Critical Behavior of the Bulk Viscosity in QCD

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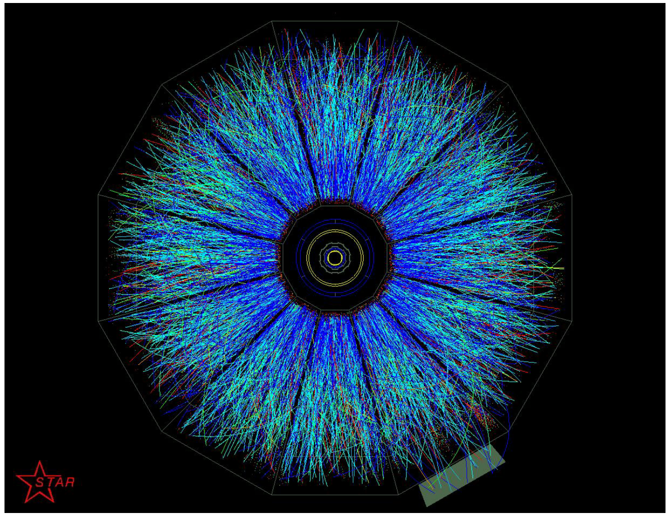
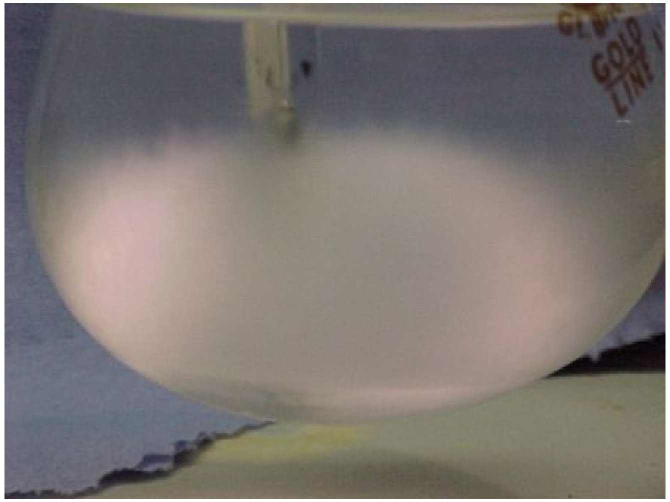
BEST COLLABORATION





with M. Martinez and V. Skokov, see [1906.11306]

Why consider critical bulk viscosity?

Model H predictions[1] (scaling with correlation length)

$$\eta \sim \xi^{0.05}\,, \qquad \kappa \sim \xi^{0.9}\,, \qquad \zeta \sim \xi^{2.8}\,.$$

Even modest values of ζ/s lead to large effects in RHICs

$$P = P_0 - \zeta(\nabla\cdot u) \simeq P_0 \left\{ 1 - \left(\frac{\zeta}{s}\right) \frac{4}{\tau_0 T} \left(\frac{\tau_0}{\tau}\right)^{2/3} \right\}$$

Backreaction: How do critical fluctuation contribute to equilibrium pressure? Replace $\zeta \sim \xi^{x_\zeta}$ estimates by $\zeta \leq f(\xi/\xi_0)$ constraints.

[1] Hohenberg, Halperin (1977); Kadanoff, Swift (1968); Onuki (1997)

Strategy

Consider an Ising-like system. Fluctuations governed by an entropy functional

$$Prob[\psi,\epsilon] \sim \exp(S[\psi,\epsilon]) \qquad\qquad S = \int d^3x\, s(\psi,\epsilon)$$

energy density ϵ, order parameter ψ

Conjugate variables

$$x^A = (\epsilon,\psi) \qquad\qquad X_A = -\frac{\partial s}{\partial x^A} = (r,h)$$

reduced temperature r, magnetic field h

QCD: Canonical pair

$$x^a = (e,n) \qquad\qquad X_a = (-\beta,\beta\mu)$$

energy density e, baryon density n
inverse temperature β, chemical potential μ

Strategy, continued

Fluctuations in the pressure

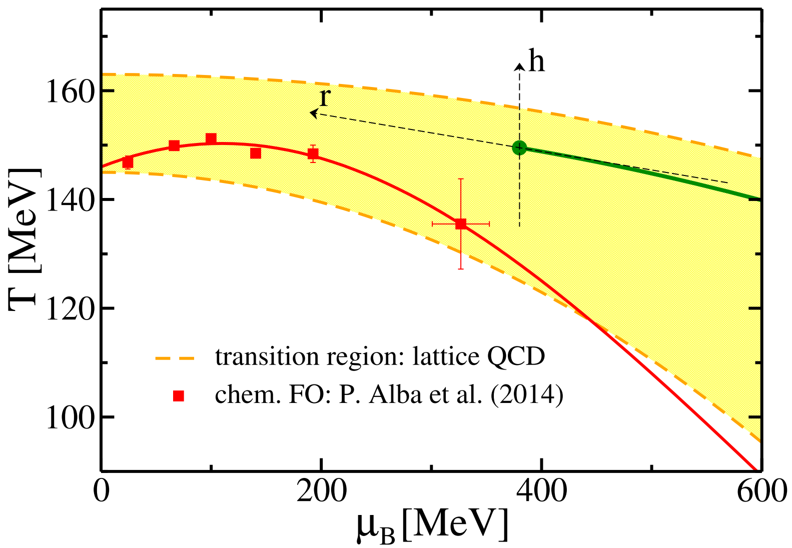
$$\delta P = \frac{e+P}{\beta}\frac{\partial s}{\partial(\delta e)} - \frac{n}{\beta}\frac{\partial s}{\partial(\delta n)}$$

