Effective Field Theory and the Nuclear Many-Body Problem

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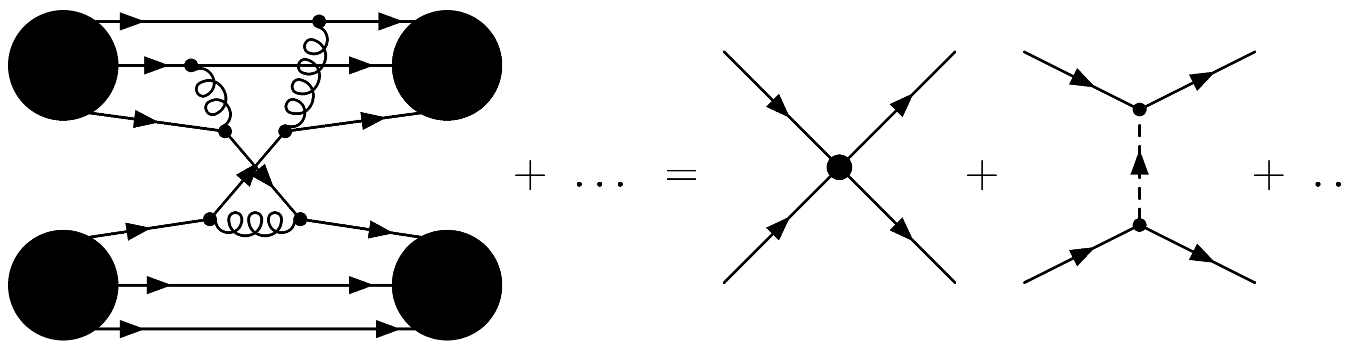
Nuclear Effective Field Theory

Low Energy Nucleons:
- Nucleons are point particles
- Interactions are local
- Long range part: pions



Advantages:
- Systematically improvable
- Symmetries manifest (Chiral, gauge, . . .)
- Connection to lattice QCD

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \ldots$$

Simplifications: neutrons only, no pions (very low energy)

Match to effective range expansion

$$p \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} \Lambda^2 \sum_n r_n \left(\frac{p^2}{\Lambda^2} \right)^{n+1}$$

Coupling constants

$$C_0 = \frac{4\pi a}{M} \qquad C_2 = \frac{4\pi a^2}{M} \frac{r}{2}$$

Few Body Physics: Successes

NN scattering: N^{3}LO potentials

External currents: $np \to d\gamma$ etc.

Three body systems: Efimov effect, Phillips line

Four body physics, . . .

The Nuclear Matter Problem is Hard: Traditional View

NN Potential has a very strong hard core

3-body forces, isobars, relativity, . . . important

Saturation density too small

The Nuclear Matter Problem is Hard: EFT View

NN Potential has a very strong hard core

Short distance behavior not relevant

3-body forces, isobars, relativity, . . . important

3-body: Yes; Isobars, relativity: Absorbed in counterterms

Saturation density too small

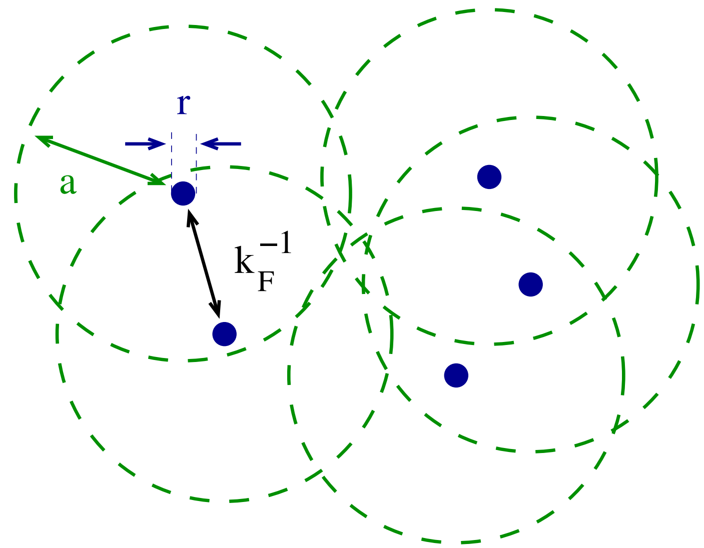
Yes: NN system and nuclear matter (?) are fine tuned

Toy Problem (Neutron Matter)

Limiting case ("Bertsch" problem)

$$(k_F a) \to \infty$$

$$(k_F r) \to 0$$



No Expansion Parameters!

