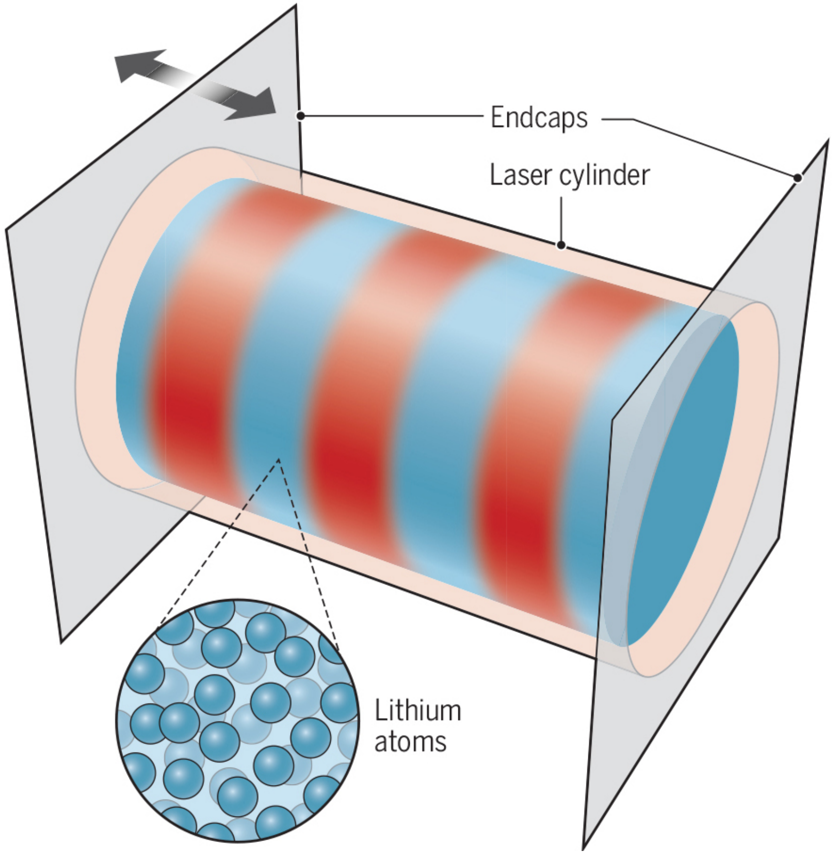
$$\eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

Phenomenology (normal phase): Two-term virial expansion works well,

$\eta \sim \eta_0 (mT)^{3/2} + \eta_1 \hbar n$

$$\eta/s|_{T_c} = 0.56 \pm 0.20$$

3. Linear Response: Sound attenuation



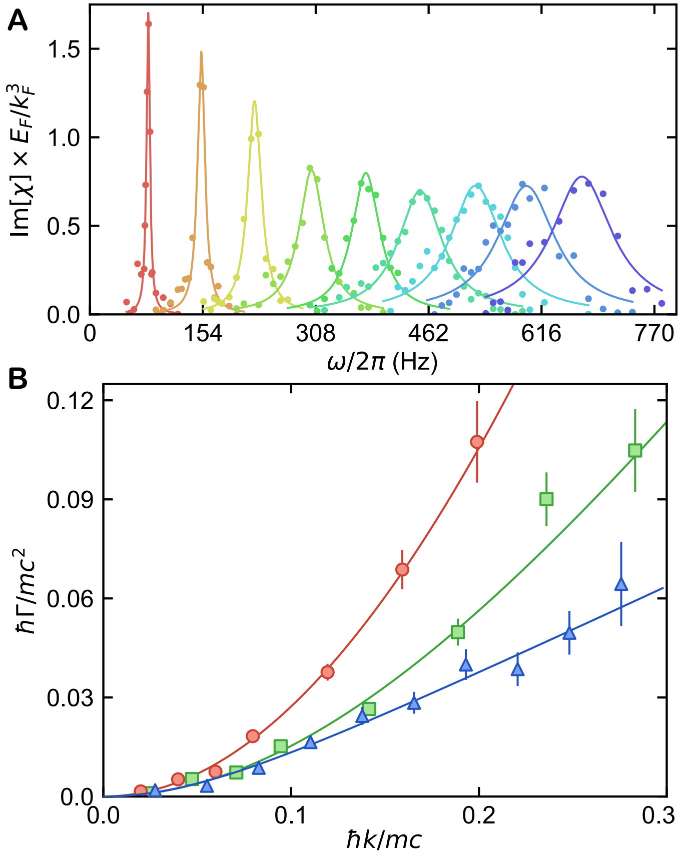
Cylindrical box, response to small harmonic drive.

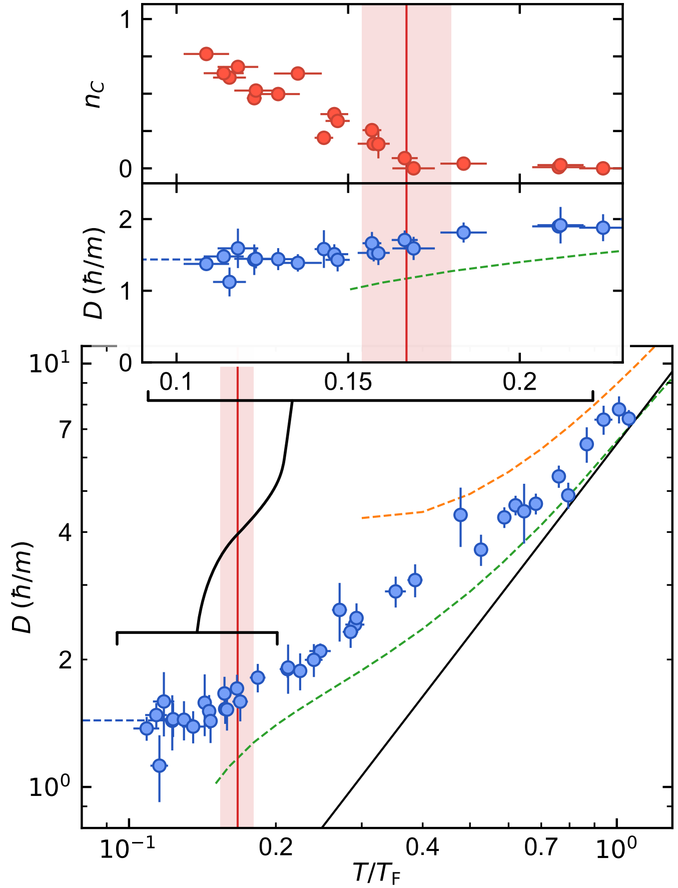
Patel et al., Science (2021)