Map QCD densities x^a onto Ising densities x^A

$$x^A = x^A(x^b) \qquad R_b^A = \frac{\partial x^A}{\partial x^b} \qquad \text{induces}: \;\; \bar{R}_A^b = \frac{\partial X_A}{\partial X_b} \quad R_b^A \bar{R}_C^b = \delta_C^A$$

Typical assumption:

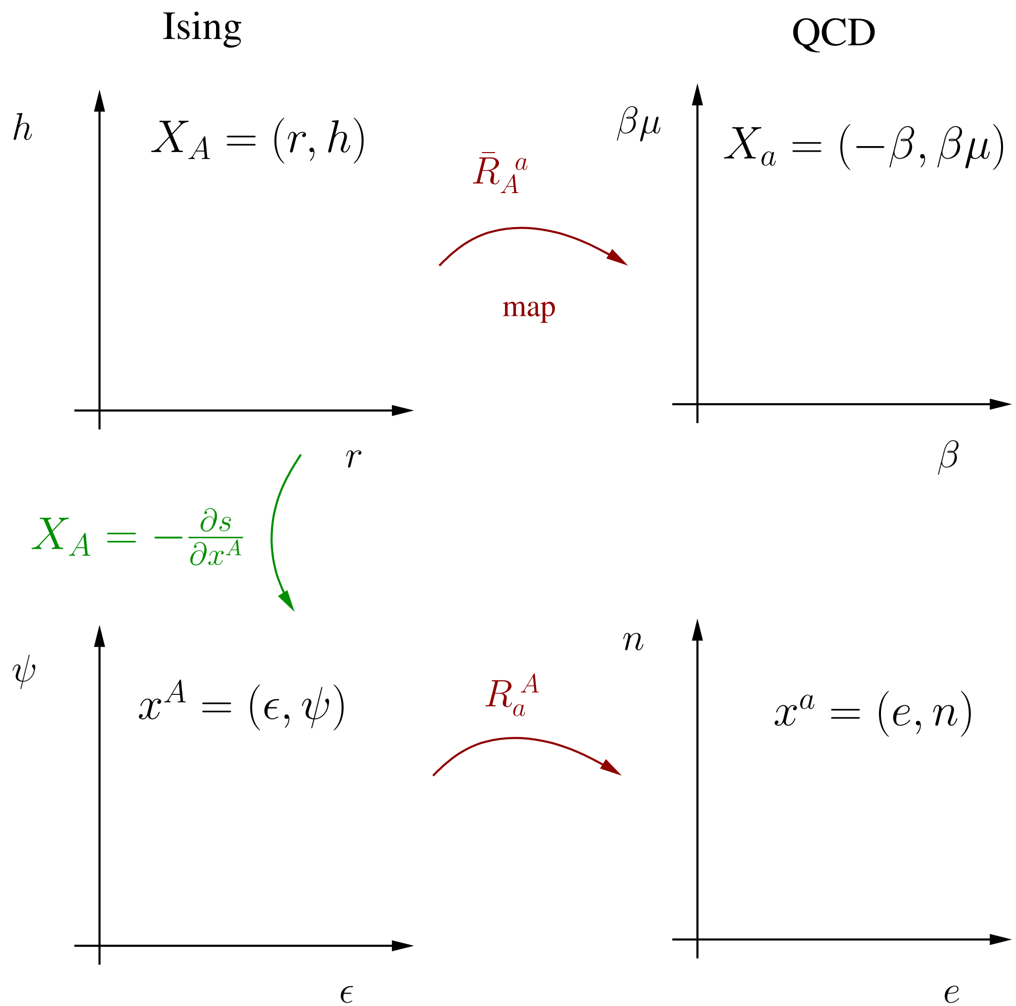
$$r \sim \mu, \quad h \sim T$$



Bluhm et al. (2017)

Mapping the Ising EOS to QCD

Map QCD variables on Ising equation of state $F(h,t)$



$$R_b^A \bar{R}_C^b = \delta_C^A$$

Strategy, continued

Express fluctuation in pressure in terms of Ising entropy

$$\delta P = \frac{e+P}{\beta}\, R_e^{\psi}\, \frac{\partial s^{Is}}{\partial \psi} - \frac{n}{\beta}\, R_n^{\epsilon}\, \frac{\partial s^{Is}}{\partial \epsilon}$$

$$\text{Main term}: \quad \frac{\partial s^{Is}}{\partial \epsilon} \sim \gamma \psi^2$$

This coupling generates (Kubo relation)

$$\zeta \sim \beta V \int dt\, \langle \delta P(0) \delta P(t) \rangle \sim \beta V (\gamma n T R_n^{\epsilon})^2 \int dt\, \langle \psi^2(0) \psi^2(t) \rangle$$

Slow order parameter relaxation $\rightarrow$ large bulk viscosity

Tri-linear coupling $s \sim \gamma\epsilon\psi^2$

Consider entropy functional

$$S[\psi, \epsilon] = -\int d^3x \left\{ \kappa(\nabla\psi)^2 + \frac{v}{2}\psi^2 + \frac{u}{4}\psi^4 + \gamma\epsilon\psi^2 + \frac{1}{2C_0}\epsilon^2 \right\} + S_0 + \frac{E}{T_0}$$

Legendre transform to obtain Gibbs free energy

$$\beta G[\psi, t] = \int d^3x \left\{ \kappa(\nabla\psi)^2 + \frac{\tilde{v}}{2}\psi^2 + \frac{\tilde{u}}{4}\psi^4 \right\}$$

$$\tilde{v} = v + 2\gamma C_0 \frac{T - T_0}{T_0^2}$$

Need $\gamma \neq 0$ to have a transition.