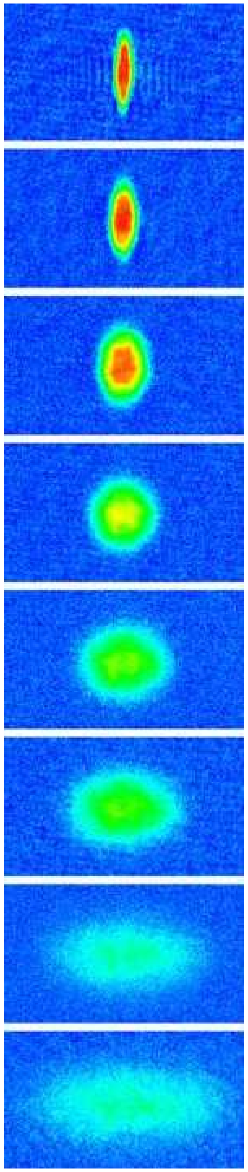
Universal properties $[E_F = k_F^2/(2m)$, $n_f = (2m\mu)^{3/2}/(3\pi^2)]$

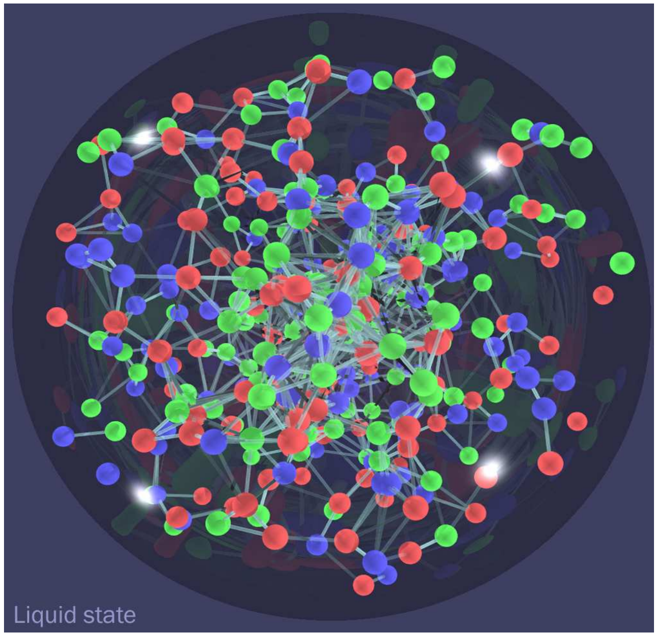
$$(E/A)|_{T=0} = \xi(E^{(0)}/A) = \xi\frac{3}{5}E_F$$

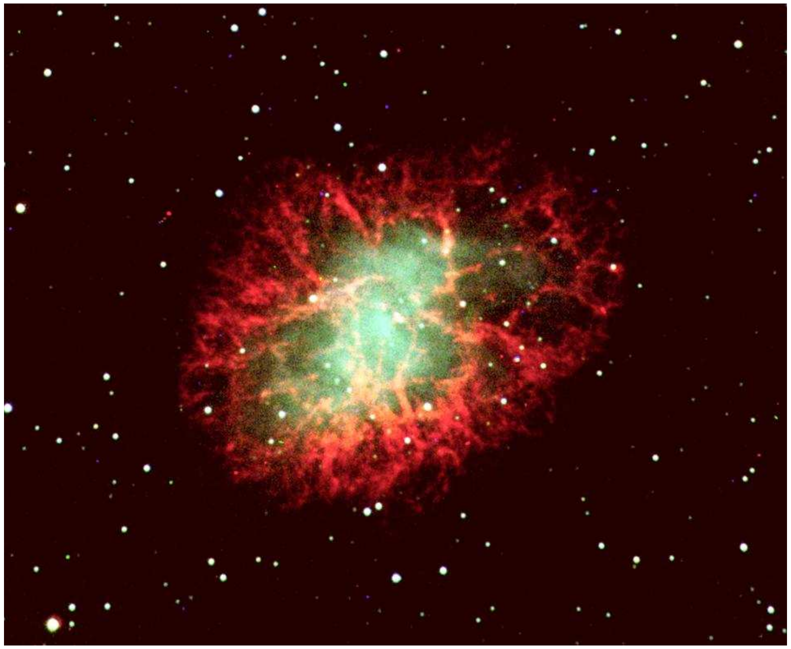
$$\Delta|_{T=0} = \zeta E_F \qquad [T_c = \zeta' T_F]$$