Sound attenuation (MIT)

Spectral response $\rho_k(\omega)$.

Sound diffusivity $D_s(T)$





Damping rate $\Gamma(k)$

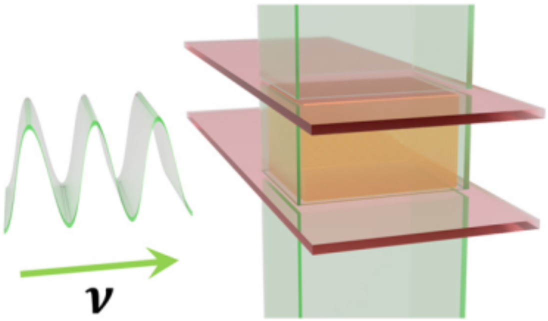
$$D_s = \frac{4\eta}{3\rho} + \frac{4\kappa T}{15P}.$$

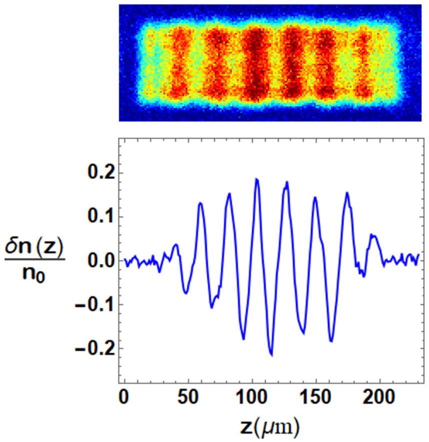
$(T/T_F = 0.36, 0.21, 0.13)$.

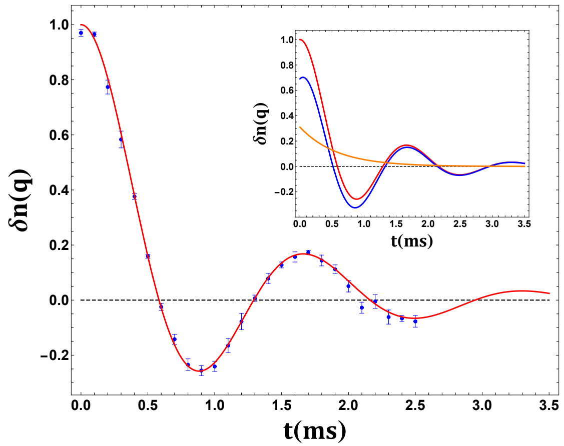
Patel et al., Science (2021)

Linear Response (NC State)

Baird et al., PRL 2019; Wang et al, PRL 2022.







$$\left.\frac{\kappa}{\eta}\right|_{T\gg T_c} = 0.93(14)\,\frac{15k_B}{4m}$$

4. Outlook: External fields, OTOCs, etc.

Can realize response to $A_0(x,t)$, as well as spatial/time variation of scattering length

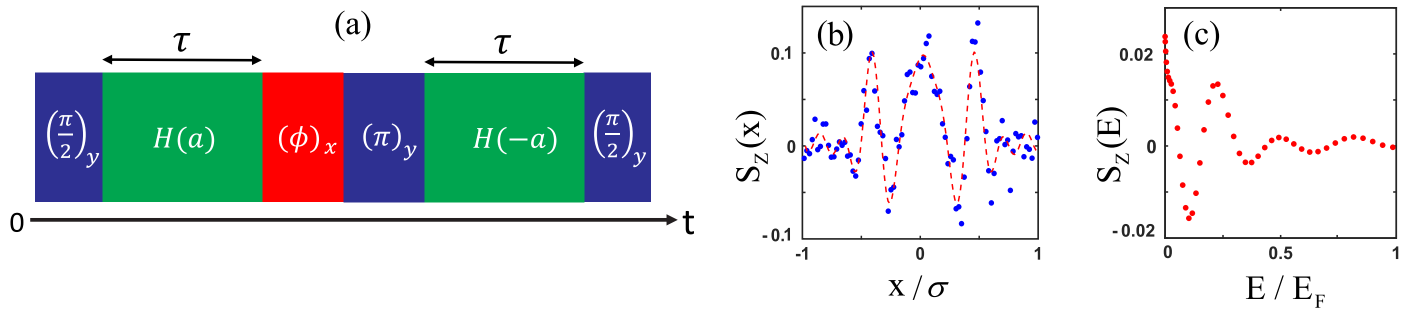
$$H' = \psi^\dagger \psi A_0(x,t), \qquad\qquad H' = C_0(x,t)(\psi^\dagger\psi)^2$$

We would like to realize non-trivial metric perturbations

$$H' = \frac{g_{xy}(x,t)}{m}\psi^\dagger \nabla_x \nabla_y \psi$$

We would also like to realize out-of-time-order correlators

$$C(t) = \langle [V(t), W(0)]^2 \rangle$$



Pegahan et al., PRL 2021.