Order parameter relaxation rate

Consider diffusive relaxation (model B)

$$\frac{\partial\psi}{\partial t} = \lambda_0 \nabla^2 \frac{\delta G}{\delta\psi} + \Theta$$

$$\langle\Theta(x,t)\Theta(0,0)\rangle = -2\lambda_0 T \delta^3(x)\delta(t)$$

Retarded correlation function

$$\Delta_R(\omega,k) = \chi_k \frac{\Gamma_k}{-i\omega + \Gamma_k}$$

Relaxation rate and susceptibility

$$\Gamma_k = \lambda_0 \chi_k^{-1} k^2 \qquad\qquad \chi_k = \frac{\xi^2}{1+(k\xi)^2}$$

Bulk viscosity

$$\zeta \sim (\gamma n T R_n^\epsilon)^2 \int d^3k \; \frac{2T\chi_k^2}{-i\omega + 2\Gamma_k}\bigg|_{\omega\to 0} \sim (\gamma n T R_n^\epsilon)^2 \xi^5$$

Refinements: Equation of State

Parotto et al. write $S(e,n) = S_{reg}(e,n) + AS_{crit}(e,n)$. Taylor expand regular part (constrained by lattice), linear map to Ising

$$\begin{aligned}\frac{T-T_c}{T_c} &= \bar{w}\left(r\bar{\rho}\sin\alpha_1 + h\sin\alpha_2\right)\\ \frac{\mu-\mu_c}{T_c} &= \bar{w}\left(-r\bar{\rho}\cos\alpha_1 - h\cos\alpha_2\right)\end{aligned}$$

Critical Gibbs Free Energy (Zinn-Justin parameterization)

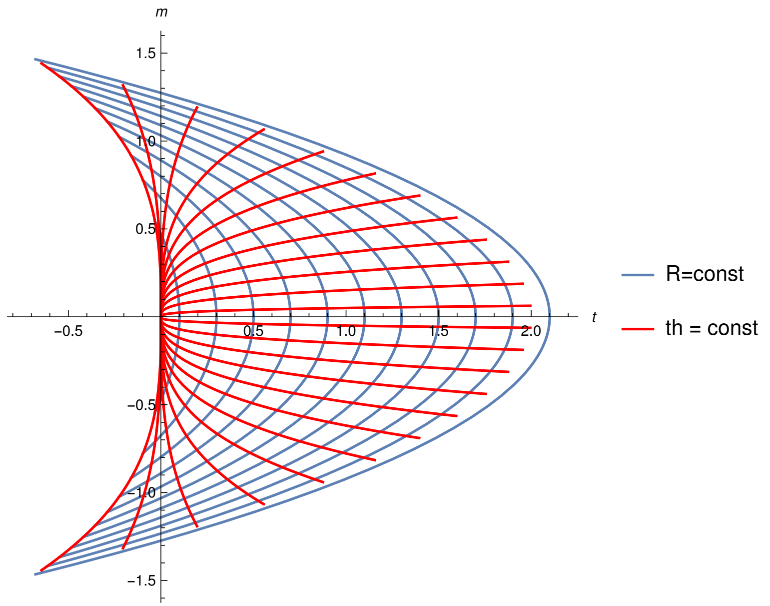
$$G[\psi, r] = h_0 M_0 R^{2-\alpha} g(\theta)$$