$$P(T,\mu) = \frac{2}{5}\mu n_f(\mu) f(T/T_F)$$

Perfect Liquids







Neutron Matter (T=1 MeV)

sQGP (T=180 MeV)

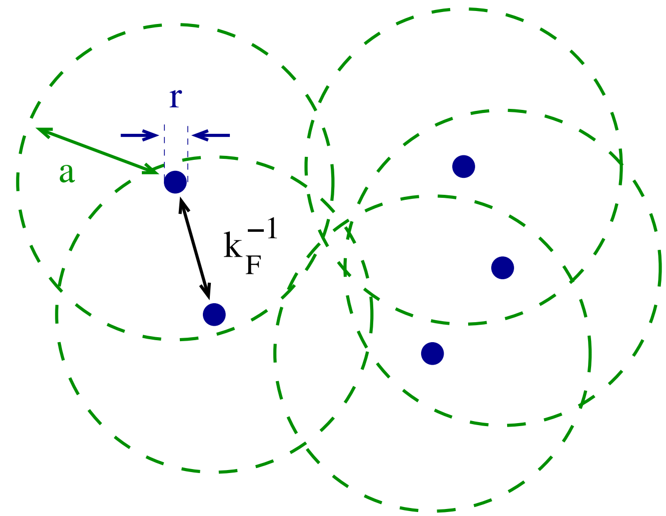
Trapped Atoms (T=1 neV)

Universality

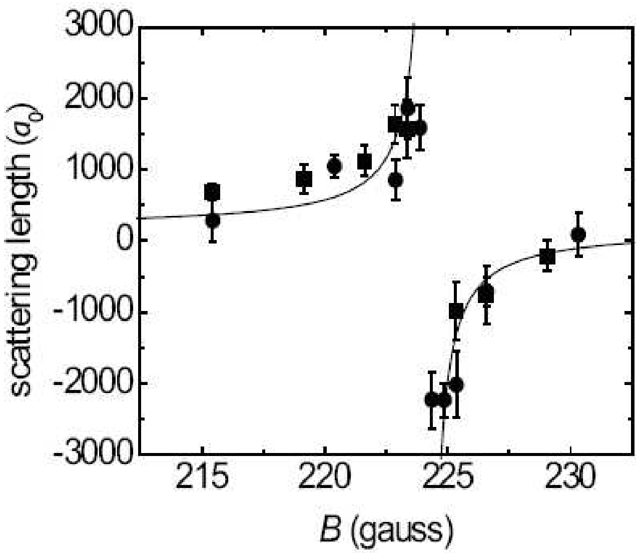
What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

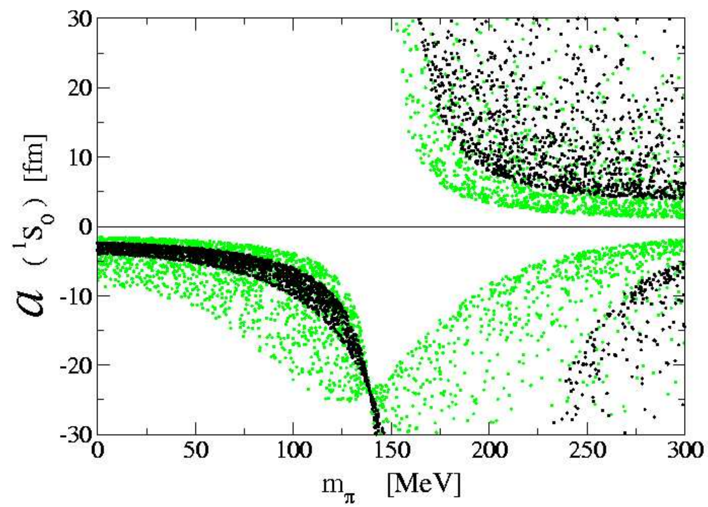
strongly correlated: $a\rho^{1/3} \gg 1$



Feshbach Resonance in ^{6}Li



Neutron Matter

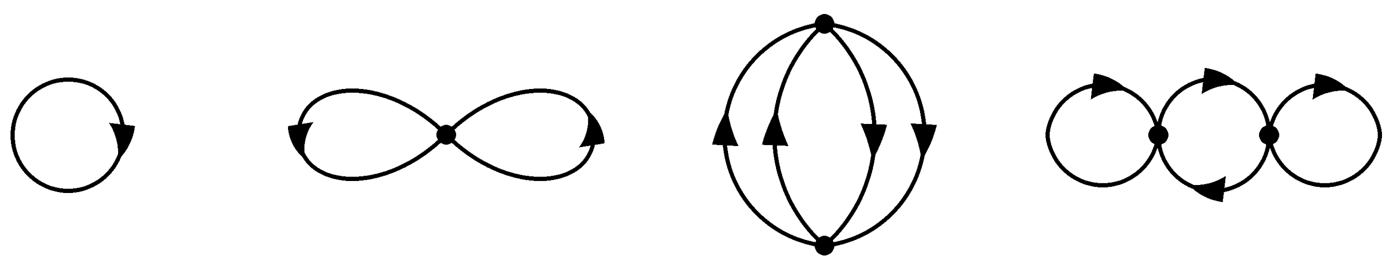


Warmup: Low Density Expansion

Finite density: $\mathcal{L} \to \mathcal{L} - \mu\psi^\dagger\psi \Rightarrow$ Modified propagator

$$G_0(k)_{\alpha\beta} = \delta_{\alpha\beta}\left(\frac{\theta(k-k_F)}{k_0 - k^2/2M + i\epsilon} + \frac{\theta(k_F - k)}{k_0 - k^2/2M - i\epsilon}\right)$$

Perturbative expansion

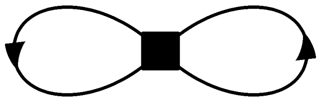


$\epsilon_F\rho$ $\qquad$ $\epsilon_F\rho\,(k_F a)$ $\qquad$ $\epsilon_F\rho\,(k_F a)^2$

$$\frac{E}{A} = \frac{k_F^2}{2M}\left[\frac{3}{5} + \left(\frac{2}{3\pi}(k_F a) + \frac{4}{35\pi^2}(11 - 2\log(2))(k_F a)^2\right) + \ldots\right]$$

Low Density Expansion: Higher orders

Effective range corrections

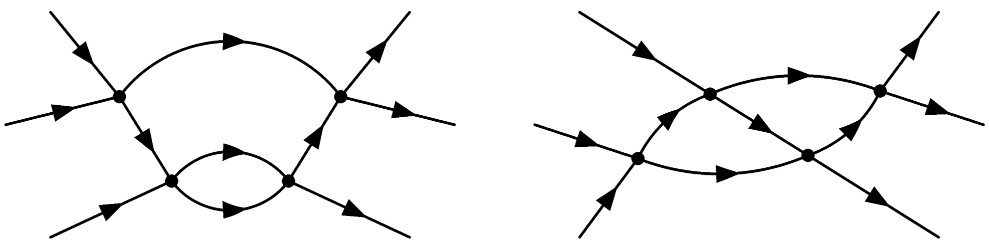


$$\frac{E}{A} = \frac{k_F^2}{2M}\frac{1}{10\pi}(k_F a)^2(k_F r)$$

Logarithmic terms

$$\frac{E}{A} = \frac{k_F^2}{2M}(g-1)(g-2)\frac{16}{27\pi^3}(4\pi - 3\sqrt{3})(k_F a)^4 \log(k_F a)$$