$$\psi = M_0 R^\beta \theta \quad r = R(1-\theta^2)$$

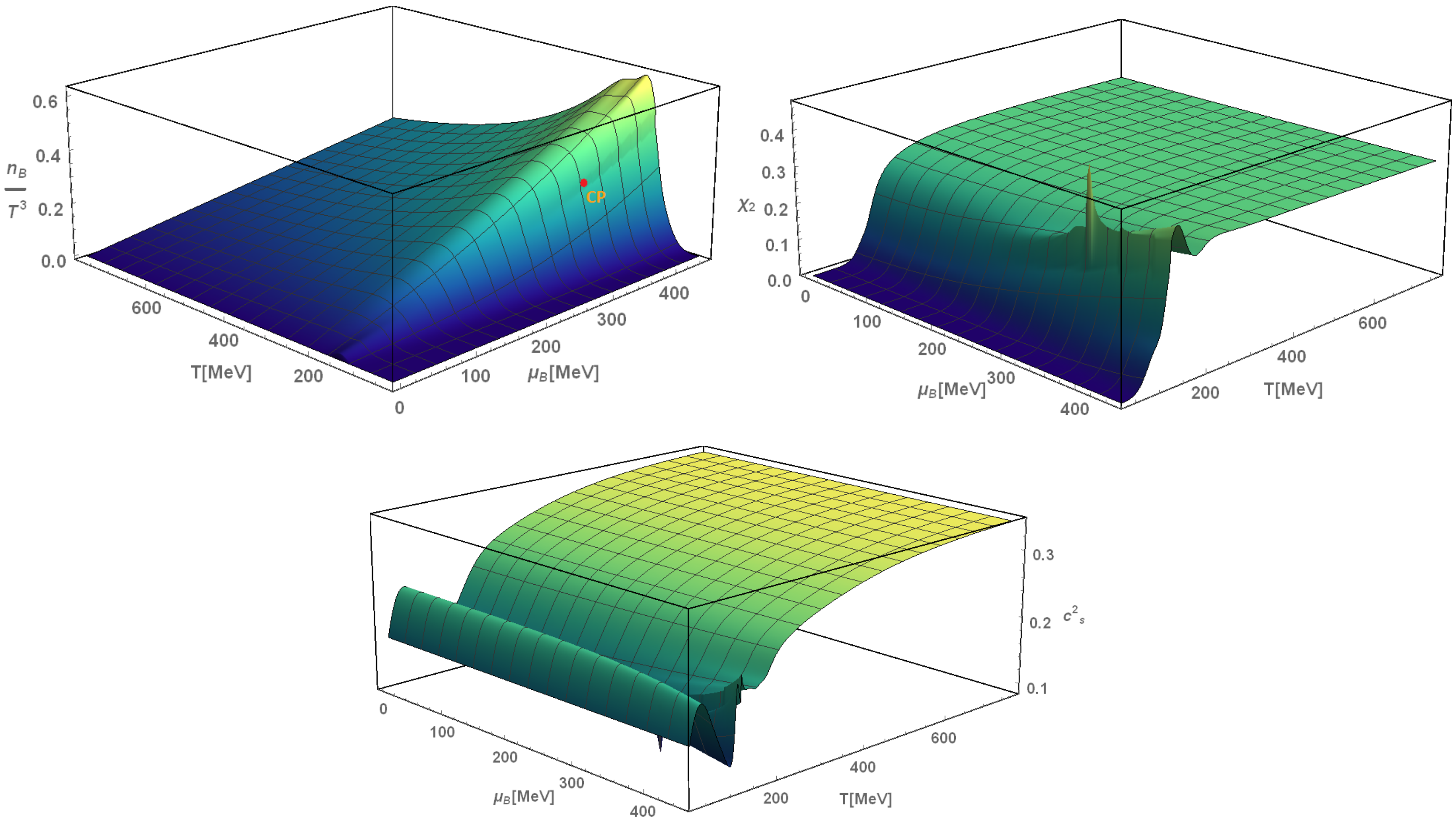
magnetic EOS

$$h = h_0 R^{\beta\delta} \tilde{h}(\theta)$$

$$\theta \in [-\theta_0, \theta_0] \quad \tilde{h}(\theta_0) = 0$$



Critical equation of state for QCD



Baryon density, compressibility, speed of sound.

Parotto et al. (2018)

Refinements, continued

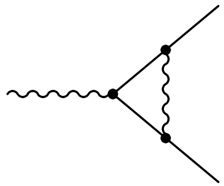
Tri-linear coupling in critical Ising equation of state

$$\gamma_\pm = \begin{cases} 0.43\, r^{1-2\beta} & r > 0 \\ 1.10\, |r|^{1-2\beta} & r < 0 \end{cases}$$

Coupling stronger in first order regime; critical exponent $1 - 2\beta \simeq 0.54$.

Result consistent with diagrammatic analysis.

$$\frac{d\gamma}{d\log\xi} \sim$$



Halperin, Hohenberg, Ma (1974)

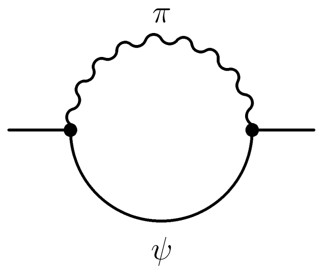
Parotto et al. map corresponds to

$$R_n^\epsilon = \frac{\bar{\rho}\bar{w}}{A}\cos\alpha_1 \simeq \frac{\bar{\rho}\bar{w}}{A}$$

Refinements: Model H

Order parameter $\psi \sim s/n$ coupled to momentum density $\vec{\pi}$

$$\frac{\partial \psi}{\partial t} = \vec{\pi} \cdot \vec{\nabla} \psi + \ldots$$



Order parameter relaxation rate governed by (non-critical) shear viscosity ("Kawasaki approximation").

$$\Gamma_k = \frac{T\xi^{-3}}{6\pi\eta_0} K(k\xi) \qquad K(x) = \frac{3}{4}\left[1 + x^2 + \left(x^3 + x^{-1}\right)\arctan(x)\right] .$$

Order parameter susceptibility

$$\chi_k = \frac{\chi_0}{1 + (k\xi)^{2-\eta}} \qquad \chi_0 = \chi_0^2 (\chi/\chi_0)^{2-\eta}$$

Define bare correlation length by $s\xi_0^3 \equiv 1$

Critical contribution to bulk viscosity

$$\frac{\zeta}{s} = \left(\frac{n}{s}\right)^2 (\gamma_\pm R_n^\epsilon)^2 (T t_0) \frac{1}{2\pi^2} \left(\frac{4\pi}{s/\eta}\right) \left(\frac{\xi}{\xi_0}\right)^{z-\alpha/\nu}$$

$z \simeq 3$ dynamical critical exponent, α/ν small.