related to log divergence in $3 \to 3$ scattering amplitude



local counterterm $D(\psi^\dagger\psi)^3$ exists if $g \geq 3$

Nonperturbative Methods

Lattice Field Theory (D. Lee's talk)

Other numerical methods: GFMC, VMC, ...

Expansion in number of species (large N)

Expansion in dimensionality (large d, $\epsilon = 4 - d$)

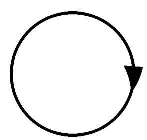
Large N approximation(s)

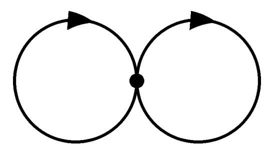
Large N gives mean field dynamics. What mean field?

Determined by symmetries of the interaction

SU(2N) symmetric interaction

$$\mathcal{L} = C_0 (\psi_f^\dagger \psi_f)^2$$



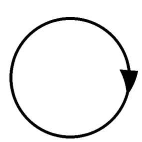


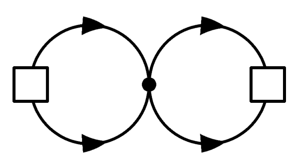
$$\rho = \frac{1}{N} \langle \psi^\dagger \psi \rangle$$

N $\quad$ $N(C_0 N)$

Sp(2N) symmetric interaction

$$\mathcal{L} = C_0 \left| \psi_f \mathcal{J}^{fg} \psi_g \right|^2$$



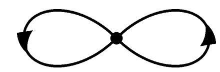


$$\Phi = \frac{1}{N} \langle \psi_f \mathcal{J}^{fg} \psi_g \rangle$$

N $\quad$ $N(C_0 N)$

Large N approximations

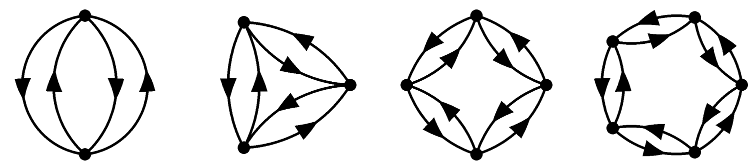
SU(2N): Hartree + ring diagrams ($x = Nk_F a/\pi$)



$N(C_0 N)$

$$\frac{E}{A} = \frac{k_F^2}{2M} \times \left[\left(\frac{3}{5} + \frac{2x}{3} \right) \right.$$

$$\left. + \frac{1}{N} R(x) + \ldots \right] \quad (\to \infty)$$



Furnstahl & Hammer (2002)

$(C_0 N)^k$

Sp(2N): BCS + fluctuations

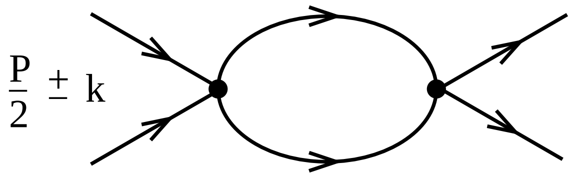
$$\frac{\Omega}{N} = -\int \frac{d^3p}{(2\pi)^3} \left\{ \sqrt{\epsilon_p^2 + \Phi^2} - \epsilon_p - \frac{m\Phi^2}{p^2} \right\} + O(1/N)$$

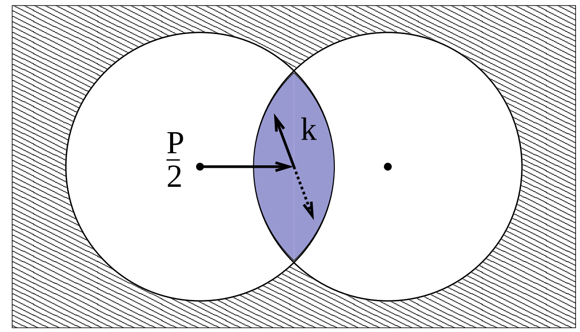
$$\xi = 0.591 - 0.312/N + \ldots = 0.279 \qquad (N = 1)$$

Sachdev (2006)

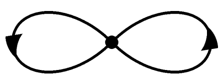
Large d Limit

In medium scattering strongly restricted by phase space

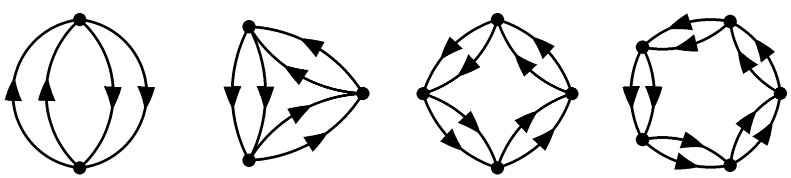




Find limit in which ladders are leading order



$(C_0/d) \cdot 1/d$



$(C_0/d)^k \cdot 1/d$

$$\lambda \equiv \left[\frac{\Omega_d C_0 k_F^{d-2} M}{d(2\pi)^d} \right]$$

$$\lambda = const \quad (d \to \infty)$$

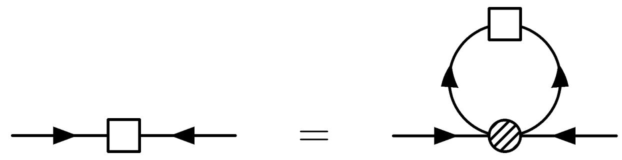
$$\xi = \frac{1}{2} + O(1/d)$$

Steele (1999), Schaefer et al (2003)

Pairing in the Large d Limit

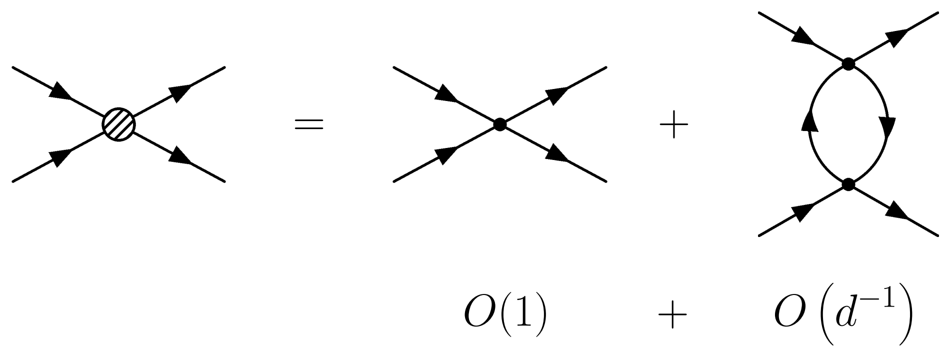
BCS gap equation

$$\Delta = \frac{|C_0|}{2} \int \frac{d^d p}{(2\pi)^d} \frac{\Delta}{\sqrt{\epsilon_p^2 + \Delta^2}}$$



Solution

$$\Delta = \frac{2e^{-\gamma} E_F}{d} \exp\left(-\frac{1}{d\lambda} \right)$$



$$\Delta = 0.375 E_F$$

Pairing energy (subleading in 1/d)

$$\frac{E}{A} = -\frac{d}{4} E_F \left(\frac{\Delta}{E_F} \right)^2 \sim \frac{1}{d}.$$