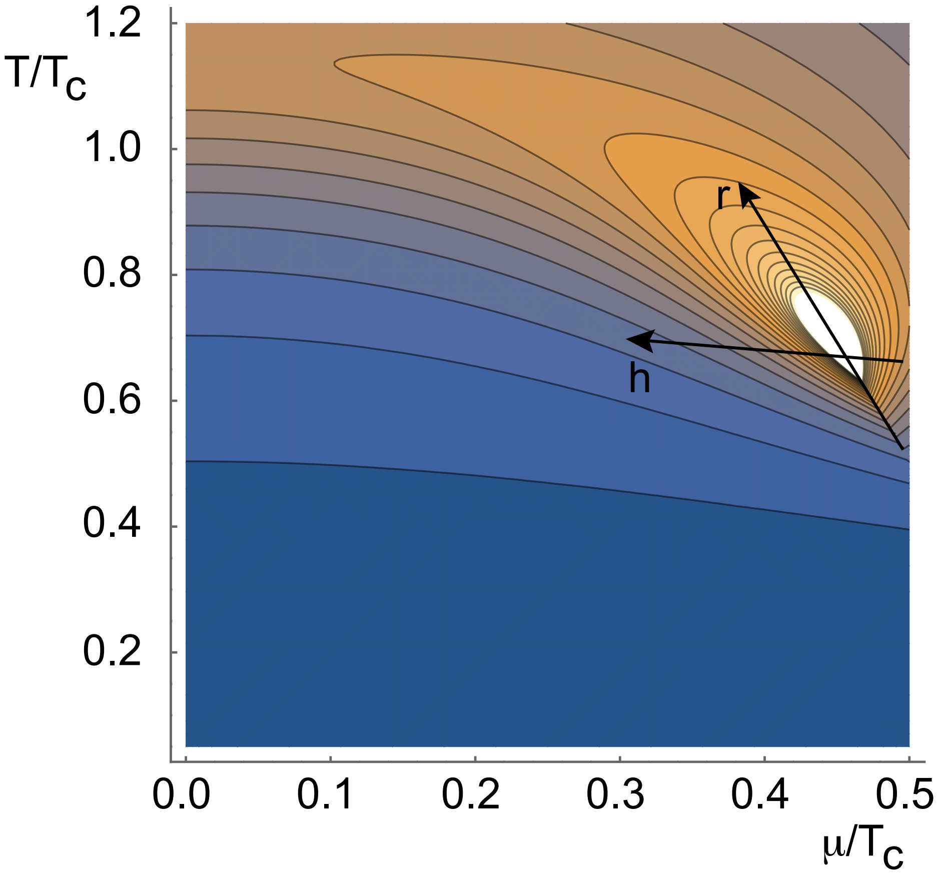
$(n/s)^2 \ll 1$ related to orientation of Ising axes.

Amplitude ratio $(\gamma_-/\gamma_+)^2 \simeq 6$.

First order regime: $\zeta \simeq 5 \cdot 10^{-4} (\xi/\xi_0)^3$

See also Stephanov & Yin 1712.10305, An et al. 1912.13456

The role of the Ising Map



Phase diagram in random matrix model, tuned to reproduce T_χ/T_{pc}

See also Pradeep & Stephanov 1905.13247

$$\frac{\zeta}{s} = \sin^2(\alpha_1)\left(\frac{4\pi}{s/\eta}\right)\left(\frac{\xi}{\xi_0}\right)^3 \begin{cases} 3.4\cdot 10^{-2} & r>0 \\ 2.2\cdot 10^{-1} & r<0 \end{cases}$$

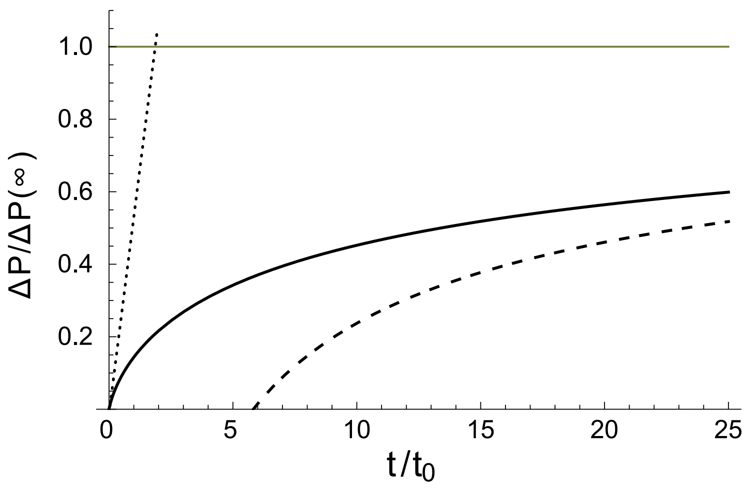
$$\sin^2(\alpha_1) \simeq 1/4$$

Bulk pressure response function

Consider response function

$$\Delta P(t) = \int^{t} dt'\, G(t-t')(\vec{\nabla}\cdot\vec{u})(t')\,,$$

$$G(t) = c\int \frac{d^3k}{(2\pi)^3}\, 2T\chi_k^2 \exp\left(-2\Gamma_k t\right)$$



Long time tail

$$\Delta P(t) \simeq \Delta P(\infty)\left\{1-\frac{4}{3}\sqrt{\frac{2}{\pi}}\left(\frac{\xi}{\xi_0}\right)^2\left(\frac{t_0}{t}\right)^{1/2}\right\}$$

Initial rise in pressure

$$\Delta P(t) = \frac{8}{3}\left(\frac{\xi_0}{\xi}\right)^4\left(\frac{t}{t_0}\right)\Delta P(\infty)$$

Summary

Critical bulk viscosity suppressed by $(n/s)^2$ and amplitude ratio.