Upper and lower critical dimension

Zero energy bound state for arbitrary d

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \qquad (r > r_0)$$

d=2: Arbitrarily weak attractive potential has a bound state

$$\xi(d{=}2) = 1$$

d=4: Bound state wave function $\psi \sim 1/r^{d-2}$. Pairs do not overlap

$$\xi(d{=}4) = 0$$

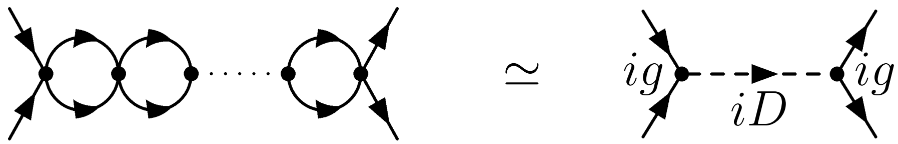
Conclude $\xi(d{=}3) \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Nussinov & Nussinov (2004)

Epsilon Expansion

EFT version: Compute scattering amplitude ($d = 4 - \epsilon$)



$$T = \frac{1}{\Gamma\left(1 - \frac{d}{2}\right)} \left(\frac{m}{4\pi}\right)^{-d/2} \left(-p_0 + \frac{\epsilon_p}{2}\right)^{1-d/2} \simeq \frac{8\pi^2\epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

$$g^2 \equiv \frac{8\pi^2\epsilon}{m^2} \qquad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

Weakly interacting bosons and fermions

Epsilon Expansion

Effective lagrangian for atoms $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$ and dimers ϕ

$$\mathcal{L} = \Psi^\dagger \left(i\partial_0 + \frac{\sigma_3 \nabla^2}{2m} \right) \Psi + \mu \Psi^\dagger \sigma_3 \Psi + \Psi^\dagger \sigma_+ \Psi \phi + h.c.$$

Perturbative expansion: $\phi = \phi_0 + g\varphi$. Free part

$$\mathcal{L}_0 = \Psi^\dagger \left[i\partial_0 + \delta\mu + \sigma_3 \frac{\vec{\nabla}^2}{2m} + \phi_0(\sigma_+ + \sigma_-) \right] \Psi + \varphi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{4m} \right) \varphi \,.$$

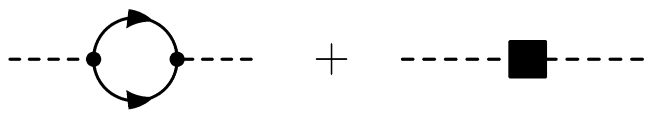
Interacting part $(g^2, \mu = O(\epsilon))$

$$\mathcal{L}_I = g\left(\Psi^\dagger \sigma_+ \Psi \varphi + h.c \right) + \mu \Psi^\dagger \sigma_3 \Psi - \varphi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{4m} \right) \varphi \,.$$

Nishida & Son (2006)

Epsilon Expansion

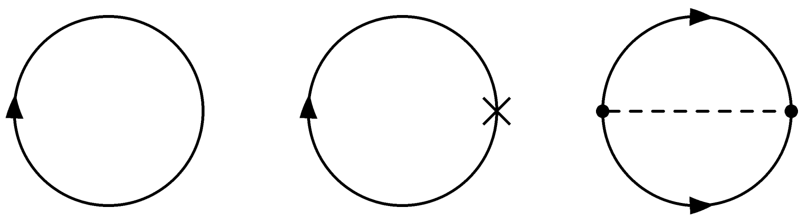
Consistency conditions



$= O(\epsilon)$

Also: tadpoles cancel

Effective potential



$O(1)$ $O(1)$ $O(\epsilon)$

$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2}\ln\epsilon - 0.0246\,\epsilon^{5/2} + \ldots$$

$$\xi(\epsilon=1) = 0.475$$

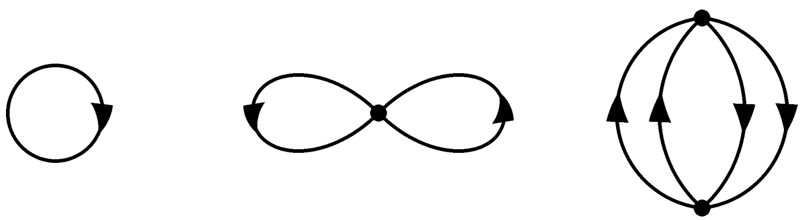
Problem: Higher order corrections large (~ 100 %)!

Near two dimensions

Scattering amplitude near d=2 ($\bar{\epsilon} = d - 2$)

$$\mathcal{A}(p_0, p) = i\frac{2\pi}{m}\,\bar{\epsilon} + O(\bar{\epsilon}^2) \qquad\qquad g^2 = \frac{2\pi\bar{\epsilon}}{m}$$

Effective potential (similar to $(k_F a)$ expansion)



$O(1)$ $\qquad O(\bar{\epsilon})$ $\qquad O(\bar{\epsilon}^2)$

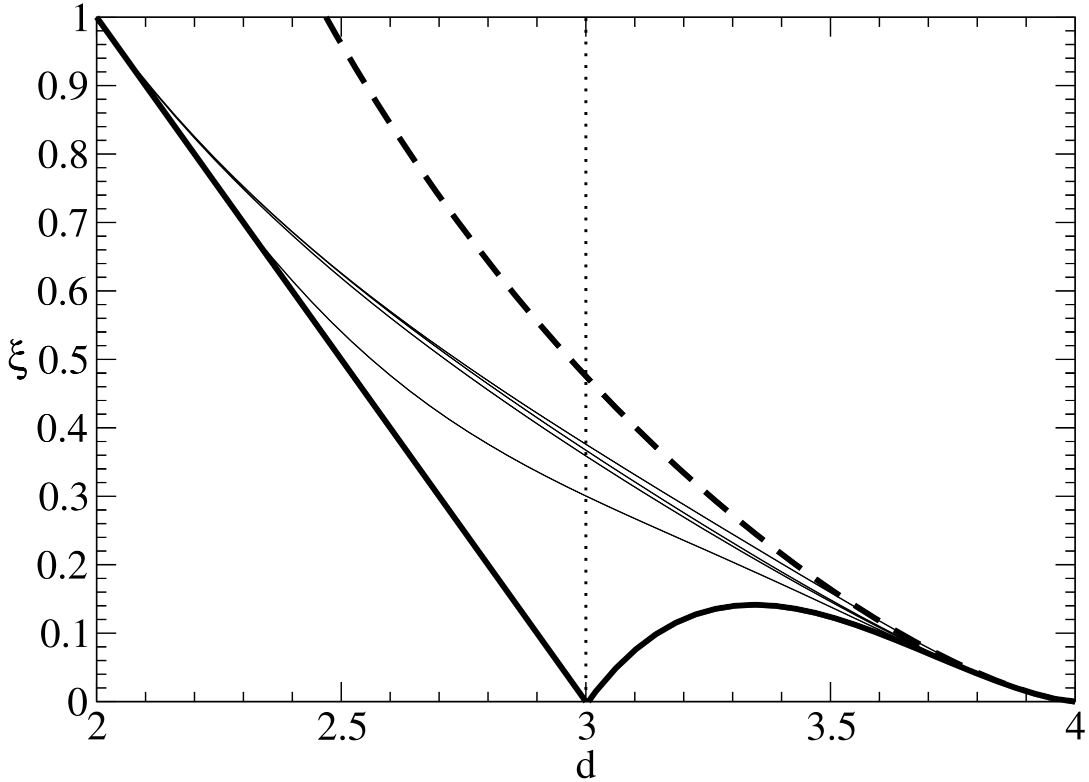
$$\xi = 1 - \bar{\epsilon} + O(\bar{\epsilon}^2)$$

$$= 0 \quad (\bar{\epsilon} = 1)$$

Superfluid gap (BCS + screening correction)

$$\Delta = \frac{2\mu}{e}\exp\left(-\frac{1}{\bar{\epsilon}}\right)$$

Combine expansions near d=2 and d=4



Conclude $\xi = (0.3 - 0.4)$

Arnold et al. (2006)

other app: Rupak (2006), Kryjevski,Rupak,S (2006)

Summary

Several systemtic approaches available

None of them is perfect, emphasize different aspects

Can be combined in interesting ways

Real nuclear matter

More perturbative. Problem becomes easier?

$1/a$, range corrections have been studied

Explicit pions, three body clusters, . . .