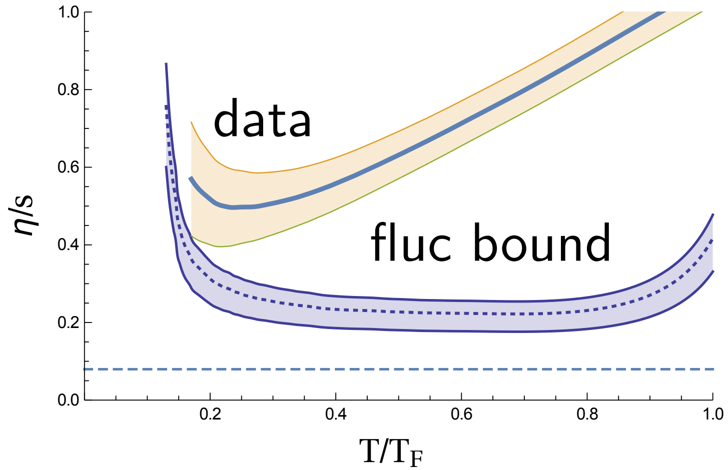
Result sensitive to orientation of Ising axes. "Non-standard" orientation leads to $\zeta_{crit} \sim \eta_0$ for $\xi \gtrsim 2\xi_0$.

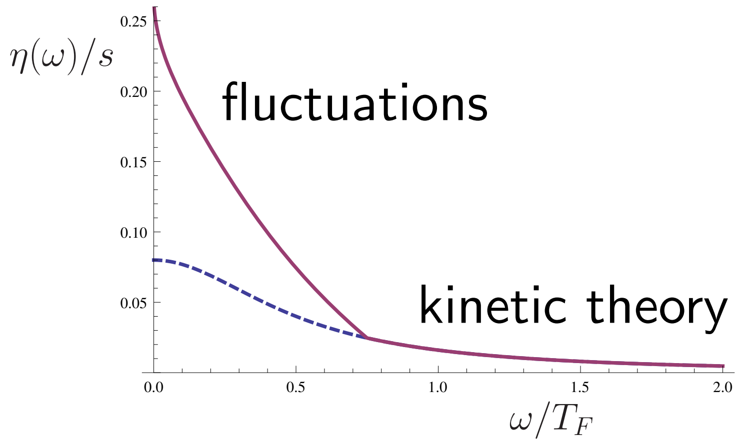
Very slow relaxation of bulk pressure near T_c.

Outlook:

Observing fluctuation effects in transport properties

Fluctuation induced bound on η/s





$$(\eta/s)_{min} \simeq 0.2$$

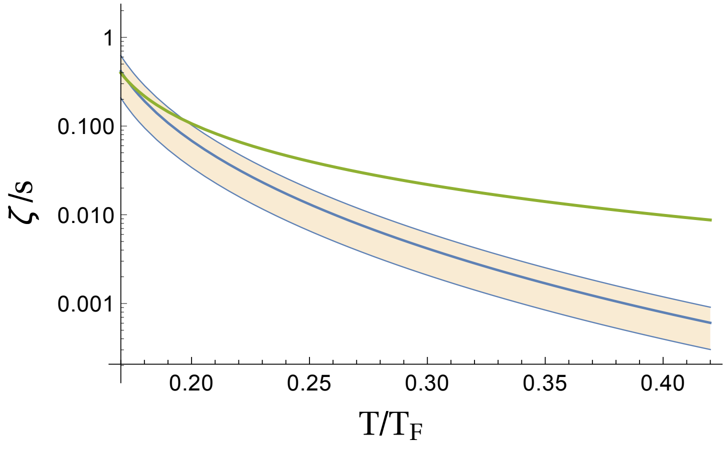
spectral function

non-analytic $\sqrt{\omega}$ term

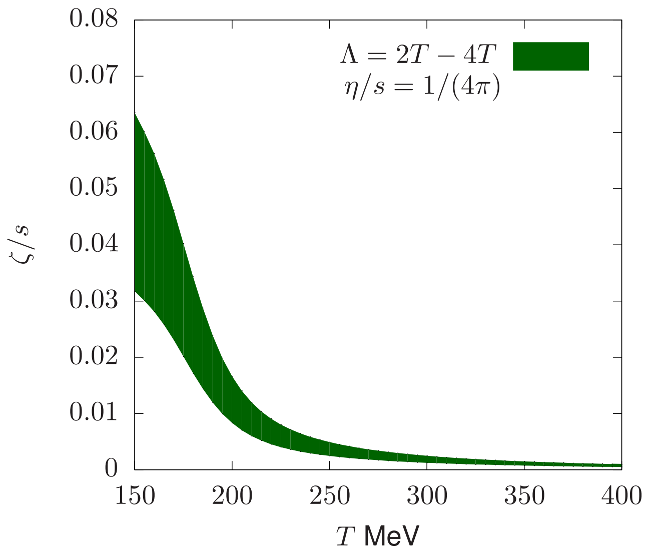
Schaefer, Chafin (2012), see also Kovtun, Moore, Romatschke (2011)

Fluctuation induced bound on ζ/s

(Detuned) Unitary Fermi Gas



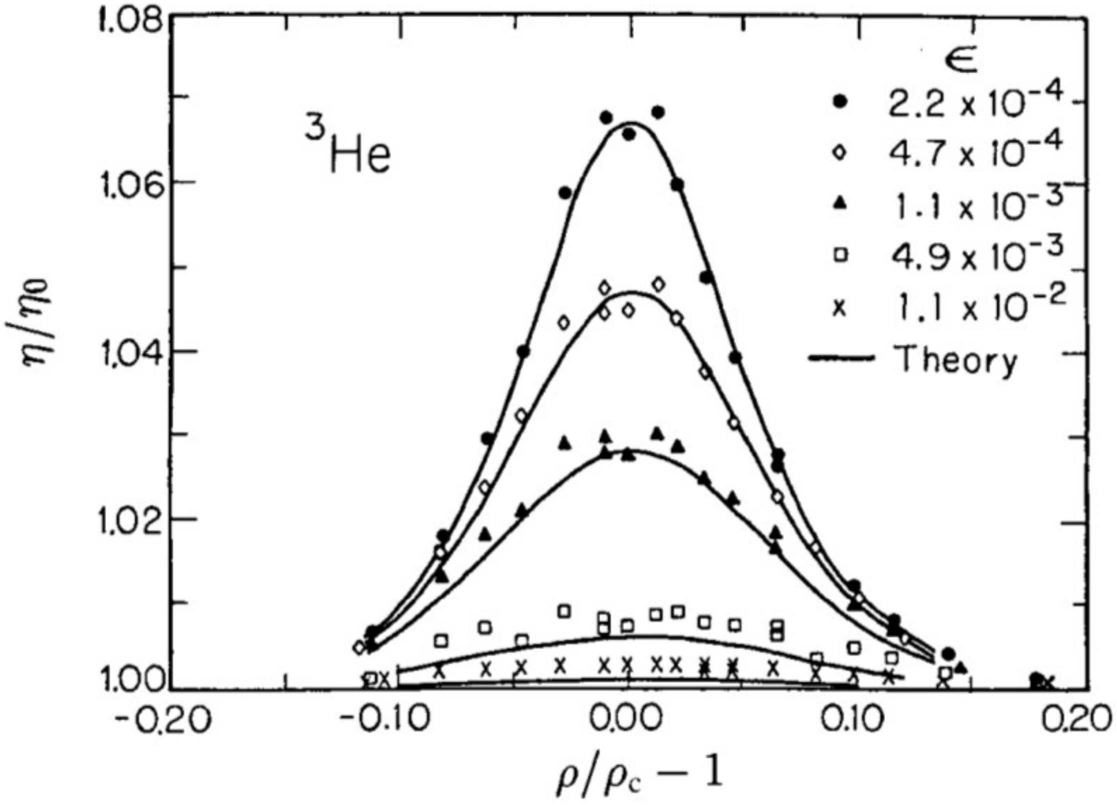
Quark Gluon Plasma



Non-relativistic fluid, M. Martinez, T. S. (2017); relativistic fluid, Akamatsu et al. (2018)

See also Kovtun, Yaffe (2003)

Critical transport (liquid-gas endpoint in helium)



Agosta, Wang, Meyer (1987)

More dramatic enhancement in thermal conductivity and bulk viscosity (sound attenuation)

Critical sound attenuation (liquid-gas endpoint in helium)

Sound Propagation in ^{3}He and ^{4}He Above the Liquid-Vapor Critical Point **905**

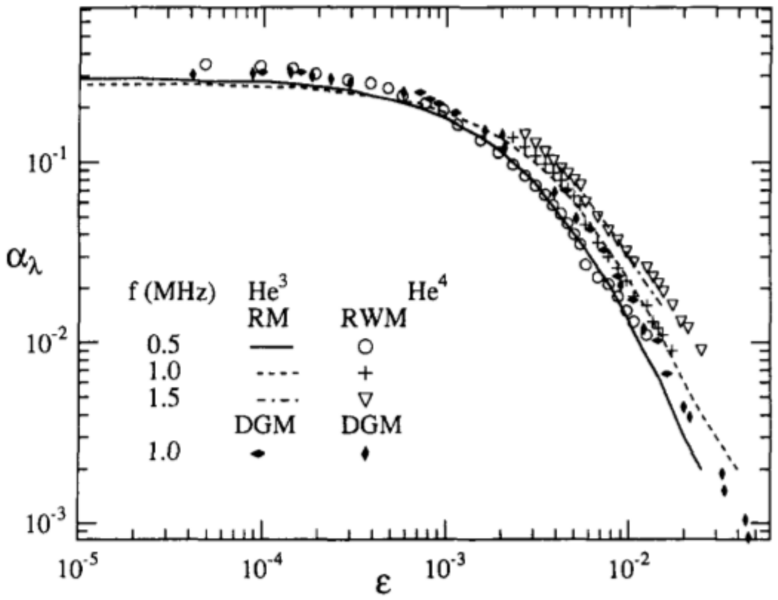
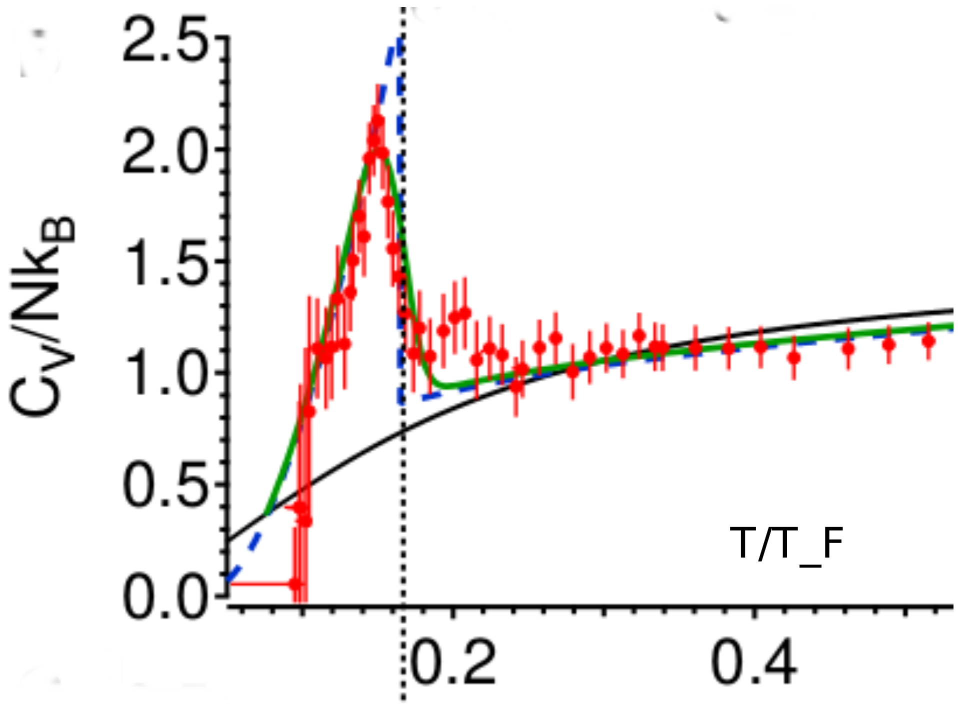


Fig. 2. Comparison of the attenuation data of He^3 and He^4 from different experiments versus ε at frequencies 0.5, 1 and 1.5 MHz (Roe *et al.* (RWM), (RM) and Doiron *et al.* (DGM)[9–11]). The lines represent the ^{3}He data of RM, while the other data are marked by symbols.

Kogan, Meyer (1998)

Sound attenuation dominated by bulk viscosity

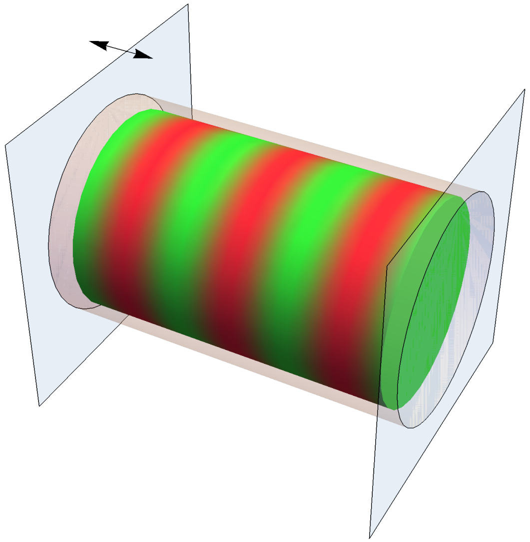
Lambda transition in ultracold Fermi gas

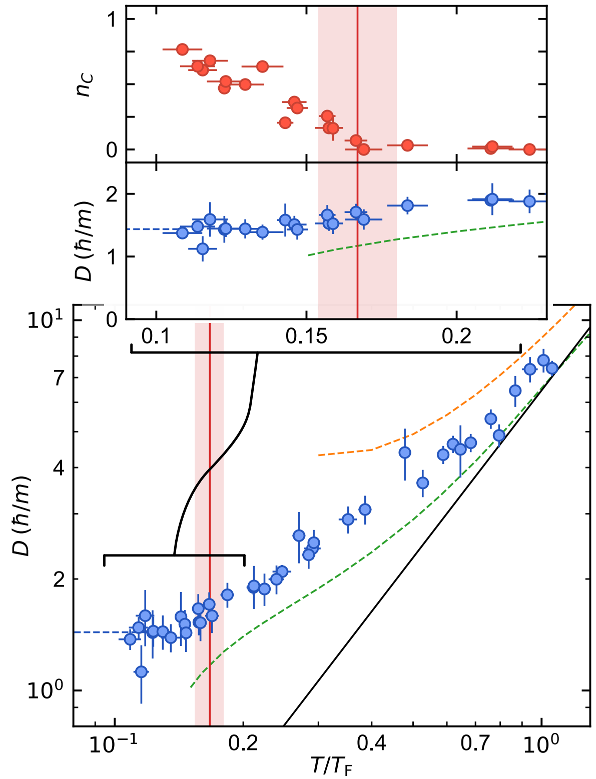


Ku et al. (2011). Critical behavior also seen in compressibility.

Dashed black line: T_c; black line: background fit; dashed blue: critical fit; green line: finite resolution effects.

Sound attenuation across lambda transition





Patel et al. (2019). No critical effects seen in sound attenuation.

Dashed black line: T_c; black line: background fit; dashed blue: critical fit; green line: finite resolution effects.